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Subject :- Advance Fluid Meachanics

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Deptt :- BE (civil)

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Q No 1

(a) **Drag:**

A body which is wholly immersed in a homogenous fluid may be subjected to two kind of forces arising from relative motion b/w body and fluid these forces are termed as drag and Lift. if the forces Parallel to the motion then it is turned as drag force.

⇒ There are two Components.

① **Pressure Drag (FP):**

It is equal to integrated of Components in direction of motion of all pressure forces extended on Surface of body.

$$FP = CP \int \frac{V^2}{2} A$$

where CP depends on shape.

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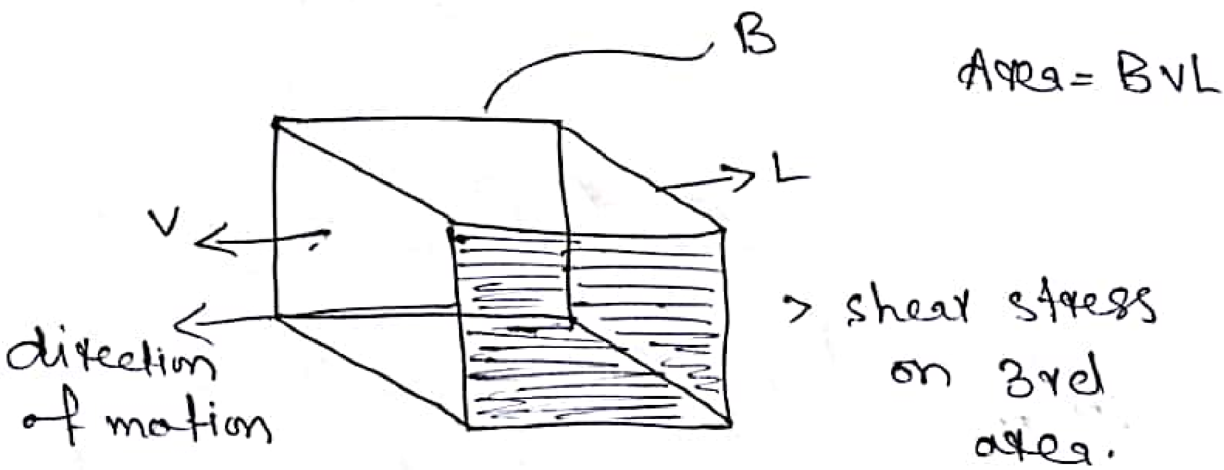
(2)

(2) Friction Drag (F_f)

it is equal to integration of components of shear stress along surface of body in direction of motion.

$$F_f = \underbrace{C_f \int \frac{v^2}{2}}_{\text{shear stress}} BL$$

Fig:



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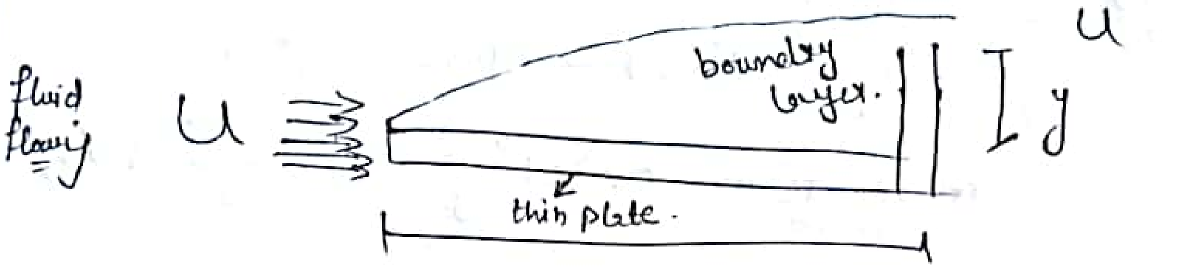
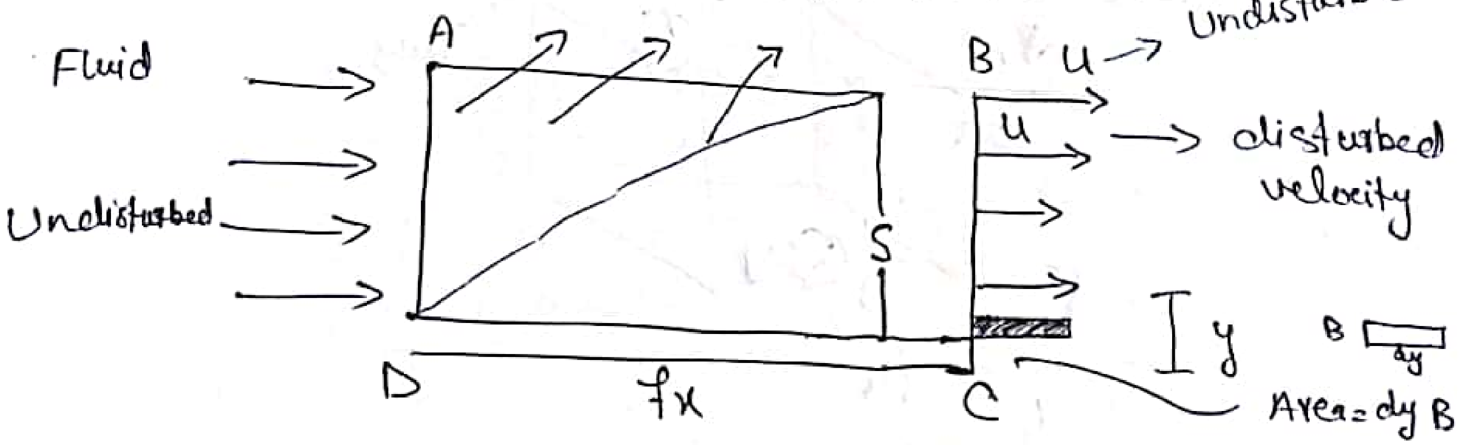
⇒ Friction Drag of Boundary Layer



⇒ Fig shows growth of boundary layer along one site of smooth plate inside the fluid.

⇒ Now consider a control volume.



> where δ is thickness of boundary layer and U is Undisturbed velocity

Thus

$-F_x$ = drag = (rate in momentum in x- direction)

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⇒ Leaving through BC + rate of momentum through AB) − rate of momentum entering through DA)

$\Delta P = P_{out} - P_{in}$

Thus according to momentum

$$\Sigma F = \frac{d}{dt}(P) = \frac{d\, mov}{dt}$$

where

$$\frac{dm}{dt} = \rho Q \quad \text{Thus}$$

$$F = \rho Q V$$

or

$$F = \rho A \cdot V \cdot V$$

$$F = \rho A V^2$$

$DA \longrightarrow \rho U\,(UBS)$

$BC \longrightarrow \rho B \int_{}^{\delta} u^2 \cdot dy$

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$$AB \longrightarrow \rho U \left(UB\delta - B \int_0^{\delta} u \cdot dy \right)$$

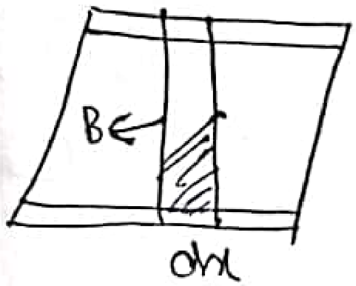
Putting value

$$Fx = \rho B \int_0^{\delta} u \, (U - u) \, dy$$

$$Fx = \rho B U^2 \delta \alpha$$ where α is function of boundary layer.

Now to find local wall shear stress

$$\tau_0 = \frac{d\,fx}{B \cdot dx - \text{area}}$$



$$Fx = \rho B U^2 \delta \alpha$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$ in general equation of shear stress.

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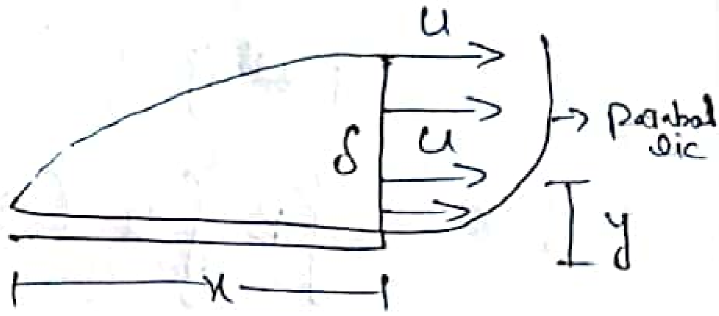
→ Laminar boundary Layer :-

$$\frac{u}{V} = F\left(\frac{y}{\delta}\right)$$



Assume

$$\eta = \frac{y}{\delta} \quad \text{or} \quad y = \eta\delta$$

Thus

$$\frac{u}{V} = f(\eta) \quad \text{or} \quad u = Uf(\eta)$$

In Case of laminar flow

$$\tau_0 = \mu\left(\frac{du}{dy}\right)$$

$$= \frac{\mu}{\delta}\left(\frac{du}{d\eta}\right) = \frac{\mu U}{\delta}\left[\frac{df}{d\eta}(\eta)\right]$$

Solving the Eq.

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$$\tau_0 = \frac{\mu U B}{\delta} \rightarrow (1)$$

As general Equation is $\tau_0 = \rho u^2 \alpha \frac{d\delta}{dx}$.

Equating both equation.

$$\frac{\mu U B}{\delta} = \rho v^2 \alpha \frac{d\delta}{dx}$$

or

$$\delta \, d\delta = \frac{\mu B}{\rho U \alpha} dx$$

Integrating the equation.

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha} x + C$$

Now at $x = 0$, $\delta = 0$ Thus $C = 0$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha} x.$$

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or

$$\delta = \sqrt{\frac{2\mu B}{\rho U \alpha}\, x} \quad \text{or} \quad \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

xing and ÷ing by "x"

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu \cancel{x}}{\rho U}} \cdot \frac{x}{\sqrt{\cancel{x}} \cdot \sqrt{x}}$$

where $\alpha = 0.135$

$\beta = 1.63$

$R_x = \frac{\rho U x}{\mu}$

$$\delta = \frac{4.91}{\sqrt{R_x}} \cdot x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}$$

Now

$$\tau_0 = \frac{\mu U B}{\delta}$$

Thus puting value

$$\boxed{\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}}$$

where R_x is local Reynold number.

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(9)

Now

$$F_g = B \int_0^{2} \frac{\tau_0 \, dx}{\text{stress}}$$

Putting values.

$$F_g = 0.664 \, B \sqrt{\rho \mu L V^3}$$

As general equation is

$$F_f = c_f \, \rho \frac{V^2}{2} BL \rightarrow$$ Equating both equation

$$c_f = 1.328 \sqrt{\frac{\mu}{\rho L V}} = \frac{1.328}{\sqrt{R}}$$

———— x ————

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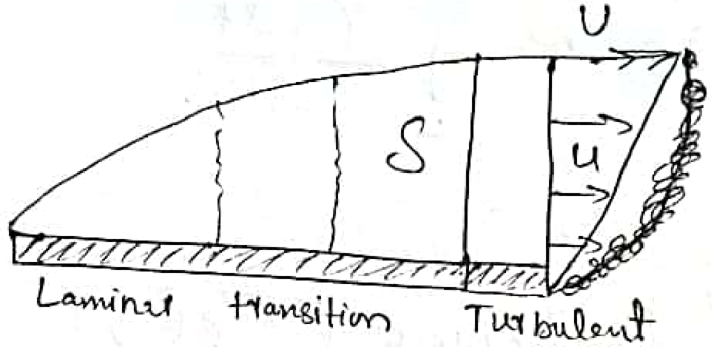
TURBULANT BOUNDARY LAYER:-



resistance is less so curve become straight

Fig show that velocity distribution in turbulent boundary layer shows a much steeper gradient near wall and flatter through out reming layer.

⇒ To shear stress is greater in turbulent than in Liminar layer.

As we have.

$$\tau_0 = \frac{f \rho v^2}{8}$$

where v denotes average velocity of pipe.

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→ now we have obtained an approximate relation b/w V and U by using pipe factor Equation of

$$\frac{V}{U_{max}} = \frac{1}{1 + 1.33\sqrt{f}}$$

Using fraction factor of 0.028 from chart which is middle critical value.

So

$$\boxed{U = 1.235 V}$$

Now we have

$$\tau_0 = f \rho \frac{V^2}{8}$$

As we know

$$f = \frac{0.316}{R^{0.25}}$$

Thus

$$\tau_0 = \frac{0.316}{\left(\frac{DV}{\nu}\right)^{1/4}} \rho \frac{V^2}{8}$$

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where

$$V = \frac{U}{1.235}$$ thus

$$\tau_0 = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{V}{1.235}\right)^2$$

$\in$ $D = 2\delta$

Thus

$$\tau_0 = \frac{0.0238\,\rho\,U^2}{\left(\frac{\delta U}{\nu}\right)^{1/4}}$$

As we have

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

Equating both and integrating for boundary condition of $x = 0$, $\delta = 0$

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Thus

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5}\left(\frac{\nu}{Ux}\right)^{1/5} x.$$

For $\alpha = 0.0972$

$$\boxed{\frac{\delta}{x} = \frac{0.377}{(R_x)^{1/5}}}$$

Putting values in equation.

$$\tau_0 = 0.0587\, \rho \frac{U^2}{2}\left(\frac{\nu}{Ux}\right)^{1/5}$$

Now

$$F_f = B\int_0^L \tau_0\, dx$$

$$F_f = 0.0735\, \rho \frac{U^2}{2}\left(\frac{\nu}{UL}\right) BL$$

As

$$F_f = c_f\, \rho \frac{V^2}{2} BL.$$

Equating Both;

$$\boxed{c_f = \frac{0.0735}{R^{1/5}}}$$

R is less than 10^7 for

$500{,}000 < R < 10^7$

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For $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.58}}}$$

Ans

End

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Q No1
Part (B)

As specific Energy

$$E = y + \frac{v^2}{2g}$$

The flow Q per unit width b can be expressed as

$$q = \frac{Q}{b}$$

Now average velocity will be

$$V = Q/A = \frac{qb}{by} = \frac{q}{y}$$

Thus

$$E = y + \frac{v^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

$$(E-y) = \frac{1}{2g}\left(\frac{q^2}{y^2}\right) \quad \text{or} \quad (E-y)\,y^2 = \frac{q^2}{2g}$$

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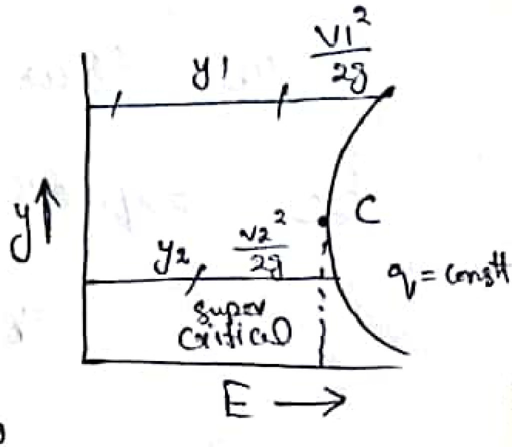
Thus plot of E vs y will be parabolic. For particular q, there will be two kind of possible values of y, for a given E.



The Equation is cubic with three roots, with third root being negative point C represents dividing point b/w two regime of flow

Thus for given q & value of E is minimum & Flow at that point is critical flow. Depth of flow at that point is critical depth y_c & velocity at that point is critical velocity V_c.

Thus

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum Specific energy $\frac{dE}{dy} = 0$

Thus

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right) = 0$$

$$\Rightarrow \frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$$\frac{q^2}{g} = y^3 \Rightarrow \left(\frac{q^2}{g}\right)^{1/3} = y_{cr}$$

Now

$$q^2 = gy^3$$

&

$$q = vy \Rightarrow v^2y^2 = gy^3$$

or

$$v^2 = gy_{cr}$$

or

$$V_{cr} = \sqrt{g\,y_{cr}}.$$

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—{Question 02}—

* Givin Data :-

Water flow at rate , $Q = 3.5 m^3/s$
Bed slope , $S_o = 0.0008$
$n = 0.0219$
width of bed is student I.D = 7690 mm

* Required :-

Depth of rectangular channel = ?
Critical Depth , $y_c = ?$
Critical velocity $V_c = ?$
Flow is critical or sub-critical = ?

* Solution :-

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2} \right) A \quad —(1)$$

Area = $7.690 \times d$

Parameter : $d + 7.690 + d$

$$\text{Hydraulic Radius} : R_n = \frac{\text{Area}}{\text{Parameter}} = \frac{7.690 \times d}{2d + 7.690}$$

we know that

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2} \right) A$$

Putting values

$$3.5 = \left(\frac{1}{0.0219} \times \left(\frac{7.690 \times d}{2d + 7.690}\right)^{2/3} \times (0.0008)^{1/2}\right)$$

$$\frac{3.5 \times 0.219}{\sqrt{0.0008}} = \left(\frac{7.690 \times d}{2d+7.690}\right)^{2/3}$$

$$(2.57)^{3/2} = \frac{7.690 d}{2d + 7.690} \times 7.690 d$$

$$4.461 \cdot (2d + 7.690) = 59.13 d^2$$

$$8.92 d + 34.30 = 59.13 d^2$$

$$59.13 d^2 - 8.92 d - 34.21 = 0$$

$$\boxed{d = 0.84 \text{ m}}$$

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* So depth of the channel is 0.840 m

$\Rightarrow$ As $q = \frac{Q}{b}$

$$q = \frac{3.5}{7.690} = 0.4551$$

$\Rightarrow$ For crictical depth :-

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_{cr} = \left(\frac{0.455}{9.81}\right)^{1/3}$$

$$\boxed{y_{cr} = 0.276\ m}$$

Now critical velocity

$$V_{cr} = \sqrt{y_{cr} \times g}$$

$$V_{cr} = \sqrt{(0.276)(9.81}$$

$$\boxed{V_{cr} = 1.645\ m/s}$$

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$$V = \frac{Q}{A}$$

$$V = \frac{3.5}{7.690 \times 0.840}$$

$$\boxed{V = 0.542 \text{ m/s}}$$

$y = 0.840$ m , $y_{cr} = 0.276$ m

$V = 0.542$ m/s , $V_{cr} = 1.645$ m/s

As $y > y_{cr}$ & $V < V_{cr}$

$\Rightarrow$ So flow is subcritical.

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— {Question 03} —

Given data :

width of smooth plate $B = 200\,mm$
$B = 0.2\,m$

length of plate $L = 800\,mm$
$L = 0.8\,m$

Specific gravity of oil $S = 0.89$
Undisturbed velocity $V = 5\,m/s$
Kinematic viscosity $\nu = 0.93 \times 10^{-4}\,m^2/s$

Required data :
Friction drag $F_f = ?$

Solution :
As we know,
The friction drag is given by the equation,

$$F_f = C_f\, \rho\, \frac{V^2}{2}\, BL \quad \text{——} \quad (1)$$

where C_f depends on viscosity.

Reynold's number is given by.

$$R = \frac{\rho V L}{\mu} = \frac{VL}{\nu} \qquad \text{where } \nu = \frac{\mu}{\rho}$$

Putting values we get

$$R = \frac{5\,m/s \times 0.8\,m}{0.93 \times 10^{-4}\,m^2/s}$$

$R = 43{,}010.75$ which is less than 500,000.

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So.

$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43011}} = 0.00640$$

Now:

As $\text{Specific Gravity} = \frac{\rho_{oil}}{\rho_{water}}$

$$\rho_{oil} = \text{Specific Gravity} \times \rho_{water}$$

$$\rho_{oil} = 0.89 \times 1000$$

$$\rho_{oil} = 890$$

Putting values in equation ①, we get

$$F_f = 0.00640 \times 0.89 \times 1000 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.392 \ N}$$

So, the friction drag on one side of smooth plate is '11.392 N'.

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