Name = Saud Khan

ID = 7774

Section = B

Date = 29-6-20

Q1

ID 7774

$Ku + Cu + Mu = Pt$

IN our case system is undomped $C = 0$ undergoing free vibration

$P(t) = 0$

Hence General eq become EOM

$Ku + Mu = 0$

$K = 3EI/L^3$

$= 3 \times 29000 \times (150)\text{ in}^4 / (10 \times 12)^3$

$K = 7.55 k/in$

$K = 90600 \text{ lb/ft}$

$M = \dfrac{7774 \text{ lb sec}^2}{32.2 \text{ ft}}$

241.42 slug

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{90600}{241.42}}$$

$$\boxed{\omega_n = 19.372 \, rad/sec}$$

$$T_n = 2\pi / \omega_n = \frac{2(3.14)}{19.372}$$

$0.324 \, sec$

Subsitituting the corresponding value in eq - 1

$90600 + 241.42 = 0$

Genral sol of EOM for undamped

$u(t) = 1/2'' = \frac{1}{24} ft$ and $u'(0) = 0$

$u(t) = \frac{1}{24} \times \cos(19.372) + 0 =$

Equivalent static force at any time "t" is

$$f_s(t) = k \cdot u(t) = \frac{90600 \times \cos 19.372t}{24}$$

$\cancel{f_s(t) = 35\ldots}$

(3)

$f_s(t) = 3775 \cos 19.372(t)$

Amplitude of dynamic displacement

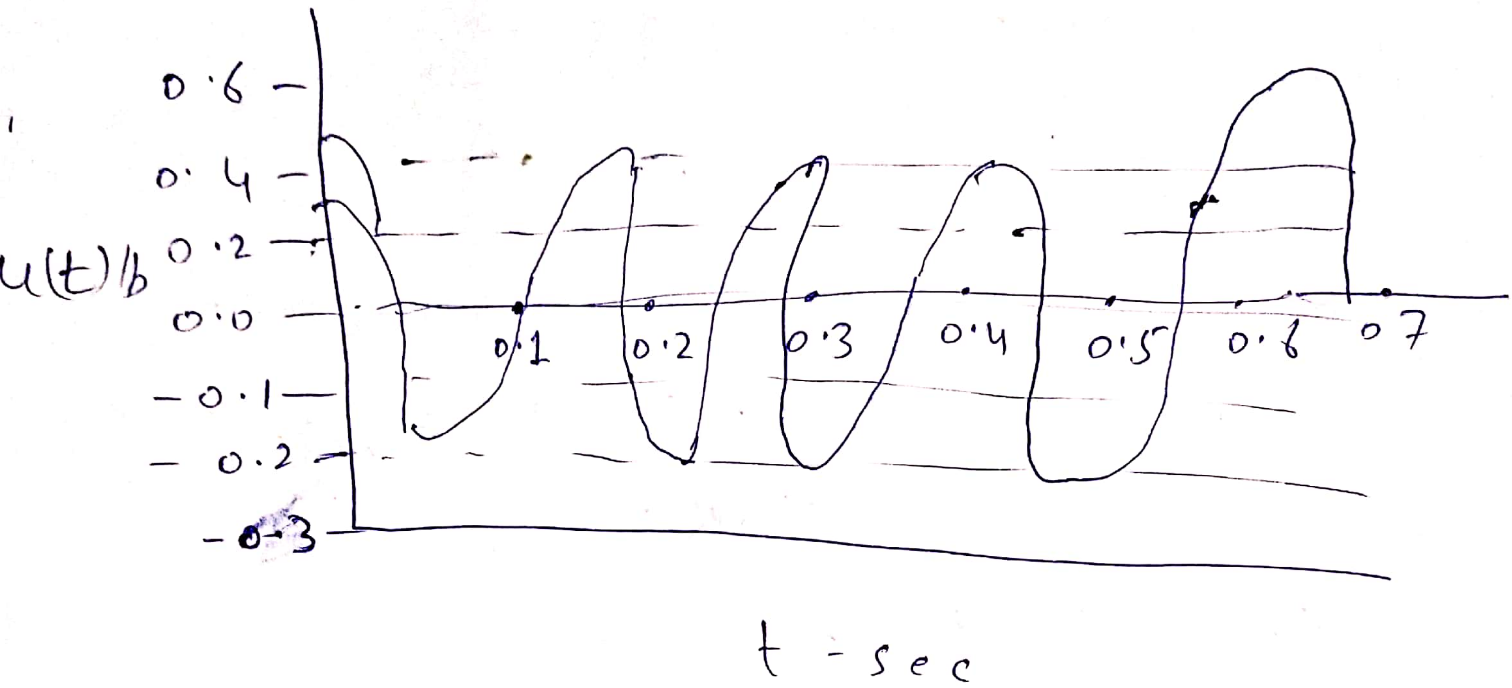
$u_0 \sqrt{\left[u(0)^2 + \left(\frac{u \cos}{\omega n}\right)^2\right.}$

$= \sqrt{\left(\frac{1}{24}\right)^2 + 0}$

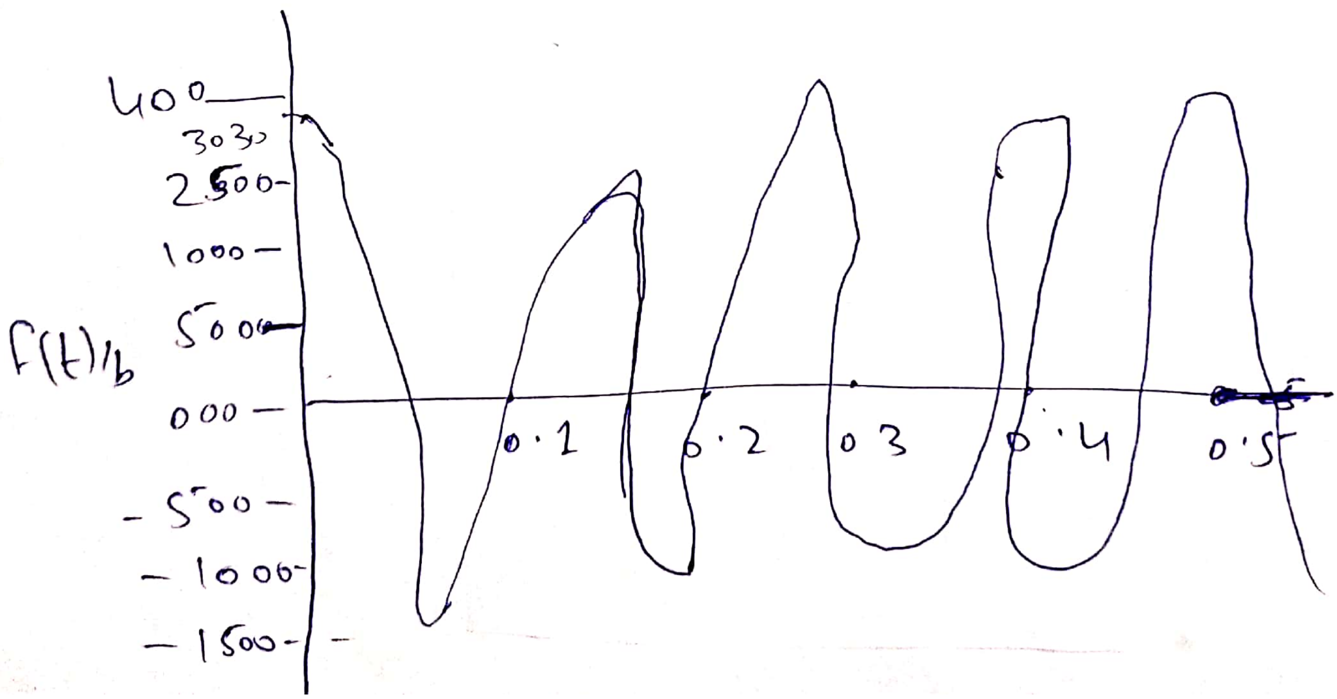
$K u_0 = 90600 \times \frac{1}{24} = 3775 \text{ lb}$

(4)

Undamped free vibration



Undamped free vibration



Scanned with CamScanner

5

Q2 ID 7774

$ku + c\dot{u} + m\ddot{u} = 0$

It is known from problem 4.1 that

$k = 90600$ lb/ft and $m = 241.42$ slug

Damping ratio of R.C = 5%

$c = \xi \times 2 m \omega_n = 0.05 \times 2 \times 241.42 \times 19.372$

$$\boxed{c = 467.678 \text{ lb-sec/ft}}$$

By subtituting values of k, c & m in eq (1) we get

$90600u + 467.678u' + 241.42\ddot{u} = 0$

Sol to the E.O.M for damped free vibration is

$u(t) = e^{-\xi\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\,\xi\omega_n\right]\sin\omega_D t\right.$

$\omega_D = 19.372$

Scanned with CamScanner

(6)

$$u t = e^{-0.05 \times 19.372 \times t} \left[\frac{1}{24} \times \cos(19.372 t) \times \frac{1}{19.372} \right.$$

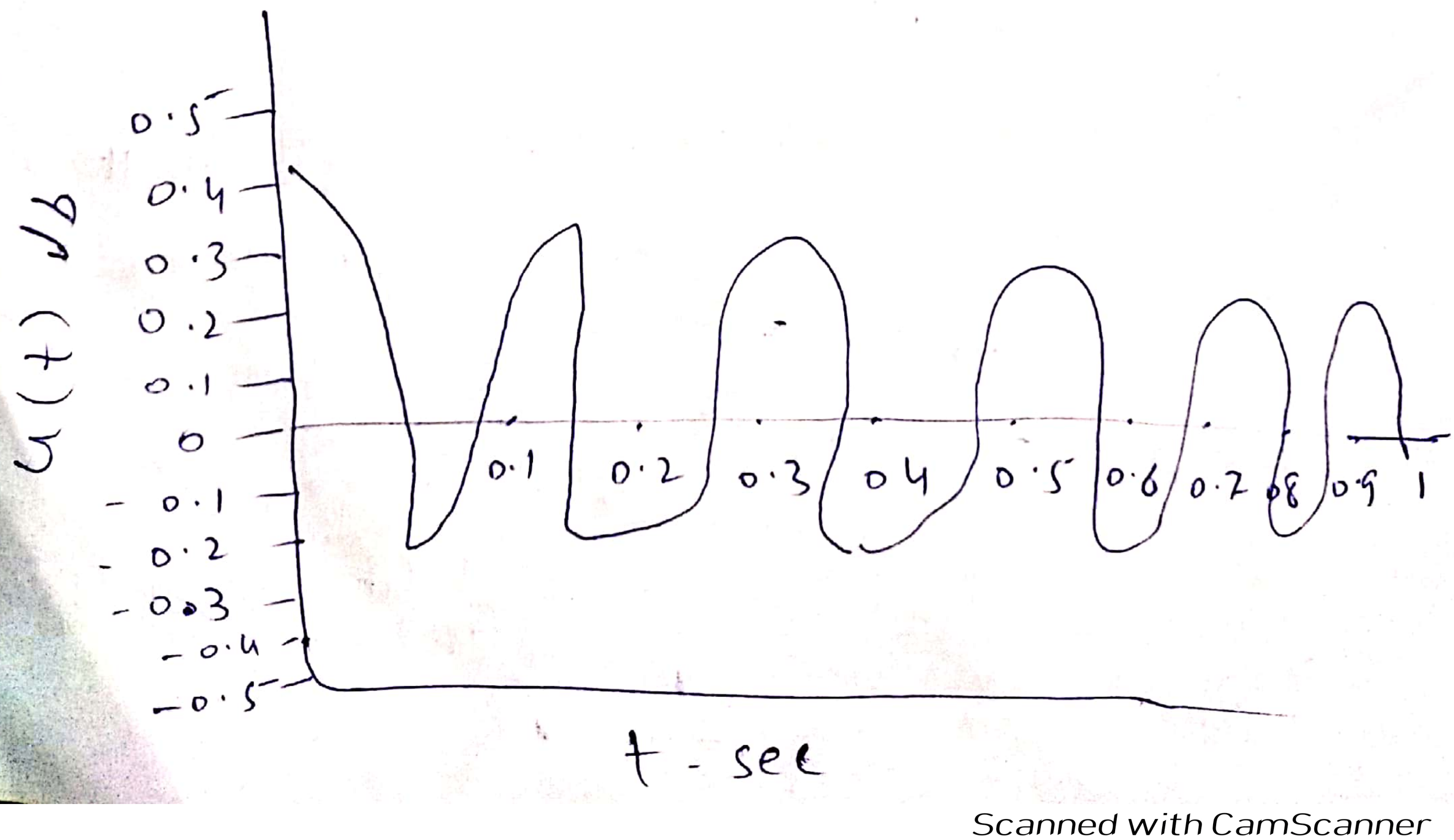
$$\left[0 + \frac{1}{24}\ 0.05 \times 19.38 \times \sin(\ldots) \right. \quad (19.37\ldots$$

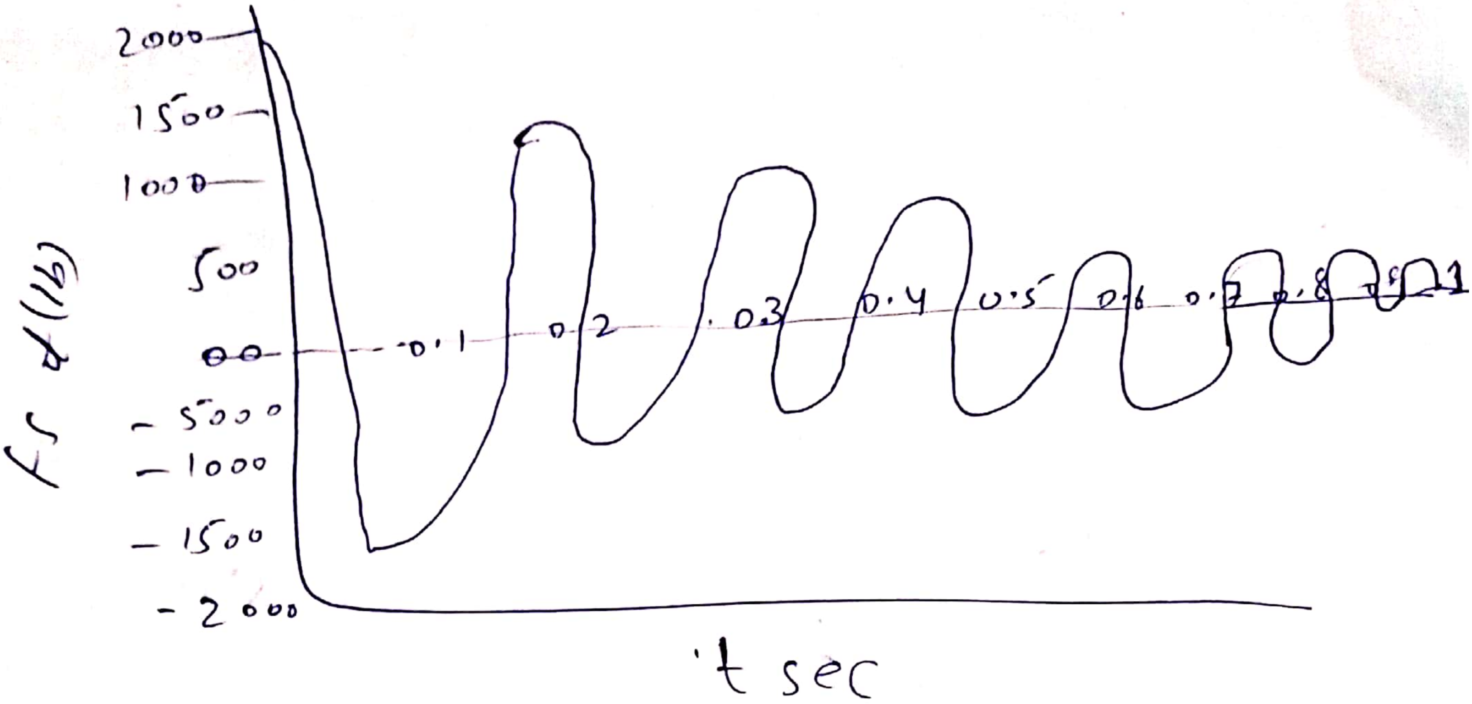
$$u t = e^{-0.968 t} \left(0.0416 \times \cos(19.372) + 0.0028 \sin(19.372 t \right.$$

$$f s(t) = K + u(t) = 90600 \times u(t)$$

$$f s = e^{-0.968 t} \left[3768.96 \cos(19.37t) + 188.48 \sin 19.372t \right)$$

$\Rightarrow$ Damped Free vibration





Decay of Response due to Damping

It can be derive that

$$\frac{u}{u_{i+1}} = e\left(\frac{2\pi \zeta \omega_n}{\omega_B}\right)$$

Taking ln

$$\delta = \ln\left[\frac{u_i}{u_{i+1}}\right] = \frac{2\pi \zeta \omega_n}{\omega_D}$$

Since $\omega_D = \omega_n \sqrt{1-\zeta^2}$

$$= \delta = \frac{2\pi \zeta \omega_n}{\omega_n \sqrt{1-\zeta^2}}$$

$$\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}}$$

For civil Engineering δ is < 0.1

$$\sqrt{1-\delta^2} = 1 \ \& \ \delta = 2\pi\delta \text{ for light damped}$$

Decay of Response due to Damping

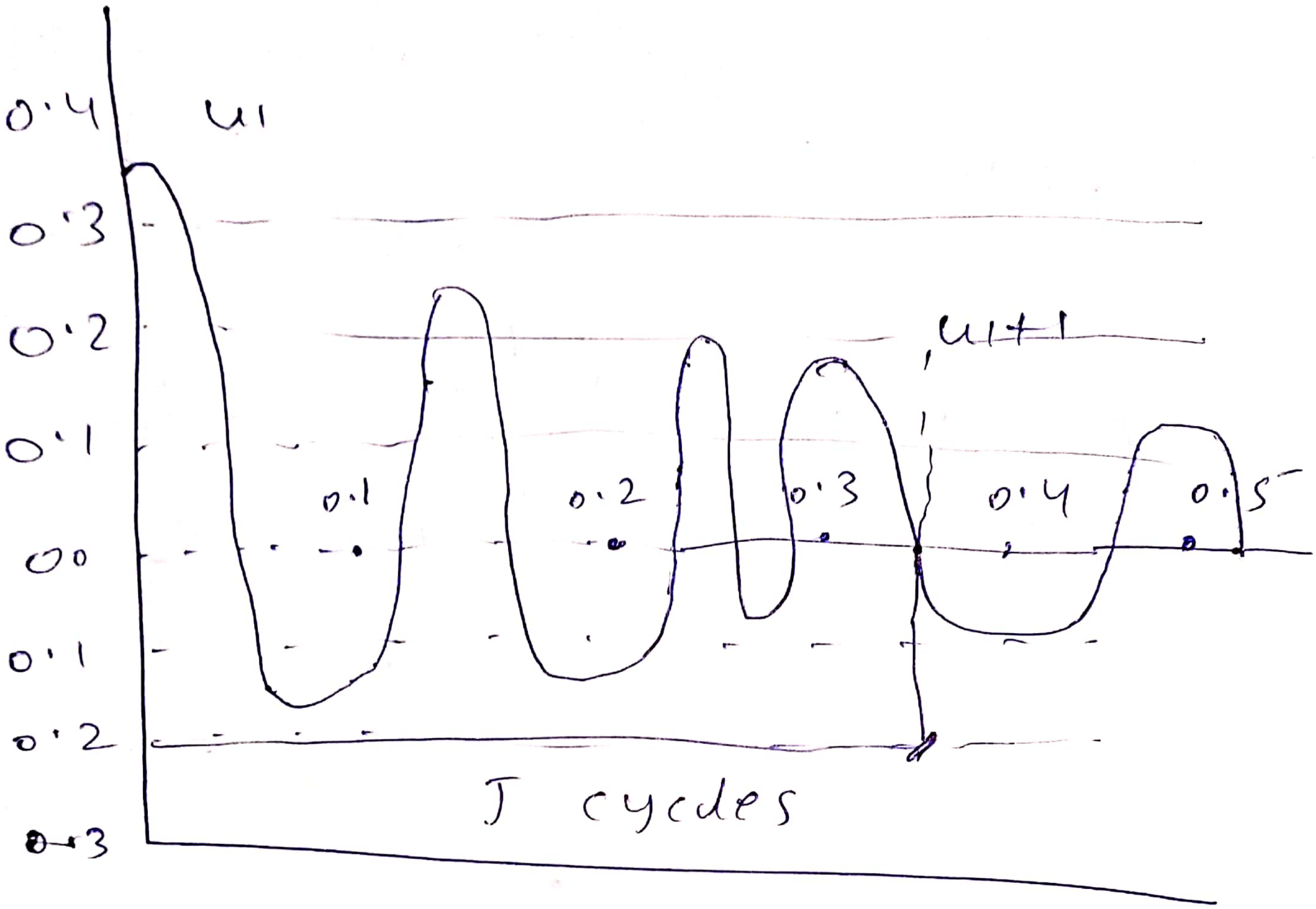
These values may be improved by taking the difference in the peak response value not at two successive peak but over a range of j peak

$$j\delta = \ln\left[\frac{u_1}{u_{1+1}}\right]$$

$$\Rightarrow \frac{1}{j}\ln\left[\frac{u_1}{u_{1+1}}\right] = 2\pi\delta$$

$$\Rightarrow J = \frac{1}{2\pi\delta}\ln\left(\frac{u_1}{u_{J+1}}\right)$$

Scanned with CamScanner


0.4
0.3
0.2
0.1
00
0.1
0.2
0.3
ui
ui+1
0.1
0.2
0.3
0.4
0.5
J cycles

Q3 Ans (10)

Sol

$$u_1 = \frac{7774}{1000}$$

$u_1 = 7.7774$ inch

After $J = 7$

u_{J+1}

$u_8 = 2.286$ cm

(a) = ξ = Damping ratio = ?

$$J = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{J+1}}\right]$$

$$7 = \frac{1}{2\pi\xi} \ln\left[\frac{7.7774}{2.286}\right]$$

$$7 = \frac{1}{2\pi\xi}(1.22398)$$

$$= \boxed{\xi = 0.0278} = 2.78\%$$

Scanned with CamScanner

(b) $T_n = ?$

7 cycles of vibration are completed in 3.57 sec

$$\frac{3.57}{7} = T_D$$

$$\boxed{T_D = 0.51 \text{ sec}}$$

Now $\omega_D = \omega_n \sqrt{1-\zeta^2}$

$$\frac{2\pi}{\omega_D} = 2\pi / (\omega_n \sqrt{1-\zeta^2})$$

$$T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0278)^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0278)^2}$$

$$\boxed{T_n = 0.5098}$$

72

(c) $K = ?$

$$K = \frac{60 \times \cos 60^\circ}{2}$$

$K = 15K/in = 180000 lb/ft$

(d) $$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(W/g)}} = \sqrt{\frac{Kg}{W}}$$

$$\Rightarrow \omega_n^2 = K \times g / W$$

$$W = \frac{K \times g}{\omega_n^2}$$

Also $\omega_n = 2\pi / T_n$

$$W = Kg \Big/ \frac{4\pi^2}{T_n^2}$$

$$W = Kg \times T_n^2 / 4\pi^2$$

$$W = \frac{180000 lb}{ft} \times \frac{32.2 ft}{sec^2} (0.51 sec)$$

13

(e) $c = ?$

it is known that $\xi = \dfrac{c}{2m\omega_n}$

$$c = \xi \times 2m\omega_n = \xi \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = \frac{0.0278 \times 4 \times \pi \left(\frac{1507530}{32.2}\right)}{0.51}$$

$$\boxed{c = 32069.67}$$

(f) No of cycle to reduce displacement amplitude from 7.774 to 0.5" $j = ?$

$$j = \frac{1}{2\pi\xi} \ln\left(\frac{7.774}{0.5}\right)$$

$$j = \frac{1}{2 \times \pi \times 0.0278} \times 2.743$$

$$\boxed{j = 15.70}$$