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Dept : BE (Electrical)

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Subject: Calculas and Analytical Geometery

Q No. 1 (a)

Ans

Given

$$\int \theta \sqrt[4]{1-\theta^2}\, d\theta$$

Solution

Let

$$1-\theta^2 = u$$

$$\frac{d}{d\theta}(1-\theta^2) = \frac{du}{d\theta}$$

$$-2\theta = \frac{du}{d\theta}$$

$$= \theta\, d\theta = -\frac{1}{2}\, du$$

Now

$$= \int (u)^{1/4} \cdot (-1/2)\, du$$

$$= -\frac{1}{2} \int u^{1/4}\, du \qquad \because 1/4 + 1 = \frac{5}{4}$$

$$= -\frac{1}{\cancel{2}} \cdot \frac{\cancel{4}}{5}\, u^{5/4} + C$$

$$= -2/5\, u^{5/4} + C$$

By back substitution

$$= -2/5\, (1-\theta^2)^{5/4} + C$$

Q 1 (b)

$$\int_0^1 x^3 (1+x^4)^3 \, dx$$

Solution

let $t = 1 + x^4$

$$\frac{dt}{dx} = 4x^3$$

$$dt = 4x^3 \, dx$$

$$\frac{dt}{4} = x^3 \, dx \longrightarrow (2)$$

So put in above eq 1

$$\frac{1}{4} \int_0^1 t^3 \, dt$$

$$\frac{1}{4} \left(\frac{t^4}{4} \right) \Big|_0^1$$

$$1/16 \, (1^4 - 0^4)$$ Now solve the limit

$$\frac{1}{16} (1) = \boxed{\frac{1}{16}}$$ Ans

A Q2 part (a)

Illustrate the centre & radius of sphere

$x^2+y^2+z^2+3x-4z+1$

<u>Solution</u>

$x^2+y^2+z^2+3x-4z+1=0$

$(x^2+3x)+y^2+z^2-4z+1=0$

$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+(y-0)^2+\left(z^2+4z+\left(\frac{-6}{2}\right)^2\right)$

$=-1+\left(\frac{3}{2}\right)^2+\left(\frac{-6}{2}\right)^2$

$\left(x+\frac{3}{2}\right)^2+(y-0)^2+(z-2)^2=\frac{21}{4}$

So,

$(x_0, y_0, z_0) =$ Centre

$=\left(\frac{-3}{2}, 0, 2\right)$

&

Radius, $\boxed{a=\sqrt{\frac{21}{4}}}$ Ans

Q2(b) The region b/w the curve $y = \sqrt{x}$, $0 \leq x \leq 4$, and the x-axis is revolved about the x-axis to generate a solid. Apply the integration. Find the volume of solid.

Solution

Given that

$$y = \sqrt{x}$$

$$0 \leq x \leq 4 \Rightarrow a \leq x \leq b$$

as

$$V = \int_a^b \pi y^2 \, dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx = \pi \frac{x^2}{2} \Big|_0^4$$

$$V = \frac{\pi}{2} ((4)^2 - 0)$$

$$= 8\pi$$

Q3)

Illustrate the vector Projection AB.

Solution

$A = 2i - 4j + \sqrt{5}k$

$B = -2i + 4j - \sqrt{5}k$

Required:-

Projection AB = ?

Solution

By dot product.

$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$= ((-2)(2) + (4)(-4) + (-\sqrt{5})(\sqrt{5}))$

$= -4 - 16 + \sqrt{-5 \times 5}$

$= -4 - 16 - \sqrt{25}$

$= -4 - 16 - 5$

$\boxed{B \cdot A = -25}$

Now

$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$= 4 + 16 + \sqrt{5 \times 5}$

$= 4 + 16 + \sqrt{25}$

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$4+16+5$

$\boxed{A \cdot A = 25}$

So,

$$Proj_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$$

putting values

$$= \left(\frac{-\cancel{25}}{\cancel{25}}\right) (2i - 4j + \sqrt{5}k)$$

$$= (-1)(2i - 4j + \sqrt{5}k)$$

$\boxed{Proj_A B = -2i + 4j - \sqrt{5}k}$ Ans

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Q4)

Find the area of region b/w graph and x-axis

$y = -x^2 + 5x - 4 \quad [0,2]$

Solution)

Given that

$y = -x^2 + 5x - 4$

&

$[a,b] = [0,2]$

As $a = 0$

$b = 2$

So,

Area under graph will be

$A = \int_a^b f(x)\,dx$

putting values

$\int_0^2 (-x^2 + 5x - 4)\,dx$

By solving integration we will get

$A = \left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right)\Big|_0^2$

$A = \left(-\frac{1}{3}(2)^3 + \frac{5}{3}(2)^2 - 4(2)\right) - (0)$

$A = \left(-\frac{1}{3}(8) + \frac{5}{2}(4) - 8\right)$

$A = -\frac{8}{3} + \frac{20}{2} - 8$

$A = \frac{-8}{3} + \frac{20}{2} - \frac{8}{1}$

$A = \frac{2 \times -8 + 3 \times 20 - 6 \times 8}{6}$

$A = \frac{-16 + 60 - 48}{6}$

$A = \frac{60 - 64}{6}$

$A = \frac{4}{6} = \frac{2}{3}$

$\boxed{A = 0.666}$ Ans

Q5 part (a)

Estimate the angle b/w A & B

$A = i - 2j - 2k$ & $B = 6i + 3j + 2k$

Solution

$A = i - 2j - 2k$

$|A| = \sqrt{(1)^2 + (-2)^2 + (-2)^2}$

$= \sqrt{1 + 4 + 4}$

$= \sqrt{9}$

$|A| = 3$

Now

$B = 6i + 3j + 2k$

$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2}$

$= \sqrt{36 + 9 + 4}$

$= \sqrt{49}$

$|B| = 7$

So,

$$Q = \cos^{-1}\left(\frac{A \cdot B}{|A|\,|B|}\right)$$

$$Q = \cos^{-1}\left[\frac{(i - 2j - 2k) \cdot (6i + 3j + 2k)}{3 \times 7}\right]$$

$$Q = \cos^{-1}\left\{\frac{(1)(6)+(-2)(3)+(-2)(2)}{21}\right\}$$

$$Q = \cos^{-1}\left(\frac{6-6-4}{21}\right)$$

$$Q = \cos^{-1}\left(\frac{-4}{21}\right)$$

$\boxed{Q = 100.97}$ Ans

Q5 (b)

Change the spherical cordinator equation for the sphere.

$$x^2+y^2+(z-1)^2 = 1$$

Solution:

$$x^2+y^2+(z-1)^2 = 1$$

$$(\rho\sin\phi\cos\theta)^2+(\rho\sin\phi\sin\theta)^2+(\rho\cos Q-1)^2 = 1$$

$$\rho^2\sin^2 Q\cos^2 Q+\rho^2\sin^2 Q\sin^2 Q+\rho^2\cos^2 Q+1-2\cos Q = 1$$

$$\rho^2\sin^2 Q(\cos^2 Q+\sin^2 Q)+\rho^2\cos^2 Q+1-2\rho\cos Q = 1$$

$$\rho^2(\sin^2\phi+\rho^2\cos^2\phi-2\rho\cos Q = 1-1$$

$$\rho^2(\sin^2 Q+\cos^2 Q)-2\rho\cos Q = 0$$

$$\rho^2 = 2\rho\cos\theta$$

$\boxed{\rho = 2\cos\phi}$ Ans.

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