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Question No (1)
=> Part (A)

Solution:-

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2$ Hence

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is

$$y_P(n) = K(-1)^n u(n)$$

Subsituting this solution into the difference equation we obtained

$$\Rightarrow K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

$\Rightarrow$ For $n = 2$, $K(1+4+4) - 2$

$K = 2/9$ The total solution is

$$y(n) = \left[C_1 2^n + C_2 n 2^n + 2/9 (-1)^n\right] u(n)$$

From the initial conditions we obtain

$$y(0) = 1, \quad y(1) = 2$$

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Than

$C_1 + 2/9 = 1$

$\Rightarrow C_1 = 7/9$

$\Rightarrow 2C_1 + 2C_2 - 2/9 = 2$

$\Rightarrow C_2 = 1/3$

$\longleftrightarrow$

Question No (1)
$\Rightarrow$ Part (B)

Solution:-
The characteristic equation is

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = 1/2 , 1/5$ Hence

$y_h(n) = C_1 \, 1/2^n + C_2 \, 1/5^n$

with $x(n) = \delta(n)$ we have

$y(0) = 2$

$y(1) - 0.7\,y(0) = 0$

$\Rightarrow y(1) = 1.4$

Hence $C_1 + C_2 = 2$

And,

$1/2\, C_1 + 1/5 = 1.4$

$1.4 = 7/5$

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$C_1 + 2/5\, C_2 = 14/5$

These equations yield

$C_1 = 10/3 \quad , \quad C_2 = -4/3$

$$h(n) = \left[\frac{10}{3} (1/2)^n - \frac{4}{3} (1/5)^n \right] u(n)$$

The step responce is

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$s(n) = \frac{10}{3} \sum_{k=0}^{n} (1/2)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$s(n) = \frac{10}{3} \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3} \left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$s(n) = \frac{10}{3} \left(\frac{1}{2}\right)^n (2^{n+1} - 1) u(n) - \frac{1}{3} (1/5)^n (5^{n+1} - 1) u(n)$$

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Question No (2)

=> Part (A)

Solution:- $X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$

Taking Inverse of Z-transform

$$\frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence

$$x(n) = [4(2)^n - 3 - n] \, u(n)$$

Question No (2)

=> part (B)

Solution:-

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$
$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Each sequence consist of four non-zero points for the purpose of illustrating the operation involved in circular convolution It is desired to graph each sequence as points on a circle

=> $x_1(n)$ with $x_2(n)$ as specified begining with

$m = 0$ we have $x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2[(-n)]N$

$x_2(-n)_4$ is simply the sequence $x_2[n]$ folded & graphed on a circle the product sequence is obtained by multiplying $x_1(n)$ with $x_2(n)$ 4 point by point. Finally we sum the values in the product sequence to obtain

$$\cancel{x_3} \; x_3(0) = 14$$

For $m = 1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)4$$

It is easily verified that $x_2(1-n)_4$ is simply the sequence $x_2(-n)_4$ rotated counter clockwise by one unit in the time.

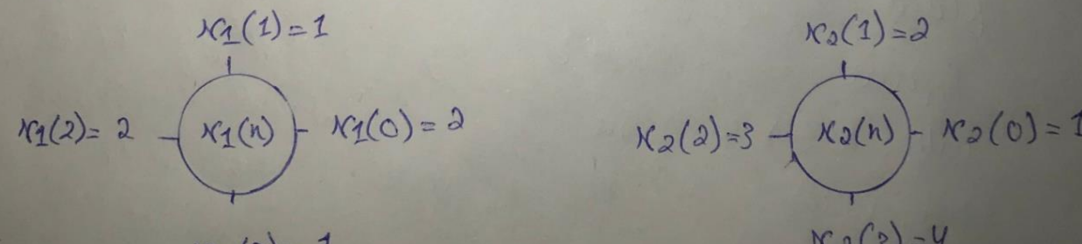
$\Rightarrow$ This rotated sequence multiplys $x_1(n)$ to yield the product sequence also finally we sum the values in the product sequence to obtain $x_3(1)$ Thus

$$x_3(1) = 16$$

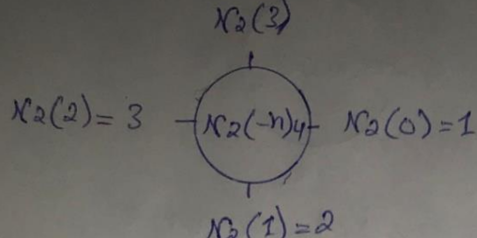
For $m=2$ we have

$$x_3(2) \sum_{n=0}^{3} x_1(n)\, x_2(2-n)_4$$

Now $x_2(2-n)_4$ is the folded sequence

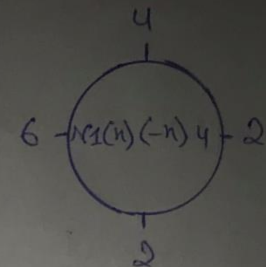


(A)

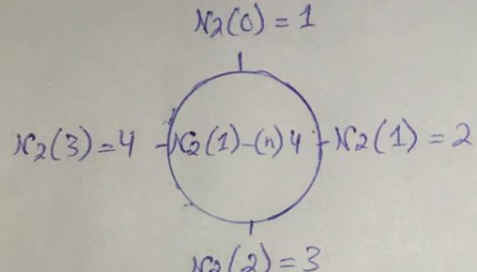


Folded Sequence

(B)

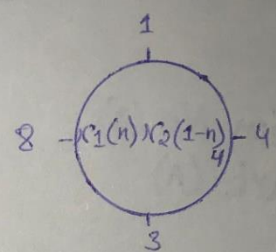


Product Sequence

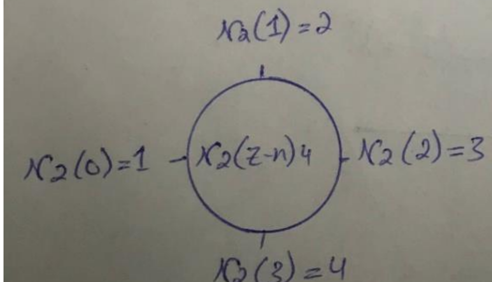


(C)

Folded Sequence rotated by one unit in time

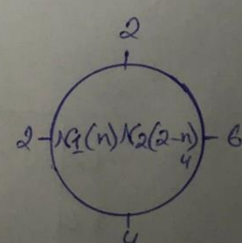


Product Sequence

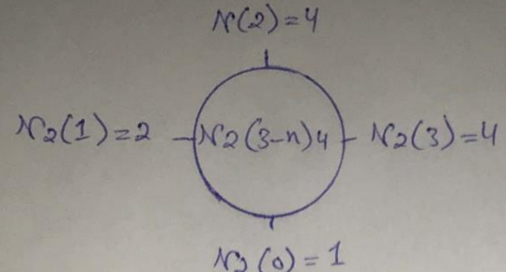


(D)

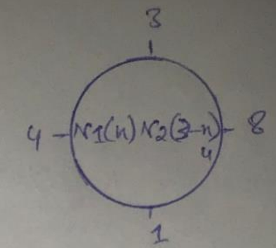
Folded frequency rotated by two unit in time



Product Sequence



(E)



Product sequence

Folded Frequency rotated by three unit in time

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Question No (3)

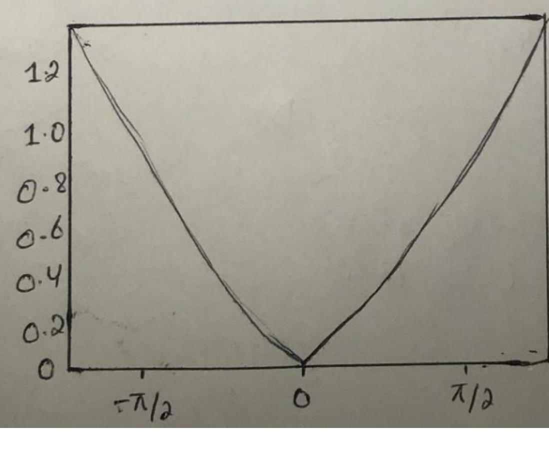
=> Part (A)

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

<u>Solution:-</u>

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $b_0 = (1-p)^2$



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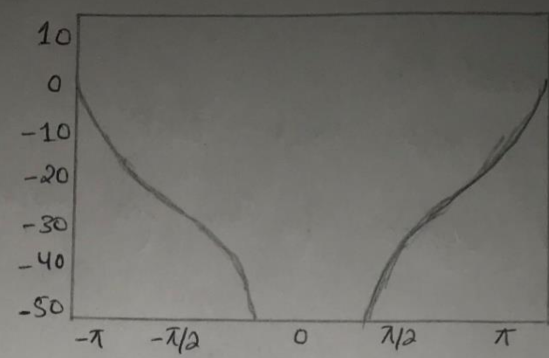

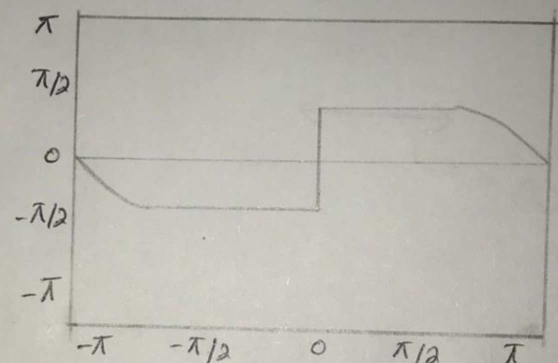

At $w = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$\Rightarrow \frac{(1-P)^2}{[1-P\cos(\pi/4)+j\sin(\pi/4)^2]}$$

$$\Rightarrow \frac{(1-P)^2}{(1-P/\sqrt{2}+jP/\sqrt{2})^2}$$

Hence

$$\frac{(1-P)^4}{[(1-P/\sqrt{2})^2(+P^2/2)^2]} \Rightarrow 1/2$$

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$\Rightarrow$ Equivalently

$$\sqrt{2}\,(1-P)^2 = 1+P^2-\sqrt{2}\,P$$

The System function for desired filter

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

$\longleftrightarrow$

Question No (3)

$\Rightarrow$ Part (B)

Solution:- The Systems must have poles at $P_{12} = re$

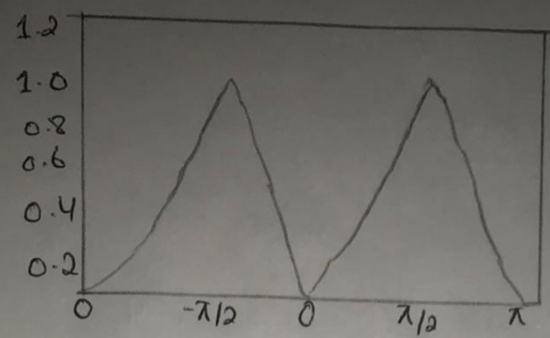
And zero at $z = 1$ & $Z = -1$

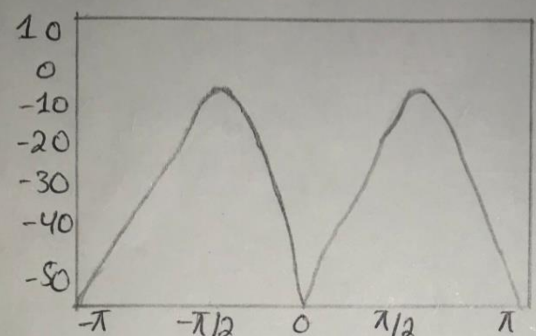
Consequently the same System function is

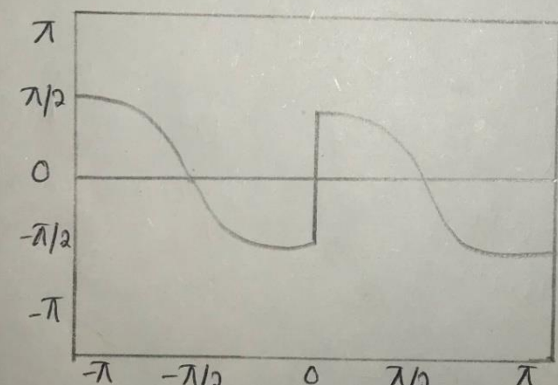
$$\Rightarrow H(z) = G\,\frac{(Z-1)\,(Z+1)}{(Z-jr)\,(Z+jr)}$$

$$\Rightarrow H(z) = \frac{G Z^2-1}{Z^2+r^2}$$

The gain factor is determined evaluating the frequency responce $H(w)$ of the filter at $w = \pi/2$







⇒ magnitude & Phase responce of a Simple bandpass filter is

$$H(z) = 0.15\left[(1-z^{-2})/(1+0.7\,z^{-2})\right]$$

The End