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SECTION B

SEMESTER 10^{th}

SUBJECT DIFFERIENTIAL EQUATION'S

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QUESTION i= $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$

i- $w = \sin(x+ct) + \cos(2x+2ct)$

Solution:

$$w = \sin(x+ct) + \cos(2x+2ct)$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) - \sin(2x+2ct) + 2$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cos(x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial t^2} = +c^2\left[-\sin(x+ct) - 4\cos(2x+2ct)\right]$$

$$c^2 . \frac{\partial^2 w}{\partial x^2}$$

Hence;

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2} .$$

ii) $w = \tan(2x + ct)$

Solution:- As $w = \tan(2x + ct)$

$$\frac{\partial w}{\partial t} = \sec^2(2x+ct)\frac{\partial}{\partial t}(2x+ct)$$

$$\frac{\partial w}{\partial t} = c\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c\,2\sec(2x+ct)\frac{\partial}{\partial t}\sec(2x+ct)$$

$$= 2c^2\sec(2x+ct)\sec(2x+ct)\tan(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = 2c^2\sec^2(2x+ct)\tan(2x+ct)$$

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 2\cdot 2\sec(2x+ct)\cdot\sec(2x+ct)\tan(2x+ct)\cdot 2$$

$$= 8\sec^2(2x+ct)\tan(2x+ct)$$

$$\Rightarrow 2c^2\sec^2(2x+ct)\tan(2x+ct) \neq c^2\,8\sec^2(2x+ct)\tan(2x+ct)$$

Not Satisfied.

Q2:- Expand the following function

$F(x) = x, \quad -\pi < x \leq 0$

$= 2x, \quad 0 \leq x \leq \pi$

Solution:

$$F(x) = \begin{cases} x ; & -\pi < x \leq 0 \\ 2x ; & 0 \leq x \leq \pi \end{cases}$$

We have to find fourier co-efficient a_0, a_n & b_n.

Now; $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)dx = \frac{1}{\pi}\int_{\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{1} 2x\,dx$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\cancel{\pi}}\left[0 - \frac{\pi^{\cancel{2}}}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$a_0 = -\frac{\pi}{2} + \pi = \pi/2 \longrightarrow eq\ 1$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So;

$$a_n = \begin{cases} \dfrac{-2}{\pi n^2} & ; \text{ if } n \text{ is odd} \\ 0 & ; \text{ if } n \text{ is even} \end{cases} \quad \text{— eq } (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx +$$

$$\frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx.$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$\Rightarrow b_n = \frac{1}{\pi}\left[-\frac{\pi \cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi \cos \pi}{n}\right] = \frac{-3\cos n\pi}{n}$

$= \frac{3(-1)^{n+1}}{n} \longrightarrow eq\ 3$

So the required fourier series is;

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2x-1)}{(2n-1)^2} + 3\sum_{n=1} (-1)^{n+1} \frac{\sin nx}{n}.$$

Q NO 3:

Solve the Initial value:

$y'' - 4y' + 13y = 8 \sin 3x \qquad y(0) = 1 \ \& \ y'(0) = 2$

Solution: As

$y'' - 4y' + 13y = 8 \sin 3x \longrightarrow eq\ 1$

Associated Homogeneous eq 1 is;

$y'' - 4y' + 13y = 0 \longrightarrow eq\ 2$

Change (2) into Auxillary equation;

put $y = m$ in eq (2)

$$m^2 - 4m + 13 = 0$$

Use quardatic formula;

$a = 1$, $b = -4$, $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(4)^2 - 4(1)(3)}}{2(1)}$$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$m = \frac{4 \pm 6i}{2}$$

$$m = 2 \pm 3i$$

$$m_1 = 2 + 3i$$

$$m_2 = 2 - 3i$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \longrightarrow (A)$$

Let

$$y_p = A\cos 3x + B\sin 3x \longrightarrow (*)$$

Diff. with respect to "x"

$$y_p' = -3A\sin 3x + 3B\cos 3x$$

Again diff. w. r. to "x"

$$y_p'' = -9A\cos 3x - 9B\sin 3x$$

put in eq (1)

$$\Rightarrow -9A\cos 3x - 12B\cos 3x + 13A\cos 3x - 9B\sin 3x$$

$$+12A\sin 3x + 13B\sin 3x - 8\sin 3x$$

$$\Rightarrow -9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x$$

$$= 8\sin 3x$$

$$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x$$

$$= 8\sin 3x$$

Comparing coefficient

$$\boxed{4B + 12A = 8} \longrightarrow (a)$$

$$4A - 12B = 0$$

$$4A = 12B$$

$$\boxed{A = 3B} \quad \text{—} \; (b)$$

put b in (A)

$$4B + 12(3B) = 8$$

$$4B + 36B = 8$$

$$40B = 8$$

$$\boxed{B = 1/5} \longrightarrow (c)$$

put c in eq (b)

$$\boxed{A = 3/5} \longrightarrow (d)$$

put eq (c) & (d) in (*)

$$y_p = \frac{B}{5} \cos 3x + \frac{1}{5} \sin 3x \quad \text{—} \; (B)$$

The General Solution is

$$y = y_c + y_p$$

$y = e^{2x}(c_1\cos 3x + c_2\sin 3x) + 3/_5\cos 3x + 1/_5\sin 3x$

Leq (c)

put $x = 0$, $y = 1$ in eq (c)

$1 = e^{x(0)}(c_1\cos 3(0) + c_2\sin 3(0) + 3/_5\cos(0)$

$1 = c_1 + 3/5$

$c_1 = 1 - 3/5$

$\boxed{c_1 = 2/5}$ —— ✱✱

Diff. (c) w. r. to "x"

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$

$-\frac{6}{5}\sin 3x + 3/_5\cos 3x \longrightarrow$ (D)

put $y' = 2$, $x = 0$ in (D)

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2c^{2x}\sin 3x$

$+ 3e^{2x}\cos 3x)$

$-6/_5\sin 3x + 3/_5\cos 3x$

put $y' = 2$, $x = 0$

$$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\cos 3(0))$$

$$-6/5 \sin 3(0) + 3/5 \cos 3(0)$$

$$2 = c_1(2) + c_2(3) - 0 + 3/5$$

$$2 = 2c_1 + 3c_2 + 3/5$$

put $c_1 = 2/5$

$$2 = 4/5 + 3c_2 + 3/5$$

$$2 = 7/5 + 3c_2$$

$$\boxed{c_2 = 3/15} \text{ — } ***$$

put $**$ & $***$ in (c)

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + 3/15 \sin 3x\right) + 3/5 \cos 3x + 1/5 \sin 3x$$

$$\boxed{y' = \frac{2}{5}e^{2x}\cos 3x + 3/15\, e^{2x}\sin 3x + 3/5 \cos 3x + 1/5 \sin 3x}$$

Hence General Solution.

QUESTION # 04:

$$(D^2 - DD')\, z = \cos x . \cos 2y$$

Solution:

We can written also as;

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = E \cos x . \cos 2y$$

Now as we have;

Corresponding A.E is $m^2 - m = 0$

when $D/D' = m$ so $m = 1$, $m = 1$.

$$C.F = \phi . y + \phi_2 (y + x)$$

Now;

$$D.I = \frac{1}{(D^2 - DD')} \cos x . \cos 2y$$

$$= \frac{1}{2} \frac{1}{D^2 - DD'} \Big[\underbrace{\cos(x+2y)}_{I} + \underbrace{\cos(x-2y)}_{II} \Big]$$

$$P.I = \frac{1}{2}\left[\frac{1}{-1+2}\cos(x+2y) + \frac{1}{-1-2}(\cos(x-2y))\right]$$

(When $\cos(ax+by)$ D^2 by $-a^2$, D'^2 by $-b^2$ and $D.D' = -ab$)

$$P.I \ \frac{1}{2}\left[\cos(x+2y) - \tfrac{1}{3}\cos(x-2y)\right]$$

The Complete Solution

$$z = \phi_1(y) + \phi_2(y+x) + \tfrac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

RESULT:

$$z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y$$