DEPARTMENT OF ELECTRICAL ENGINEERING

ELECTROMAGNETIC FIELD THEORY

Spring Semester

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Assignment:

Solve problem 4.1, 4.2, 4.3, 4.5, and 4.7 of course book.

Problems:-

(4.1) The value of E at $P(\rho=2, \phi=40^\circ, z=3)$ is given as $E = 100a_\rho - 200a_\phi + 300a_z$. V/m. Determine the incremental work required to move a 20-μC charge a distance of 6μm in the direction of: (a) a_ρ; (b) a_ϕ; (c) a_z; (d) E; (e) $G = 2a_x - 3a_y + 4a_z$.

a) in the direction of a_ρ: The incremental work is given by $dW = -qE \cdot dL$, where in this case, $dL = d\rho\, a_\rho = 6\times10^{-6} a_\rho$. Thus

$$dW = -(20\times10^{-6}C)(100\ V/m)(6\times10^{-6}m)$$

$$= -12\times10^{-9}\ J = \underline{-12\ nJ}$$

b) in the direction of a_ϕ: In this case $dL = 2d\phi\, a_\phi = 6\times10^{-6} a_\phi$, and so

$$dW = -(20\times10^{-6})(-200)(6\times10^{-6})$$

$$= 2.4\times10^{-8}\ J \quad J = \underline{24\ nJ}$$

c) in the direction of a_z=

Here, $dL = dz\,a_z = 6\times10^{-6} a_z$, and so

$$dW = -(20\times10^{-6})(300)(6\times10^{-6}) = 3.6\times10^{-8}\,J$$

$J = \underline{-36\,nJ}$

d) in the direction of E: Here, $dL = 6\times10^{-6} a_E$, where

$$a_E = \frac{100a_\rho - 200a_\phi + 300a_z}{[100^2 + 200^2 + 300^2]^{1/2}}$$

$$= 0.267a_\rho - 0.535a_\phi + 0.802a_z$$

Thus

$$dW = -(20\times10^{-6})[100a_\rho - 200a_\phi + 300a_z]\cdot[0.267a_\rho - 0.535a_\phi + 0.802a_z](6\times10^{-6})$$

$$= \underline{-44.9\,nJ}$$

e) In the direction of $G = 2a_x - 3a_y + 4a_z$:

In this case, $dL = 6\times10^{-6} a_G$, wher

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2+3^2+4^2]^{1/2}} = 0.371a_x - 0.557a_y + 0.743a_z$$

So now,

$$dW = -(20\times10^{-6})[100a_\rho - 200a_\phi + 300a_z]\cdot[0.371a_x - 0.557a_y + 0.743a_z](6\times10^{-6})$$

$$= -(20\times10^{-6})[37.1(a_\rho\cdot a_x) - 55.7(a_\rho\cdot a_y) - 72.2(a_\phi\cdot a_x) + 111.4(a_\phi\cdot a_y) + 222.9](6\times10^{-6})$$

where at P, $(a_\rho\cdot a_x) = (a_\phi\cdot a_y) = \cos(40^\circ) = 0.766$, $(a_\rho\cdot a_y) = \sin(40^\circ) = 0.643$, and $(a_\phi\cdot a_x) = -\sin(40^\circ) = -0.643$. Substituting these results in

$$dW = -(20\times10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9](6\times10^{-6}) = \underline{-41.8\,nJ}$$

(4.2) Let $E = 400a_x - 300a_y + 500a_z$ in the neighborhood of point (6, 2, −3). Find the incremental work done in moving a 4-C charge a distance of 1mm in the direction specified by:

a) $a_x + a_y + a_z$; where,

$$dW = -qE \cdot dL = -4\,(400a_x - 300a_y + 500a_z) \cdot \frac{a_x + a_y + a_z}{\sqrt{3}}\,(10^{-3})$$

$$= -\frac{(4 \times 10^{-3})}{\sqrt{3}}\,(400 - 300 + 500) = \underline{-1.39\ J}$$

b) $-2a_x + 3a_y - a_z$: The computation is similar to that of part a, but we change the direction :

$$dW = -qE \cdot dL = -4(400a_x - 300a_y + 500a_z) \cdot \frac{(-2a_x + 3a_y - a_z)}{\sqrt{14}} (10^{-3})$$

$$= -\frac{(4 \times 10^{-3})}{\sqrt{14}} (-800 - 900 - 500) = \underline{2.35 \, J}$$

(4.3) If $E = 120a_\rho$ V/m, find the incremental amount of work done moving a $5 \mu m$ charge a distance of 2mm from:

a) $P(1, 2, 3)$ toward $Q(2, 1, 4)$: The vector along this direction will be $Q - P = (1, -1, 1)$ from which $a_{PQ} = [a_x - a_y + a_z]/\sqrt{3}$.

$$dW = -qE \cdot dL = -(50 \times 10^{-6}) \left[120a_\rho \cdot \frac{a_x - a_y + a_z}{\sqrt{3}} \right] (2 \times 10^{-3})$$

$$= -(50\times10^{-6})(120)[(a_\rho\cdot a_x)-(a_\rho\cdot a_y)]\frac{1}{\sqrt{3}}(2\times10^{3})$$

At P, $\phi = \tan^{-1}(2/1) = 63.4°$. Thus $(a_\rho\cdot a_x)$
$= \cos(63.4) = 0.447$ and $(a_\rho\cdot a_y)$
$= \sin(63.4) = 0.894$. Substituting
these, we obtain $dW = \underline{3.1\mu J}$.

b) Q (2, 1, 4) toward P (1, 2, 3): A little through is in order here: Note that the field has only radial component and does not depend on ϕ or z. Note also that P and Q are at the same radius ($\sqrt{5}$) from the z axis. but have different ϕ and z coordinates. We could as well position the two points at the same z location and the problem would not change. If this where, then moving along a straight line between P and Q would thus involve moving along a chord of a circle whose radius is $\sqrt{5}$ Halfway along this line is a point of symmetry in the field (make a skech to see this). This means that when starting from either point, the initial force will be the same. Thus the answer is $dW = \underline{3.14\,J}$ as in part a. This is also found by going through the same procedure as in part a, but with the direction (roles of P and Q) reversed.

(4.5) Compute the value of $\int_A^P G \cdot dL$ for $G = 2y a_x$ with $A(1,-1,2)$ and $P(2,1,2)$ using the path:

(a) straight-line segments $A(1,-1,2)$ to $B(1,1,2)$ to $P(2,1,2)$: In general we would have,

$$\int_A^P G \cdot dL = \int_A^P 2y\,dx$$

The change in x occurs when moving between B and P, during which $y = 1$. Thus

$$\int_A^P G\cdot dL = \int_B^P 2y\,dx = \int_1^2 2(1)\,dx = 2$$

b) Straight-line segments $A(1,-\frac{1}{2},2)$ to $C(2,-1,2)$ to $P(2,1,2)$: In thus case the change in x occurs when moving from A to C, during which $y=-1$. Thus

$$\int_A^P G\cdot dL = \int_A^C 2y\,dx = \int_1^2 2(-1)\,dx = -2$$

(4.7) Let $G = 3xy^2 a_x + 2z a_y$. Given the initial point $P(2,1,1)$ and a final point $Q(4,3,1)$, find $\int G\cdot dL$ using the path:

(a) Straight line: $y = x-1, z=1$: we obtain:

$$\int G\cdot dL = \int_2^4 3xy^2\,dx + \int_1^3 2z\,dy = \int_2^4 3x(x-1)^2\,dx + \int_1^3 2(1)\,dy = 90$$

(b) Parabola: $6y = x^2 + 2, z = 1$: we obtain:

$$\int G \cdot dL = \int_2^4 3xy^2 \, dx + \int_1^3 2z \, dy = \int_2^4 \frac{1}{12} x (x^2+2)^2 \, dx + \int_1^3 2(1) \, dy = 82$$