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SECTION = A

SUBJECT = A.Fluid. Mechanics

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EXAM = FINAL TERM

(Q No #01 part #A)

PART # (A) Define Drag with Its Components, equation for friction Drag. Cofficient both In laminar & Turbulant boundry layers?

DRAG:- A body which as wholly Immersed In a homogenous fluid may be subjected to Two kind of force arising from relative motion B/w ~~body~~ body & fluid that those forces are termed as ~~dray~~ drag & lift. If the forces parallel ~~to~~ to the motion

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Then It is Termed as drag force.

There are Two Components

(I) PRESSURE DRAG (Fp)

It is equal to Integration of Components in direction of motion of all pressure forces Exerted on surface of body

$$F_p = C_p \rho \frac{V^2}{2} A$$ where C_p depends on shape

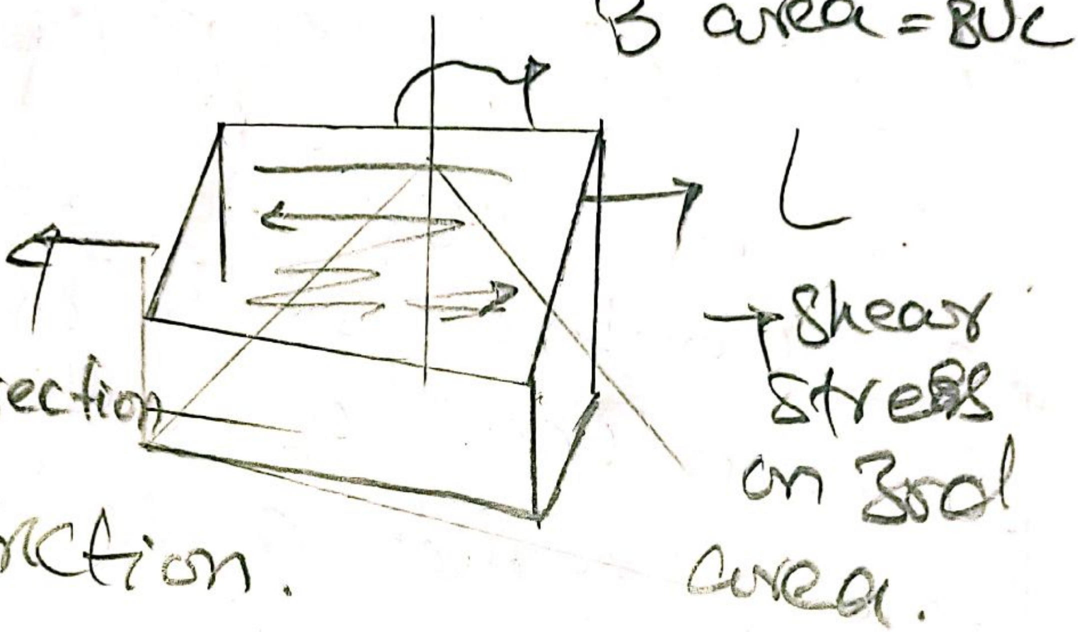
(II) FRICTION DRAG (Ff)

It is equal to Integration of Components of shear stress along surface of body in direction of motion.

01/03

$$F_f = C_f \rho \frac{V^2}{2} BC$$

Fig:-7



→ FRICTION DRAG OF B. LAYER

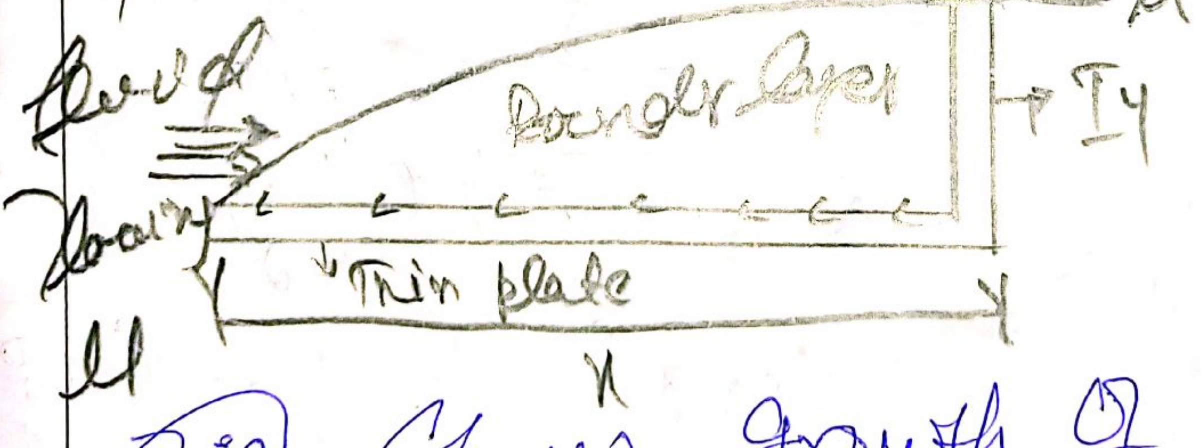


Fig Shows growth of boundry layer along one side of smooth plate Inside the fluid.

~~D - F~~ F_x
where δ is ~~th~~ Thickness of boundry layer ~~U~~ is undisturbed velocity

Thus $-F_x$ = drag =
(rate in momentum in x- direction).

(Leaving Through BC + rate of momentum Through AB) - rate of momentum entering Through (DA).

$\Rightarrow \Delta P = P_{out} - P_{in}$

Thus according to momentum

$$\Sigma F = \frac{d(P)}{dt} = \frac{d\,m_{in}}{dt}$$

where

$$\frac{dm}{dt} = \rho Q - \text{Thus}$$

$$F = \rho Q V$$

$$F = \rho A \cdot V \cdot V$$

$$F = \rho A V^2$$

$DA \rightarrow \rho U (UB\delta)$

$BC \rightarrow \rho B \int_1^{\delta} u^2 \cdot dy$

$AB \rightarrow \rho U (UB\delta - B \int^{\delta} u \cdot dy)$

putting value:

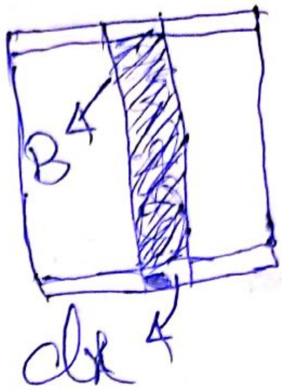
$$F_x = \rho B \int_0^{\delta} u (U - u) dy$$

$f_x = \rho B U^2 \delta x$ (when α is formation of boundary layer.)

Now to find local wall shear →

stress.

$$\tau_0 = \frac{d f_x}{B\text{-}dx\text{-}area}$$



$$F_x = \int B U^2 \rho \alpha$$

$$\tau_0 = \rho u^2 \alpha \frac{d\delta}{dx}$$

In general equation of shear stress.

LAMINAR BOUNDRY LAYER:-

$$\frac{u}{U} = F\left(\frac{y}{\delta}\right)$$

Assume

$$\eta = \frac{y}{\delta} \text{ or } y = \eta\delta$$

Thus $\frac{u}{U} = f(\eta)$ or $u = U f(\eta)$

In Case of laminar flow

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$$\tau_0 = \mu\left(\frac{du}{dy}\right)$$

$$= \frac{\mu}{\delta}\left(\frac{du}{dx}\right) = \frac{\mu u}{\delta}\left[\frac{df(\eta)}{d\eta}\right]$$

Solving the equation:

$\Rightarrow \tau_0 = \frac{\mu u B}{\delta} \rightarrow$ (1)

$\Rightarrow$ The generala equation is $\tau_0 = \rho u^2 \alpha \frac{ds}{dx}$

$\Rightarrow$ equating both equation:

$$\Rightarrow \frac{\mu u B}{\delta} = \rho u^2 \alpha \frac{d\delta}{dx}$$

$\delta\gamma$.

$$\Rightarrow \int \delta\, d\delta = \frac{\mu B}{\rho u \alpha}\, dx$$

#01/08

Integrating The equation

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho \mu \alpha} x + C$$

Now at $x=0$, $\delta=0$

Thus $C=0$

$$\frac{\delta^2}{2} = \frac{\mu B x}{\rho \mu \alpha}$$

or

$$\delta = \sqrt{\frac{2\mu B x}{\rho \mu \alpha}} \text{ or } \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho v}}$$

~~xing~~ $\times$ing & $\div$ing by "x"

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho v}} \cdot \frac{x}{\sqrt{x \cdot x}}$$

Where $\alpha = 0.135$

$\beta = 1.63$

$$R_x = \frac{\rho U x}{\mu}$$

$$\delta = \frac{4.91}{\sqrt{R_x}} \cdot x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}$$

Now $\tau_0 = \frac{\mu U \beta}{\delta}$

Thus putting value.

$$\boxed{\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}}$$

Where R_x is local Reynold Number Now

$$F_D = B \int_0^{L} \underbrace{\tau_0}_{\text{Stress}} dx$$

Putting value

$$F_f = 0.664 B \sqrt{\rho \mu L V^3}$$

As general equation is

$$F_f = C_f \rho \frac{V^2}{2} BL$$ → equating both equation.

$$C_f = 1.328 \sqrt{\frac{\mu}{\rho V}} = \frac{1.328}{\sqrt{R}}$$

TURBULANT BOUNDRY LAYER

That velocity distribution in Turbulant boundry layer shows a much steeper gradient near wall & (flatter Through) out ~~remaining~~ layer. The shear stress is greater in Turbulant Than in laminar layer.

As we have

$\tau_0 = f \rho \frac{V^2}{8}$ where V donates Average velocity

Now we have obtained an approximate relation b/w pipe U & U by using pipe factor equation of

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}}$$

using friction factor of 0.028 from chart which is middle critical value.

So $\boxed{U = 1.235U}$

Now we have

$\tau_0 = f\rho\frac{V^2}{8}$ as we know

$$f = \frac{0.316}{R^{0.25}}$$

Thus $\tau_0 = \frac{0.316}{(DU/\nu)^{\frac{1}{4}}} \frac{\rho U^2}{8}$

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where $v = \frac{U}{1.235}$ Thus

$$\tau_0 = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{U}{1.235}\right)\right)^{\frac{1}{4}}} \cdot \frac{\rho}{8}\left(\frac{U}{1.235}\right)^2$$

& $D = 2\delta$

Thus $\tau_0 = \frac{0.0023\,\rho U^2}{\left(\frac{\delta U}{\nu}\right)^{\frac{1}{4}}}$

As we know

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

Equating both & integrating for boundary condition of $x=0, \delta=0$

Thus $\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5}\left(\frac{\nu}{Ux}\right)^{\frac{1}{5}} x$

For $\alpha = 0.0972$

$$\delta/x = \frac{0.377}{(R_x)^{\frac{1}{5}}}$$

#01/13

$$\frac{\delta}{x} = \frac{0.377}{(R_x)^{\frac{1}{5}}}$$

putting values in equation

$$\tau_0 = 0.0587 \rho \frac{U^2}{2} \left(\frac{\nu}{Ux}\right)^{\frac{1}{5}}$$

Now $F_D = B\int_0^L \tau_0 \, dx$

$$F_D = 0.0735 \rho \frac{U^2}{2} \left(\frac{\nu}{UL}\right) BL$$

As $F_D = C_D \rho \frac{U^2}{2} BL$

equating both

$$C_f = \frac{0.0735}{R^{\frac{1}{5}}}$$

R is less Than 10^7 to $500000 < R < 10^7$

For $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.58}}}$$

Answer.

(Q #01 Part-B)

→ Equation for Critical Depth, Critical velocity of rectangular Section of a Channel?

As Specific Energy, $E = y + \frac{V^2}{2g}$

→ Depth of flow at that point is Critical depth y_c & velocity at that point is Critical velocity V_c.

Thus, → $E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$

For minimum Specific Energy

$\frac{dE}{dy} = 0$ Thus:→

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right) = 0$$

$$= \frac{q^3}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$$= \frac{q^2}{g} = y^3 \Rightarrow \boxed{\left(\frac{q^2}{g}\right)^{\frac{1}{3}} = y_{cr}}$$

Now

$$q^2 = gy^3$$

$$q = vy \Rightarrow v^2y^2 = gy^3$$

or

$$v^2 = gy_{cr}$$

OR $\boxed{v_{cr} = \sqrt{gy_{cr}}}$ Ans.

#07/01

08

Find Depth of Rectangular Channel
F.rate = 3.5 m^3/s with ~~bed~~
bed Slope = 0.0008 &
$n = 0.0219$
weidth of Bed = 7724 mm

Required: Critical Depth = ? (Rectangular channel)
Critical Velocity = ?
Sub Critical or Super Critical ?

Solution:

Manning Equation

$$Q = \left(\frac{1}{n} R_n^{2/3} S_0^{\frac{1}{2}}\right) A \rightarrow ①$$

#08/02

Solution:

$\text{Area} = 7.724 \times d$

$= 7.724\, d$

$\text{Perimeter} = d + 7.724 + d$

$= 7.724 + 2d$

Hydraulic Radius $R_h = \dfrac{\text{Area}}{\text{Perameter}}$

$= \dfrac{7.724\, d}{2d + 7.724}$

put value on eq → ①

$Q = \left[\dfrac{1}{n} R_h^{2/3} S_0^{\frac{1}{2}}\right) A$

$3.5 = \left[\dfrac{1}{0.0219} \times \left(\dfrac{7.724\, d}{2d + 7.724}\right)^{2/3} \times (0.0008)^{\frac{1}{2}} \times 7.724 d$

$$\Rightarrow \frac{3.5 \times 0.0219}{(0.0008)^{\frac{1}{2}}} = \left(\frac{7.724}{2d+7.724}\right)^{\frac{2}{3}} \times 7.724d$$

$$\Rightarrow 2.709 = \sqrt[2/3]{\left(\frac{7.724d}{2d+7.724}\right)^{2/3}} \times 7.724d$$

$$\Rightarrow 2.709 = \frac{7.724d}{2d+7.724} \times 7.724d$$

$$\Rightarrow (2.709)(2d+7.724) = 59.66d$$

$$\Rightarrow (5.418d + 20.924 = 59.66d$$

$$\Rightarrow 20.924 = (59.66d - 5.418d)$$

$$\Rightarrow 20.924 = 54.242d$$

$$\Rightarrow d = \frac{20.924}{54.242}$$

$$\Rightarrow \boxed{d = 0.0387m}$$ Answer.

#02/04

⇒ So the Depth of channel ⇒ is $\boxed{0.387 m}$

Now

As q = discharge per unit width

$$q = \frac{Q}{b}$$

$$= \frac{3.5}{7.724}$$

$$q = 0.453$$

⇒ CRITICAL DEPTH ⇒ (Y_{cr})

Using equation

$$Y_{cr} = \left(\frac{q^2}{g}\right)^{1/3} \Rightarrow \left(\frac{(0.453)^2}{9.81}\right)^{1/3}$$

$$= \left(\frac{0.2052}{9.81}\right)^{\frac{1}{3}} \Rightarrow Y_{cr} = 0.156m$$

#02/05

$Y_{cr} = 0.156 m$

→ Critical velocity, V_{cr}

using equ = $V_{cr} = \sqrt{g Y_{cr}}$

→ $V_{cr} = \sqrt{(9.81)(0.\cancel{2}56)}$

→ $V_{cr} = 0.765\ m/s$ | $V = \frac{Q}{A} \Rightarrow \frac{3.5}{7.724 \times 0.387 m}$

$\boxed{V = 0.175 m}$

→ Y @ Rd. are Same

→ $Y = 0.387 m$ $Y_{cr} = 0.0156\ m$ $V_{cr} = 1.0756\ m/s$

As $Y > Y_{cr}$ & $V < V_{cr}$

So, flow is SubCritical

03/01

(Q #03)

DATA :-

Friction Drag on one side of a Smooth plate 200mm wide & 800 mm length longitudanaly.

oil specific Gravity = 0.89

The u.disturb velocity = 5 m/s

Kinematic Viscosity = 0.93×10^{-4} m²/s

Required Data Friction drag on one side of a smooth plate, F_f = ?

SOLUTION :-

Check the flow

As $\nu = 0.93 \times 10^{-4}$ m²/s

$$R = \frac{LU}{\nu} = \frac{(0.8)(5)}{0.93 \times 10^{-4}}$$

$R = 43010.75 < 500,000$

Thus flow is laminar.

Now

$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010.75}}$$

$$C_f = 6.403 \times 10^{-3}$$

$$C_f = 0.0064$$

$$\Rightarrow F_f = C_f \, \rho \frac{V^2}{2} BL$$

$$= (0.0064)(\text{soil} \times \gamma\text{water}) \times \frac{(5)^2}{2} \times (0.2)(0.8)$$

$$= (0.0064)(0.89 \times 1000) \times \frac{5^2}{2} \times (0.2)(0.8)$$

$$\Rightarrow \boxed{F_f = 11.392\,N}$$

Answer.