NAME = Badrul Zaman

ID = 13685

(1)

4.1 The value of E at $P(\rho=2, \phi=40°, z=3)$ is given as $E = 100a_\rho - 200a_\phi + 300a_z$ V/m Determine the incremental work require to move a $20\mu C$ charge a distance of $6\mu m$:

a) in the direction of a_ρ: The incremental work is given by $dW = -qE \cdot dL$, where in this case $dL = d\rho a_\rho = 6\times10^{-6}$ a_ρ. Thus

$dW = -(20\times10^{-6}C)(100 V/m)(6\times10^{-6}m) = -12\times10^{-9} J = -12 nJ$

b) in the direction of a_ϕ: in this case $dL = 2d\phi a_\phi = 6\times10^{-6}$ a_ϕ, and so $dW = -(20\times10^{-6})(-200)(6\times10^{-6})$

$= 2.4\times10^{-8} J = 24 nJ$

c) in the direction of a_z: Here, $dL = dz a_z = 6\times10^{-6} a_z$, and so $dW = -(20\times10^{-6})(300)(6\times10^{-6}) = -3.6\times10^{-8} J = -36 nJ$

d) in the direction of E: Here, $dL = 6\times10^{-6} a_E$, where

$$a_E = \frac{100a_\rho - 200a_\phi + 300a_z}{[100^2 + 200^2 + 300^2]^{1/2}} = 0.267a_\rho - 0.535a_\phi + 0.802a_z$$

Thus. (27)

$$dW = -(20\times10^{-6})[100a_\rho - 200a_\phi + 300a_z]\cdot[0.267a_\rho - 0.802a_z](6\times10^{-6}) = \underline{-44.9\,J}$$

(e) In the direction of $G = 2a_x - 3a_y + 4a_z$: In this case, $dL = 6\times10^{-6}\,a_G$, where

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2+3^2+4^2]^{1/2}} = 0.371a_x - 0.557a_y + 0.743a_z.$$

So Now

$$dW = -(20\times10^{-6})[100a_\rho - 200a_\phi + 300a_z]\cdot[0.371a_x - 0.557a_y + 0.743a_z](6\times10^{-6}) = -(20\times10^{-6})[37.1(a_\rho\cdot a_x) - 55.7(a_\rho\cdot a_y) - 74.2(a_\phi\cdot a_x) + 111.4(a_\phi\cdot a_y) + 222.9](6\times10^{-6})$$

(4.2) Let $E = 400a_x - 300a_y + 500a_z$ in the neighborhood of point $P(6,2,-3)$ find the incremental work done in moving a 4-c charge a distance of 1 mm in the direction specified by:

a) $a_x + a_y + a_z$: we write

$$dW = -qE\cdot dL = -4(400a_x - 300a_y + 500a_z)\cdot\frac{(a_x+a_y+a_z)}{\sqrt{3}}(10^{-3})$$

$$= \frac{(4\times10^{-3})}{\sqrt{3}}(400-300+500) = \underline{-1.39\,J}$$

ⓑ $-2a_x+3a_y-a_z$: The computation is similar to that of part a. but we change the direction:

$$dW = -qE\cdot dL = -4(400a_x - 300a_y + 500a_z)\cdot \frac{(-2a_x+3a_y-a_z)}{\sqrt{14}}(10^{-3})$$

$$= -\frac{(4\times10^{-3})}{\sqrt{14}}(-800-900-500) = \underline{2.35\ J}$$

4.3 if $E = 120a_\rho$ V/m, find the incremental amount of work done is moving a $50\mu m$ charge a distance of 2 mm from:

a) $P(1,2,3)$ toward $Q(2,1,4)$: The vector along this direction will be $Q-P = (1,-1,1)$ from

$$dW = -qE\cdot dL = -(50\times10^{-6})\left[120a_\rho \frac{(a_x-a_y+a_z)}{\sqrt{3}}\right](2\times10^{-3})$$

$$= (50\times10^{-6})(120)[(a_\rho\cdot a_x)-(a_\rho\cdot a_y)]\frac{4}{\sqrt{3}}(2\times10^{-3})$$

At P. $\varphi = \tan^{-1}(2/1) = 63.4°$. Thus $(a_\rho\cdot a_x) = \cos(63.4) = 0.447$ and $(a_\rho\cdot a_y) = \sin(63.4) = 0.894$. Substituting these, we obtain $dW = 3.1\ \mu J$.

4.5

Compute the value of $\int_A^P G \cdot dL$ for $G = 2y a_x$ with $A(1,-1,2)$ and $P(2,1,2)$ using the path:

ⓐ Straight line segments $A(1,-1,2)$ to $B(2,1,2)$

in General we could have

$$\int_A^P G \cdot dL = \int_A^P 2y\,dx.$$

The Change in x occurs when moving between B and P during which $y = 1$ Thus

$$\int_A^P G \cdot dL = \int_B^P 2y\,dx = \int_1^2 2(1)\,dx = \underline{2}$$

ⓑ Straight line segments $A(1,-1,2)$ to $C(2,-1,2)$ to $P(2,1,2)$ in this case the change in x occurs

$$\int_A^P G \cdot dL = \int_A^C 2y\,dx = \int_1^2 2(-1)\,dx = \underline{-2}$$

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4.7 Repeat problem 4.6 for $G = 3xy^2 a_x + 2z a_y$
Now things are different in that the path does matter:

a) straight line: $y = x - 1, Z = 1$: we obtain

$$\int G \cdot dL = \int_2^4 3xy^2 dx + \int_1^3 3x(x-1)^2 dx + \int_1^3 2(1) dy = \underline{90}.$$

b) parabola: $6y = x^2 + 2, z = 1$,
we obtain

$$\int G \cdot dL = \int_2^4 3xy^2 dx + \int_1^3 2z\, dy = \int_2^4 \frac{1}{12} x(x^2+2)^2 dx + \int_1^3 2(1) dy = \underline{80 + 2 = 82}$$

The End: