NAME	M. Afnan
ID	7895
Section	"A"
Semister	4^{th}
paper	Differential Equastion
Sign	
Department	Civil Engineering

(i) $w = \sin(x+ct) + \cos(2x+2ct)$

Given

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2} \longrightarrow (1)$$

Now $\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}\left[\sin(x+ct) + \cos(2x+2ct)\right]$

$$= \frac{\partial}{\partial t}\left(\sin(x+ct)\right) + \frac{\partial}{\partial t}\left(\cos(2x+2ct)\right)$$

$$\frac{\partial w}{\partial t} = c\cos(x+ct) - 2c\sin(2x+2ct)$$

Now $\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left[c\cos(x+ct) - 2c\sin(2x+2ct)\right]$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)}$$

Now $\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left[\sin(x+ct) + \cos(2x+2ct)\right]$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - 2\sin(2x+2ct)$$

$$\Rightarrow \frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left[\cos(x+ct) - 2\sin(2x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

(1) $\Rightarrow$ $-c^2\sin(x+ct) - 4c^2\cos(2x+2ct) = c^2 \left[-\sin(x+ct) - 4\cos(2x+2ct)\right]$

$$\cancel{-c^2\sin(x+ct)} - 4c^2\cos(2x+2ct) = \cancel{-c\sin(x+ct)} - 4c^2\cos(2x+2ct)$$

$$0 = 0 \quad (\text{Satisfied})$$

(ii) $w = \tan(2x+ct)$

Now $\frac{\partial w}{\partial t} = c\sec^2(2x+ct)$

& $\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(c\sec^2(2x+ct)\right)$

$$\Rightarrow \quad c^2 \cdot 2 \sec^2(2x+ct)\tan(2x+ct)$$

Now $$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct)$$

(1) $$\Rightarrow \quad 4c^2 \cancel{\sec^2(2x+ct)\tan(2x+ct)} = 4c^2 \cancel{\sec^2(2x+ct)\tan(2x+ct)}$$

$$0 = 0$$

(satisfied)

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Given function is

$$f(x) = \begin{cases} x \; ; & -\pi < x \leq 0 \\ 2x \; ; & 0 \leq x \leq \pi. \end{cases}$$

we have to find the four series co-efficient a_0, a_n and b_n

Now $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx = \frac{1}{\pi} \int_{-\pi}^{0} x\, dx$

$$+ \frac{1}{\pi} \int_{0}^{\pi} 2x\, dx$$

$$= \frac{1}{\pi} \left[\frac{x^2}{2} \right]_{-\pi}^{0} - \frac{2}{\pi} \left[\frac{x^2}{2} \right]_{0}^{\pi}$$

$$= \frac{1}{\pi} \left[0 - \frac{\pi^2}{2} \right] + \frac{2}{\pi} \left[\frac{\pi^2}{2} - 0 \right]$$

$$a_0 = -\frac{\pi}{2} + \pi$$

$$\boxed{a_0 = \frac{\pi}{2}} \longrightarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi} 2x\cos nx)dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$= \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos nx}{n^2}\right] + \frac{2}{\pi}$$

$$\left[\frac{\cos nx}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$\boxed{a_n = \begin{cases} \frac{-2}{\pi n^2} ; & \text{if } n \text{ is odd} \\ 0 ; & \text{if } n \text{ is even} \end{cases}} \rightarrow (2)$$

$$b_n = \frac{1}{\pi}\left[-\frac{\pi \cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi \cos n\pi}{n}\right] = \frac{-3\cos n\pi}{n}$$

$$= \frac{3(-1)^{n+1}}{n}$$

So the required fourier Series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2x-1)x}{(2n-1)^2} + 3\sum (-1)^{n+1}$$

$$\frac{\sin nx}{n}$$

Q3

$$y'' - 4y' - 13y = 8\sin 3x \quad y(0) = 1$$
$$y'(0) = 2$$

Sol: Associated Homogeneous eq of 1 is

$$y'' - 4y' + 13y = 0 \longrightarrow (2)$$

change (2) into Auxiliary equation

put $y = m$ in eq (2)

$$m^2 - 4m + 13 = 0$$

use quadratic formula

$a = 1$, $b = -4$, $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$\Rightarrow \quad m = \frac{4 \pm \sqrt{16-52}}{2}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm \sqrt{36i}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

$$m_1 = 2 + 3i$$

$$m_2 = 2 - 3i$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \rightarrow (1)$$

Let $y_p = A \cos 3x + B \sin 3x \rightarrow (X)$

Differentiate with respect to "x"

$$y_p' = -3A \sin 3x + 3B \cos 3x$$

$\Rightarrow$ Again Differential w.r.t "x"

$$y''_p = -9A\cos 3x - 9B\sin 3x \qquad 2$$

put in eq (1)

$$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos 3x) + 13(A\cos 3x + B\sin 3x) = 8\sin 3x$$

$$\Rightarrow -9A\cos 3x - 12B\cos 3x + 13A\cos 3x - 9B\sin 3x + 12A\sin 3x + 13B\sin 3x = 8\sin 3x$$

$$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$$

$$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$$

Comparing Co-efficient

$\sin 3x \Rightarrow \quad 4B - 12A = 8 \rightarrow (a)$

$\cos 3x \Rightarrow \quad 4A - 12B = 0 \qquad 4A = 12B$

$$\Rightarrow \boxed{A = 3B} \rightarrow (b)$$

$\Rightarrow$ Put B in (a)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$\boxed{B = 1/5} \rightarrow (c)$

Put c in (b)

$\Rightarrow \boxed{A = 3/5} \rightarrow (d)$

Put (c) and (d) in (K)

$y_p = \frac{3}{5} \cos 3x + \frac{1}{5} \sin 3x \rightarrow (B)$

The G. Sol is

$y = y_c + y_p$

$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5} \cos 3x$

$+ 1/5 \sin 3x \rightarrow (C)$

Now we need to find the values of c_1 and c_2

for this,

put $x = 0$ and $y = 1$ in (C)

$$1 = e^{x(2)}\left(c_1 \cos 3(0) + c_2 \sin 3(0)\right) + \frac{3}{5} \cos 3(0) + \frac{1}{5} \sin 3(0)$$

$$1 = \left(c_1(1) + c_2(0)\right) + \frac{3}{5}(1) + \frac{1}{5}(0))$$

$$1 = c_1 + 3/5$$

$$c_1 = 1 - \frac{3}{5}$$

$$\boxed{c_1 = 2/5} \longrightarrow **$$

Differentiate (C) w.r.t "x"

$$y' = c_1\left(2e^{2x}\cos 3x - 3e^{2x}\sin 3x\right) + c_2$$
$$\left(2e^{2x}\sin 3x + 3e^{2x}\cos 3x\right)$$
$$-\frac{6}{5}\sin x + \frac{3}{5}\cos 3x \longrightarrow (1)$$

Put $y' = 2,\ x = 0$ in (1)

$$y' = c_1\left(2e^{2x}\cos 3x - 3e^{2x}\sin 3x\right)$$
$$+ c_2\left(2e^{2x}\sin 3x + 3e^{2x}\cos 3x\right)$$
$$-\frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$$

Put $y' = 2,\ x = 0$

$$2 = c_1\left(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)\right)$$
$$+ c_2\left(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)\right)$$
$$-\frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0)$$

$$2 = c_1(2) + c_2(3) - 0 + \frac{3}{5}$$

$$2 = 2c_1 + 3c_2 + \frac{3}{5}$$

put $c_1 = \frac{2}{5}$

$$\Rightarrow \quad 2 = \frac{4}{5} + 3c_2 + \frac{3}{5}$$

$$2 = \frac{7}{5} + 3c_2$$

$$3c_2 = 2 - \frac{7}{5}$$

$$3c_2 = \frac{3}{5}$$

$$\boxed{c_2 = 3/15} \longrightarrow ***$$

put (**) and (***) in (C)

$$y = e^{2x}\left(2/5 \cos 3x + \frac{3}{15} \sin 3x\right)$$

$$+ \frac{3}{5} \cos 3x + \frac{1}{5} \sin 3x$$

$$\boxed{y = \frac{2}{5} e^{2x} \cos 3x + \frac{3}{15} e^{2x} \sin 3x + \frac{3}{5} \cos 3x + 1/5 \sin 3x}$$

↳ Required General solution.

"

Sol :

It is already in Symbolic form

$(D^2 - DD')z = \cos x \cos 2y \rightarrow (a)$

Put AE $D^2 - DD' = 0$

As we know

$\frac{D}{D'} = m$ i.e $D = m, D' = 1$

$\Rightarrow m^2 - m = 0$

$m = 0, 1$

Therefore $C.F = f(y) + f_2(y+x)$

From eq (a)

$$PI = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$= \frac{1}{2} \frac{1}{D^2 - DD'} 2 \cos x \cos 2y$$

As

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$C.f = f_1(y-x) + x f_2(y-x)$$

$$PI = \frac{1}{D^2 + 2DD' + D'^2}\left[2(y-x) + \sin(x-y)\right]$$

$$= \frac{1}{(D+D')^2}\left[2(y-x) + \sin(x-y)\right]$$

By General Method

$$m = -1, \quad y - x = c$$

$$= \frac{1}{D+D'} \int \left[2c + \sin(-c)\right] dx$$

$$= \frac{1}{D+D'} \left[2cx - (\sin c)x\right]$$

Replacing c by $y-x$

$\Rightarrow$ Again put $y - x = c$

$= \int (2xc - x \sin c)\, dx \Rightarrow cx^2 - \frac{x^2}{2} \sin c$

$= x^2(y-x) - \frac{x^2}{2} \sin(y-x) = x^2 y - x^3$

$+ \frac{x^2}{2} \sin(x-y)$.

Hence the required

Solution is

$z = C.F + P.I = f_1(y-x) + x f_2(y-x) + x^2 y$

$- x^3 + \frac{1}{2} x^2 \sin(x-y)$

The End