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PAPER :- Intro to structural dynamics & Earth quake Engg

Submitted to :-

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Question. No: 1

Solution:

The general E.O.M. SDOF System

$$Ku + c\dot{u} + m\ddot{u} = p(t)$$

in our case system is undamped (C=0) undergoing free vibration p(t) = 0.

Hence general equation of motion

$$Ku + m\ddot{u} = 0 \quad \text{---(1)}$$

$$K = 3EI/L^3$$

$$= \frac{3 \times 29000\ K/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$$\boxed{K = 7.55\ K/in}$$

in order to eliminate the chances of mistake during calculation of it is more appropriate to use fundamental units like lb, ft, sec or kg, m, sec

$K = 7.55\ K/m = 90625\ lb/ft$

$m = \frac{7277\ lb\,sec^2}{32.2 ft} = \boxed{225.50\ slug}$

$W_n = \sqrt{K/m} = \sqrt{\frac{90625}{225.50}} = \boxed{20.04\ rad/sec}$

$T_n = \frac{2\pi}{W_n} = \frac{2 \times 3.14}{20.04} = \boxed{~~0.314~~\ sec}$ $\boxed{0.313\ sec}$

Subsitting the corresponding value in eq (1)

90625 ~~90625~~ $u + 225.5\ddot{u} = 0$

$u_t = u(0)\cos(W_n t) + \frac{\dot{u}(0)}{W_n} \sin(W_n t)$

$u(0) = 1/2'' = 1/24$ and $\dot{u}(0) = 0$

$u(t) = (1/24) \times \cos(20.04t) + 0 = 1/24 \times \cos(20.04t)$

Equilent static force at any time "t" is

$F_s(t) = Ku(t) =$ ~~90625~~ $\frac{90625\cos(20.04t)}{24}$

$F(t) = 3776 \cos(80.04t)$

Amplitude of dynamic displacement u_0 for undamped undamped free vibration is

$$u_0 = \sqrt{\left[u(0)^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2\right]}$$

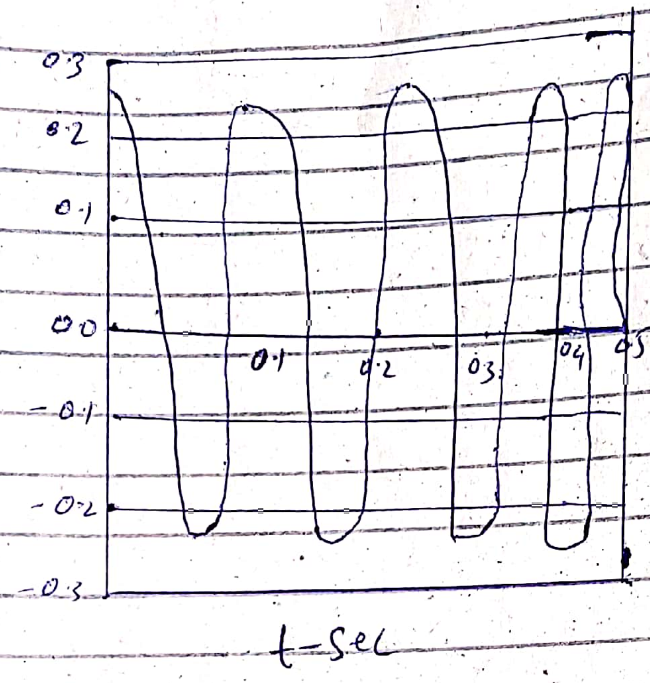
$$= \sqrt{\left(\left(\frac{1}{24}\right)^2 + 0\right)}$$

$$= \boxed{\frac{1}{24} \text{ ft}}$$

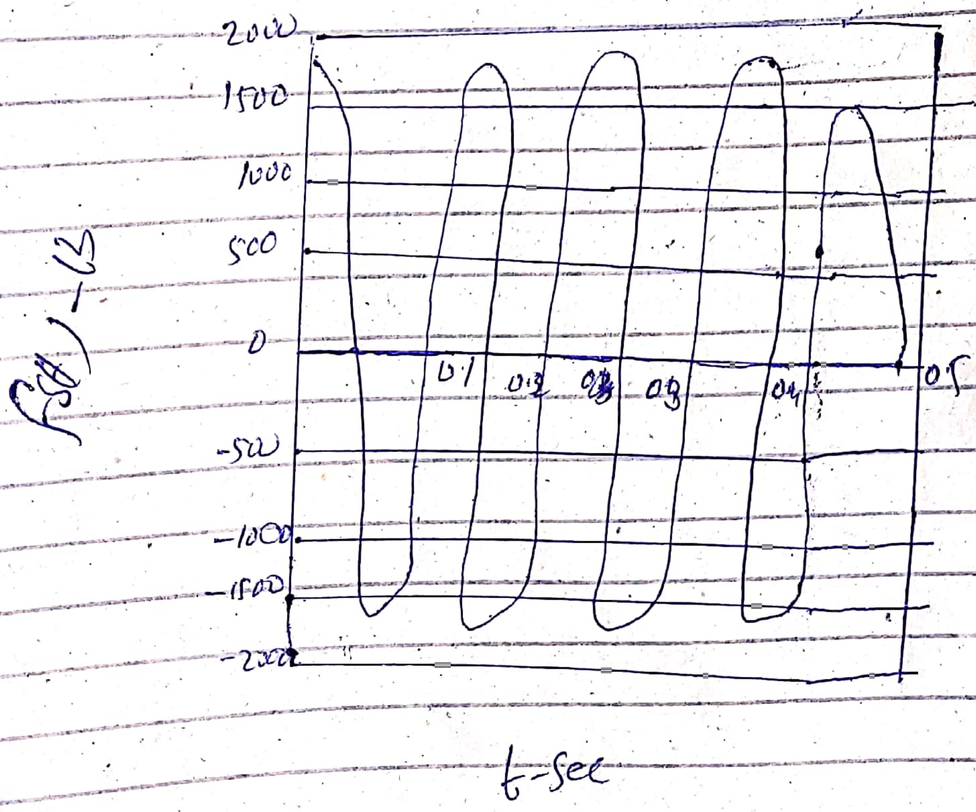
Amplitude equilent Static Force

$K u_0 = 90625 \times \frac{1}{24} = \boxed{3776 \text{ lb}}$

undempadest free Vibration



undamped free Vibration



Question No: 2

Solution:

E.O.M for damped free vibration

$Ku + C\dot{u} + m\ddot{u} = 0$ —— (1)

$K = 90625$ lb/ft

$m = 225.50$ lb/sec²/ft

$C = \zeta \times 2 m \omega_n =$

$= 2 \times 225.50 \times 20.04 \times 0.025$

~~2×225.50×0.025~~

~~C = 11.275 lb.sec/ft~~

$= 225.95$ lb.sec/ft

By substituting value of k and m in (1)

$90625u + 225.95\dot{u} + 225.50\ddot{u} = 0$

Solution to the E.O.M for damped free vibration

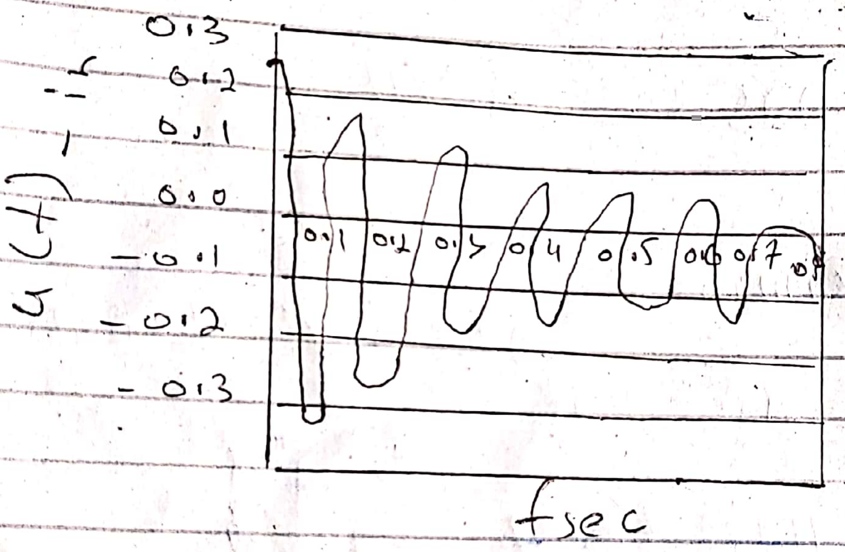
$u(t) = e^{-\zeta \omega_n t} \left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D} \left[\dot{u}(0) + u(0)\zeta\omega_n \right] + \sin(\omega_D t) \right]$

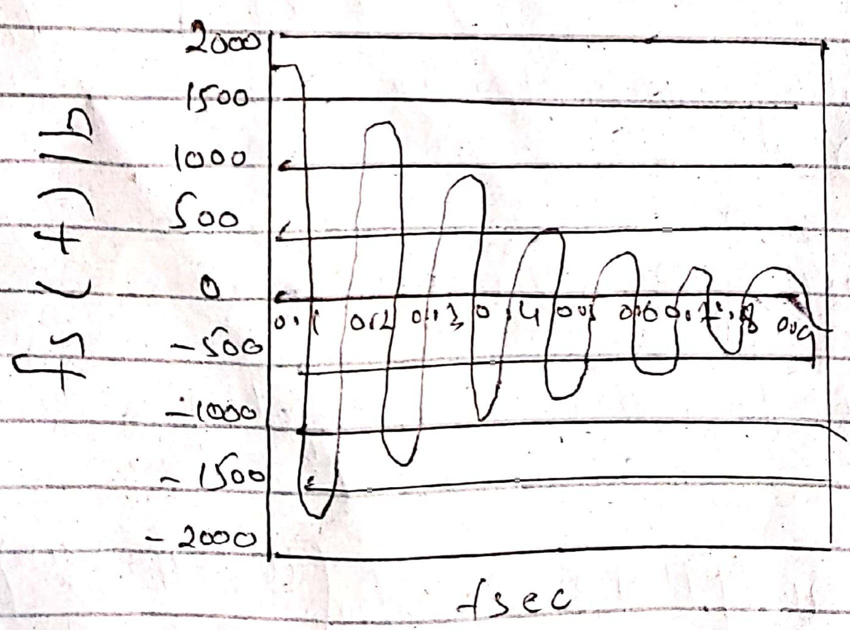
$\omega d = 20.04 \ rad/sec$

$$u(t) = e^{-0.025 + 20.04}\left[\frac{1}{24}\cos(20.04t) + \frac{1}{20.04} + \left[0 + \frac{1}{24} \times 0.0025 \times 20.9\right.\right.$$

$$\times \sin(20.04)\,]$$

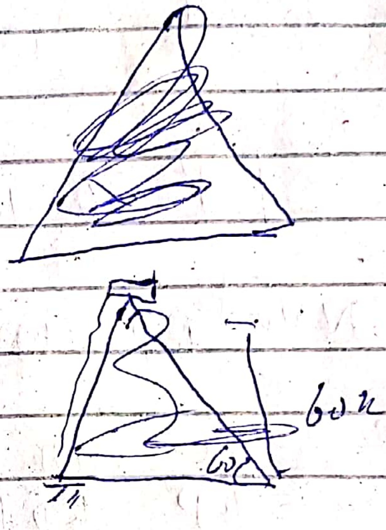
$$u(t) = e^{0.501}\left[0.041\cos(20.04t) + 0.0.0.355(20.04t)\right]$$





Q No. 3

Ans:



cycle = 7

~~T = [illegible]~~

T = 3.57 sec

Amplitude of displacement

(1) Damping ratio

$u_i = 7.277$

$$\zeta = \frac{1}{2\pi j} \ln\left[\frac{u_i}{u_{j+1}}\right]$$

$$7 = \frac{1}{(2)(3.14)\zeta} \ln\left[\frac{7.270}{0.09}\right]$$

$$43.96\zeta = \ln(8.077)$$

$$\zeta = \frac{2.08}{43.96} = \boxed{\zeta = 0.047}$$

(b) $T_n = ?$

7 cycle of vibration are completed is 3.57 sec
the required to complete one cycle $\frac{3.57}{7} = T_p$

$T_p =$ ~~0.51~~ sec 0.51 sec

Now $w_p = w_n \sqrt{1-\xi^2}$

$$\frac{2\pi}{w_p} = \frac{2\pi}{w_n \sqrt{1-\xi^2}}$$

$$T_p = \frac{T_n}{\sqrt{1-\xi^2}}$$

$= 0.51 \sqrt{1-(0.04)^2}$

$= 0.51 \sqrt{0.998}$

$T_n = 0.509$ sec

(c) $K = ?$

$$K = \frac{60 \cos 60^\circ}{7.27}$$

$K = 4.126$ K/in

~~[crossed out]~~ 4943904/21

$\omega_n = \sqrt{k/m} = k/(\omega/g)$

$= \sqrt{kg/\omega}$

$\omega = kg/\omega_n^2$

$\omega = 2\pi/T_n$

$\omega = \frac{kg\, T_n^2}{4\pi^2}$

$\omega = \frac{49439\ lb}{ft} \times \frac{32.2\ ft}{s^2}$

$\omega = 1591.93\ (0.00658)$

$= 10475.72\ lb$

$\omega = 10.45\ k$

$C = ?$

$\zeta = \frac{C}{2m\omega_n}$

$C = \zeta 2m\omega_n$

$\zeta 2m(2\pi/T_n)$

$C = \frac{(0.0475)4(3.14)\left(\frac{10475.7}{32.2}\right)}{0.5}$

$C = 380.31\ lb/in$

No of cycle to reduce displacement

Amplitude from 7.27 to 0.5 in $j = ?$

$$j = \frac{1}{2\pi\xi} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$= \frac{1}{2\pi(0.04)} \ln\left(\frac{7.27}{0.5}\right)$$

$$j = \frac{1}{0.251}(2.67)$$

$$\boxed{J = 3.98} \text{ or } 9 \text{ cycles}$$