Assignment

Q.No.1

(i). The order of matrix A is **m × p** and the order of B is **p × n.**

Then the order of matrix AB is?

ANS: The order of matrix AB is **m × n.**

(ii). The number of non-zero rows in an Echelon form?

ANS: The number of non-zero rows in an Echelon form is the Rank of the matrix.

(iii). If B=$\begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then a=?

Solution:

As |B|=0 ,

SO $\begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = 0$

(1 * a)-(2*4)=0

a-8=0

a=8 ANS

(iv). If A=$\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then |A| = ?

Solution:

|A| = $\begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$$|A|=(2i * -i)-(i * i)$$

$$|A|= -2i^2 - i^2 \qquad i^2 = -1$$

$$|A|= -3(-1)$$

|A|= 3 ANS.

(v). The matrix A=$\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

ANS: The matrix A=$\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is Scalar Matrix; because the diagonal elements are same and non-diagonal are zero.

(vi). Solution of $\frac{dy}{dx}$ +2xy=y?

Ans. Solution of $\frac{dy}{dx}$ +2xy=y is $\mathbf{\ln y + x^2 + x = c}$.

(vii). The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

ANS: Order is 1, while the degree is 0 because the differential equation in not polynomial in derivative.

(viii). The order and degree of differential equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is?}$$

ANS: Order is 2, while the degree is 0 because the differential equation in not polynomial in derivative.

(ix). The differential equation $2\frac{dy}{dx}$ + x^2y + = 2x+3, y(0)=5 is ?

ANS: The differential equation $2\frac{dy}{dx}$ + x^2y + = 2x+3, y(0)=5 is ,

$\frac{1}{2}\ \mathbf{ln}\ |5 - \frac{y}{x}|$ **- x=0**

(x). $\begin{vmatrix} 1 & a & a2 \\ 1 & b & b2 \\ 1 & c & c2 \end{vmatrix}$ **is?**

Solution:

$\begin{vmatrix} 1 & a & a2 \\ 1 & b & b2 \\ 1 & c & c2 \end{vmatrix}$ **= $1(bc^2 - b^2c) - a(c^2 - b^2) + a^2(c-b)$**

$\begin{vmatrix} 1 & a & a2 \\ 1 & b & b2 \\ 1 & c & c2 \end{vmatrix}$ **= $bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$**

$\begin{vmatrix} 1 & a & a2 \\ 1 & b & b2 \\ 1 & c & c2 \end{vmatrix}$ ***= $a^2(c-b) + b^2(a-c) + c^2(b-a)$ ANS.***

Q.N0.2

(i). Express the Determinant

$$\begin{vmatrix} a & b & c \\ a2 & b^2 & c^2 \\ a3 & b^3 & c^3 \end{vmatrix}$$

As the product of factors which are linear in a, b, c.

Solution:

$$\begin{vmatrix} a & b & c \\ a2 & b^2 & c^2 \\ a3 & b^3 & c^3 \end{vmatrix} = a(c^3b^2 - b^3c^2)\ -b(c^3a^2 - a^3c^2)\ +c(b^3a^2 - a^3b^2)$$

$$\begin{vmatrix} a & b & c \\ a2 & b^2 & c^2 \\ a3 & b^3 & c^3 \end{vmatrix} = ab^2c^3 - a\,b^3c^2 - a^2b\,c^3 + a^3b\,c^2 + a^2\,b^3c - a^3b^2c$$

$$\begin{vmatrix} a & b & c \\ a2 & b^2 & c^2 \\ a3 & b^3 & c^3 \end{vmatrix} = c^3(ab^2 - a^2b) - b^3(a\,c^2 - a^2c) + a^3(b\,c^2 - b^2c)$$ ***ANS.***

(II). Find the Eigen value $A=\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Solution: The Characteristic matrix for A is,

$$A-\lambda I=\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A-\lambda I = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$A-\lambda I = \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

$$A-\lambda I= -\begin{bmatrix} \lambda-2 & 1 & 1 & 0 \\ 1 & \lambda-3 & 1 & 1 \\ 1 & 1 & \lambda-3 & 1 \\ 0 & 1 & 1 & \lambda-2 \end{bmatrix}$$

$$\lambda I-A= \begin{bmatrix} \lambda-2 & 1 & 1 & 0 \\ 1 & \lambda-3 & 1 & 1 \\ 1 & 1 & \lambda-3 & 1 \\ 0 & 1 & 1 & \lambda-2 \end{bmatrix}$$

The Characteristic equation is given by, If

$|\lambda I-A|=0$

$$\begin{vmatrix} \lambda-2 & 1 & 1 & 0 \\ 1 & \lambda-3 & 1 & 1 \\ 1 & 1 & \lambda-3 & 1 \\ 0 & 1 & 1 & \lambda-2 \end{vmatrix} = 0$$ Add all columns to the first one,

$$\begin{vmatrix} \lambda & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \lambda & \lambda-3 & \mathbf{1} & \mathbf{1} \\ \lambda & \mathbf{1} & \lambda-3 & \mathbf{1} \\ \lambda & \mathbf{1} & \mathbf{1} & \lambda-2 \end{vmatrix} = 0$$ factor out λ in the first column,

$$\lambda \begin{vmatrix} 1 & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ 1 & \lambda-3 & \mathbf{1} & \mathbf{1} \\ 1 & \mathbf{1} & \lambda-3 & \mathbf{1} \\ 1 & \mathbf{1} & \mathbf{1} & \lambda-2 \end{vmatrix} = 0$$ R4-R1,

$$\lambda \begin{vmatrix} 1 & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ 1 & \lambda-3 & \mathbf{1} & \mathbf{1} \\ 1 & \mathbf{1} & \lambda-3 & \mathbf{1} \\ 0 & \mathbf{0} & \mathbf{0} & \lambda-2 \end{vmatrix} = 0$$ R3-R1,

$$\lambda \begin{vmatrix} 1 & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ 1 & \lambda-3 & \mathbf{1} & \mathbf{1} \\ 0 & \mathbf{0} & \lambda-4 & \mathbf{1} \\ 0 & \mathbf{0} & \mathbf{0} & \lambda-2 \end{vmatrix} = 0$$ R2-R1,

$$\lambda \begin{vmatrix} 1 & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ 0 & \lambda-4 & \mathbf{0} & \mathbf{1} \\ 0 & \mathbf{0} & \lambda-4 & \mathbf{1} \\ 0 & \mathbf{0} & \mathbf{0} & \lambda-2 \end{vmatrix} = 0$$ factor λ multiply back to the first column,

$$\begin{vmatrix} \lambda & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ 0 & \lambda-4 & \mathbf{0} & \mathbf{1} \\ 0 & \mathbf{0} & \lambda-4 & \mathbf{1} \\ 0 & \mathbf{0} & \mathbf{0} & \lambda-2 \end{vmatrix} = 0$$

$\lambda - 2 = 0$, $\lambda - 4 + 1 = 0$, $\lambda - 4 + 1 = 0$, $\lambda + 1 + 1 = 0$

$\boldsymbol{\lambda} = \mathbf{2}, \boldsymbol{\lambda} = \mathbf{3}, \boldsymbol{\lambda} = \mathbf{3}, \boldsymbol{\lambda} = -\mathbf{2}$ are the Eigen values.

Q.No.3

The rate of change in the form of differential equation is given by,

(x^2+ $3y^2$)dx – 2xydy =0, Find the general solution at x= 2 and y=6.

Solution: Given (x^2+ $3y^2$)dx – 2xydy =0,

We can write it as,

(x^2+ $3y^2$)dx –=2xydy By cross multiplication,

$$\frac{dy}{dx} = \frac{2xy}{x^2 + 3y^2}$$

$$\text{Let } y = vx\,,\ \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{2xy}{x^2 + 3y^2}$$

$$v + x\frac{dv}{dx} = \frac{2x(vx)}{x^2 + 3(vx)^2}$$

$$v + x\frac{dv}{dx} = \frac{2vx^2}{x^2 + 3v^2x^2}$$

Taking common x^2 from nominator and denominator,

$$v + x\frac{dv}{dx} = \frac{x^2(2v)}{x^2(1 + 3v^2)} \text{ So,}$$

$$v + x\frac{dv}{dx} = \frac{(2v)}{(1 + 3v^2)}$$

$$x\frac{dv}{dx} = \frac{(2v)}{(1 + 3v^2)} - v$$

After taking LCM,

$$x\frac{dv}{dx}=\frac{(2v)-v-3v^3}{(1+3v^2)}$$

$$x\frac{dv}{dx}=\frac{v-3v^3}{(1+3v^2)}$$

After converting into separable form,

$$\frac{1+3v^2}{(v-3v^3)}\ dv = \frac{1}{x}dx$$

Taking Integration Both sides,

$$\int \frac{1+3v^2}{(v-3v^3)}\ dv=\int \frac{1}{x}dx$$

After doing Partial of $\frac{1+3v^2}{(v-3v^3)} = v + \frac{2v}{(3v^2+1)}$

$$\int v + \frac{2v}{(3v^2+1)}\ dv = \ln x + c$$

$$\frac{v^2}{2}+\int \frac{2v}{(3v^2+1)}\ dv = \ln x + c$$

multiply and divide 3 to left side,

$$\frac{v^2}{2}+\frac{1}{3}\int \frac{6v}{(3v^2+1)}\ dv = \ln x + c$$

$$\frac{v^2}{2}+\frac{1}{3}\ln(3v^2+1)=\ln x + c\text{-------------(A)}$$

Putting $v=\frac{y}{x}$ in (A)

$$\frac{(\frac{y}{x})^2}{2}+\frac{1}{3}\ln(3(\frac{y}{x})^2+1)=\ln x + c$$

$$\frac{(y)^2}{2x^2}+\frac{1}{3}\ln((\frac{3y}{x})^2+1)=\ln x + c\text{-----(B)}$$

As x=2 and y=6

$$\frac{(6)^2}{2(2)^2}+\frac{1}{3}\ln((\frac{3*6}{2})^2+1)=\ln 2 + c$$

$$\frac{36}{8}+\frac{1}{3}\ln(82)=0.69+c$$

4.5+0.33(4.40)-0.69=c

C=5.24 Putting in B,

$$\frac{(y)^2}{2x^2}+\frac{1}{3}\ln((\frac{3y}{x})^2+1)=\ln x+5.24$$

Required Particular solution.

The End