Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title: Electrical Network Analysis **Module:** 4th
Instructor: **Dr. Shehryar Shafique** **Total Marks:** 30

Student Details

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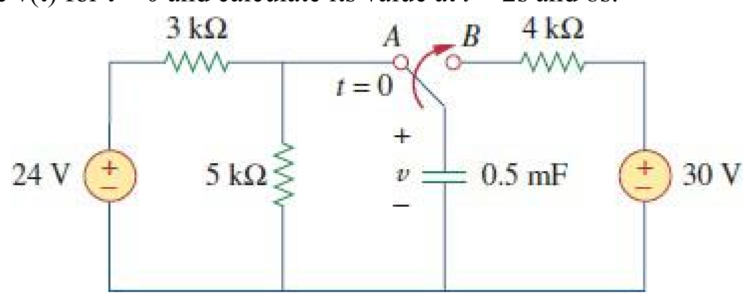
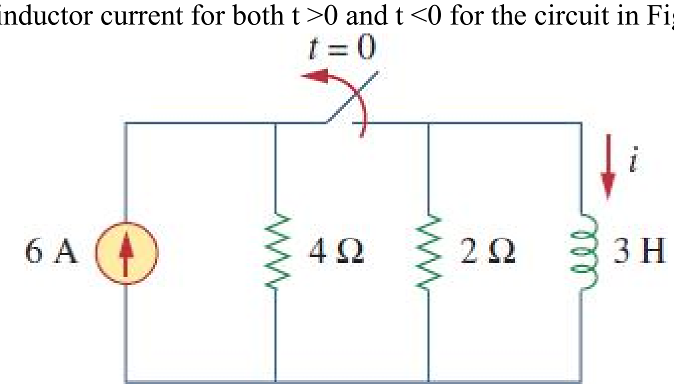
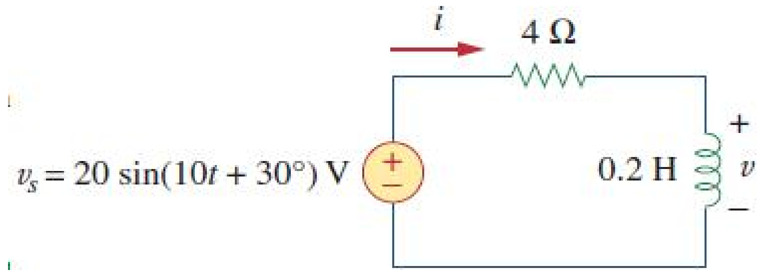
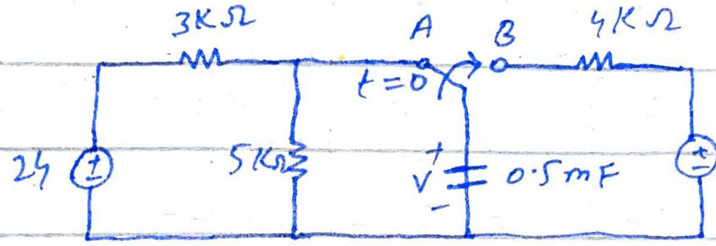
Q1.		The switch in Fig. 1 has been in position *A* for a long time. At t = 0 the switch moves to *B*. Determine v(t) for t > 0 and calculate its value at *t* = 2s and 8s.  **Figure 1**	Marks 06 CLO 01
Q2.		Determine the inductor current for both t >0 and t <0 for the circuit in Fig. 2.  **Figure 2**	Marks 06 CLO 01
Q3.		A series RLC circuit is described by ___ + __ +_ = Find the response when L = 0.5 H , R = 4 Ω and C = 0.2 F. Let $i(0) = 1$, $di(0)/dt = 0$	Marks 06 CLO 01

Q4.		A series RLC circuit has R = 100Ω, L = 240 H and C = 10mF. If the input voltage is v(t) = 10cos 2t, find the current flowing through the circuit.	Marks 06 CLO 03
Q5.		Find v(t) and i(t) in the circuit shown in figure 3.  **Figure 3**	Marks 06 CLO 03

Q1 :- The switch in Fig. 1 has been in position A for a long time. At t = 0 the switch move to B. Determine v(t) for t > 0 and calculate it's value at t = 2s and 8s

Ans :-



Sol

$$v(0) = \frac{5}{5+3}(24) = 15V$$

using the fact the capacitor voltage cannot change instantaneously

$$v(0) = v(0^-) = v(0^+) = 15V$$

For t > 0 the switch in position B. The thievens resistance connected to the capacitor is $R_{Th} = 4K\Omega$ and time constt is

$$\tau = R_{Th}C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2s$$

Since the capacitor acts like open circuit to dc at steady state

$v(\infty) = 30 V$. Thus.

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

(2) (4)

$= 30 + (15 - 30)e^{-t/2} = (30 - 15e^{-0.5t})$

At $t = 2s$

$v(2) = 30 - 15e^{-0.5(2)}$

$v(2) = 30 - 15e^{-1}$

$v(2) = 30 - 15(0.3678)$

$v(2) = 30 - 5.517$

$\boxed{v(2) = 24.483}$

At $t = 8s$

$v(8) = 30 - 15e^{-0.5(8)}$

$v(8) = 30 - 15e^{-4}$

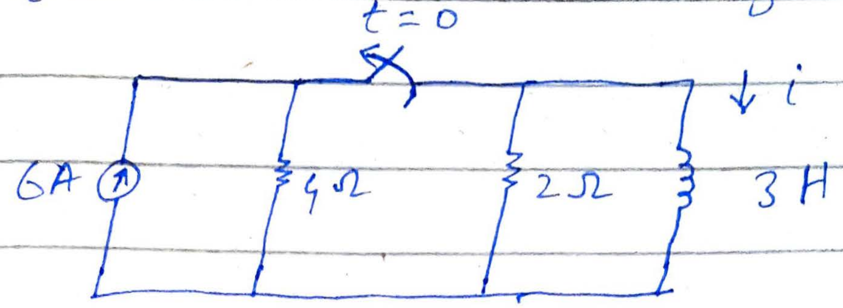
$v(8) = 30 - 15(0.0183)$

$v(8) = 30 - 0.2745$

$v(8) = 29.7255$

$\times$ $\times$

Q2:- Determine the Inductor current for both $t > 0$ and $t < 0$ for the circuit in fig. 2



Sol

For $t < 0$

The switch is closed and inductor acts as short circuit therefore inductor current is

$$\boxed{i = 6A}$$

For $t > 0$

The switch is open and time constant $\tau = L/R$

$$\tau = \frac{3}{2}$$

Now the inductor current

$$i(t) = 6e^{-t/\tau}$$

$$i(t) = 6e^{-t/(3/2)}$$

$$i(t) = 6e^{-2t/3}\, u(t) \text{ A}$$

Q3 :- A series RLC circuit is described by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Find response when $L = 0.5H$, $R = 4\Omega$ & $C = 0.2F$

Let $i(0) = 2$, $di(0)/dt = 0$

Sol

Given data

$L = 0.5H$

$R = 4\Omega$

$C = 0.2F$

$i(0) = 2A$

$\frac{di(0)}{dt} = 0$

voltage of the given RLC circuit is described by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Divided by L

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{10}{L}$$

For R.H.S of the equation multiply by $\frac{C}{C}$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{10C}{LC}$$

$C = 0.2F$, thus

$$\frac{d^2 i}{dt^2} + \frac{R\,di}{L\,dt} + \frac{i}{LC} = \frac{2}{LC}$$

subsititute,

$$\frac{d^2 i}{dt^2} + 8\frac{di}{dt} + 20\,i = 20 \quad \ldots (1)$$

general equation of a RLC circuit is given by

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}\,i = \frac{I_s}{LC} \ldots (2)$$

compare (1) & (2)

$$\frac{R}{L} = 8 \rightarrow (3)$$

$$\frac{1}{LC} = 20 \rightarrow (4)$$

$$\frac{I_s}{LC} = 20 \rightarrow 5$$

From eq 3 α is given by

$$\alpha = \frac{R}{2L} = \frac{8}{2} = 4 \text{ rad/s} \rightarrow (6)$$

$$w_0 = \sqrt{\frac{1}{LC}}$$

From eq (4)

$$w_0 = \sqrt{10} \text{ rad/s} \rightarrow 7$$

$\therefore$ The circuit is over damped

$$S_1 = -\alpha + \sqrt{\alpha^2 - w_0^2}$$

$$= -4 + \sqrt{4^2 - \sqrt{10}^2}$$

$$= -4 + \sqrt{6} \text{ rad/s}$$

And

$$S_2 = -\alpha - \sqrt{\alpha^2 - w_0^2}$$

$$= -4 + \sqrt{4^2 - \sqrt{10}^2}$$

$$= -4 - \sqrt{6} \text{ rad/s}$$

From eq 5 the steady state current is given by

$$I_s = 20 \times LC = 20 \times 0.5 \times 0.2 = 2A \rightarrow 8$$

$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t > 0 \;-\; 9$

$t = 0,$

$i(0) = I_s + A_1 + A_2$

$1 = 2 + A_1 + A_2$

Thus

$A_1 + A_2 = -1 \longrightarrow 10$

From eq (9) find $\frac{di(t)}{dt}$

$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$

$t = 0,$

$\frac{di(0)}{dt} = A_1 s_1 + A_2 s_2$

Substitute the value

$(-4 + \sqrt{6}) A_1 + (-4 - \sqrt{6}) A_2 = 0 \longrightarrow (11)$

Solve (10) & (11) simultaneously

$A_1 = -1.326$

$A_2 = 0.326$

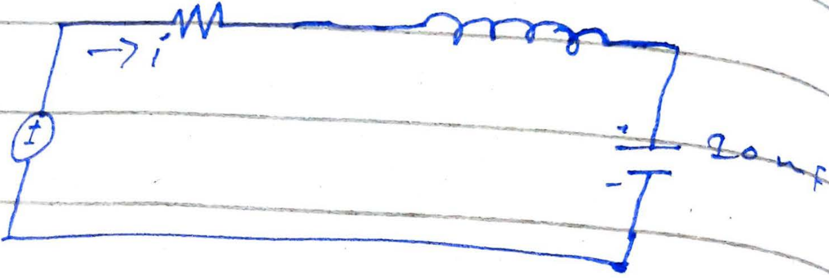
Substitute in 9

$i(t) = 2 - 1.326 e^{(-4+\sqrt{6})t} + 0.326 e^{(-4-\sqrt{6})t}$ A

$i(t) = 2 - 1.326 e^{(-4+\sqrt{6})t} + 0.0326 e^{(-4-\sqrt{6})t}$ A

Q4:- A series RLC circuit has $R = 200\ \Omega$, $L = 240\ H$ & $C = 20\ mF$. If the input voltage is $v(t) = 20 \cos 2t$, find the current flowing through the circuit.

circuit diagram :-



Input voltage is $= $

$v(t) = 20 \cos 2t\ V$

Here

Amplitude $= v_m = 20\ V$

Angular frequency, $\omega = 2$ rad/s

phase angel, $\theta = 0°$

So phase for the voltage $v(t)$

$v(t) = 20 \angle 0°\ V$

(10) (9)

Now for inductive reactance

$X_L = \omega L$

so $\omega = 2$ rad/s , $L = 240H$

$X_L = (2)(240)$

$= 480\ \Omega$

Now for capacitive reactance

$X_C = \frac{1}{\omega C}$

$\omega = 2$ rad/s , $C = 10$ mF

$\frac{1}{2(10 \times 10^{-3})}$

$\frac{1}{2 \times 10^{-2}}$

$\frac{1 \times 10^{2}}{2}$

$\frac{100}{2}$

$X_C = 50\ \Omega$

(6)

Now for Impedence

$$Z = R + jX_L - jX_C$$

$R = 100\,\Omega$, $X_L = 480\,\Omega$, $X_C = 50$

$$Z = (100 + j480 - j50)\,\Omega$$

$$(100 + j430)\,\Omega$$

Represent Z in phase form

$$Z = (100 + j430)\,\Omega$$

$$= \sqrt{(100)^2 + (430)^2} \angle \tan^{-1}\left(\frac{430}{100}\right)$$

$$\sqrt{10000 + 184900} \angle \tan^{-1}(4.3)$$

$$= 441.47 \angle 76.90^\circ\,\Omega$$

for current flowing

$$i = \frac{v(t)}{Z}$$

(11)

$$v(t) = \frac{20\angle 0^\circ \text{ V}}{441.47\angle 76.9^\circ \Omega}$$

$$\frac{20}{441.47}\angle[0-76.9]\text{A}$$

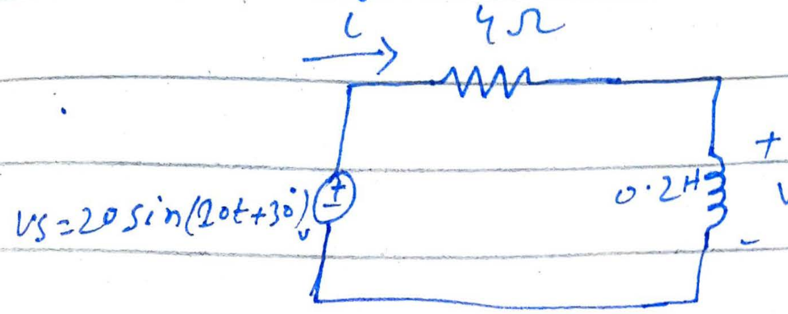
$$= 22.6\times 10^{-3}\angle -76.9^\circ \text{ A}$$

general expression for (i)

$$i = 22.6\cos(2t - 76.9^\circ)\text{ mA}$$

* * *

Q5:- Find v(t) and i(t) in the circuit shown in figure 3



Sol

From the voltage source

$(V_s) = 20 \sin(10t + 30^\circ)$ V

$v_s = 20 \cos(10t + 30^\circ - 90^\circ)$ V

$v_s = 20 \cos(10t - 60^\circ)$ V

$V_s = 20 \angle -60^\circ$ V

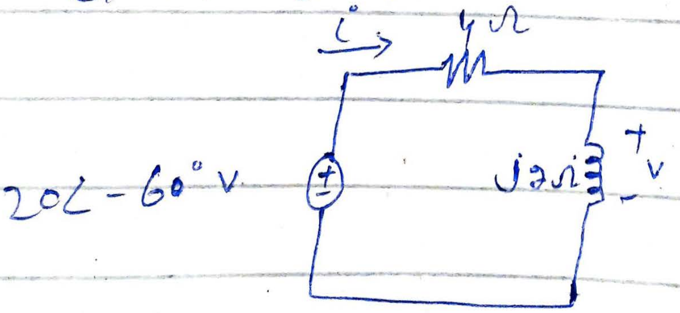
$\omega = 10$ rad/sec

$X_L = j \omega L$

$0.2\,H = j \times 10 \times 0.2$

$0.2\,H = j2\,\Omega$

Given circuit can be represent as.



From the circuit diagram

$Z = 4 + j2 \ \Omega$

Hence, the current is,

$$I = \frac{20 \angle -60^\circ}{4 + j2}$$

$$I = \frac{20 \angle -60}{\sqrt{4^2 + 2^2} \angle \tan^{-1}\left(\frac{2}{4}\right)}$$

$$I = \frac{20 \angle -60^\circ}{4.472 \angle 26.57^\circ}$$

$$I = 4.472 \angle -86.57^\circ$$

convert into time domain

$i(t) = 4.472 \cos(20t - 86.57^\circ)$

$i(t) = 4.472 \sin(10t - 86.57^\circ + 90^\circ)$

$i(t) = 4.472 \sin(10t + 3.43^\circ) A$

From the circuit voltage across the inductor is,

$v = j2 \times i$

$v = j2 \times (4.472\angle -86.57^\circ)$

convert polar form to rectangular form we get

$v = j2 \times (0.26756 - j4.464)$

$V = 8.928 + j0.53512$

from rectangular to polar form

$$\Rightarrow v = \left(\sqrt{(8.926)^2 + (0.53512)^2}\right) \angle \tan^{-1}\left(\frac{0.5312}{8.928}\right)$$

Now into time domain

$$v(t) = 8.944 \cos(10t + 3.4^\circ)$$

$$v(t) = 8.944 \sin(10t + 3.4^\circ + 90^\circ)$$

$$v(t) = 8.944 \sin(10t + 93.4^\circ) \text{ V}$$

Thank you