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Final Term Assignment

LINEAR ALGEBRA

Total Marks: 50

Submitted to :

Sir.Muhammad Shakeel

Submitted by :

Muhammad Islam

ID = 6844

BS (SE) Section B (8^{th} semester)

(1)

Q - No 1:→ Determine if The following system is Consistent or not

Solution

My ID = 6844

$$x_1 - 4x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

$$= \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 5 & -5 & 10 \end{array}\right]$$

$$R_3 - 5R_1 = \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 40 & -10 & 10 \end{array}\right]$$

$$R_2/4,\ R_3/10 = \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -1 & 1 \end{array}\right]$$

$$R_3 - 4R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -15 & 15 \end{array}\right]$$

so That consistent because of This traingle

$$-15x_3 = -15$$

$$x_3 = 1$$

$$x_2 - 4x_3 = 4$$

$$x_2 = 4 + 4x_3$$

$$x_2 = 8$$

$$x_1 - 8x_2 + x_3 = 0$$

$$x_1 = 8x_2 - 8_3$$

$$x_1 = 64 - 1$$

$$x_1 = 65$$

So

$$x_1 = 65$$
$$x_2 = 8$$
$$x_3 = 1$$

→ **Answer**

(3)

Q No 2:- Find The inverse of

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 10-4 \\ 5 & -2 & 7 \end{bmatrix}$$ by Adjoint Methode

SOLUTION :>

Find A^{-1}

My ID = 6844
4 Digits = (4)

$$|A| = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= 3 \times \begin{vmatrix} -1 & 4 \\ -2 & 7 \end{vmatrix} - 4 \times \begin{vmatrix} 2 & 4 \\ 5 & 7 \end{vmatrix} + 5 \times \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$= 3 \times (-1 \times 7 - 4 \times (2)) - 4 \times (2 \times 7 - 4 \times 5) - 5 \times (-2) \times (-2) - (-1) \times 5)$$

$$= 3 \times (-7+8) - 4 \times (14-20) + 5 \times (-4+5)$$

$$= 3(1) - 4(-6) + 5(1) = 3+24+5 = 32$$

$$Adj(A) = Adj \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{vmatrix} -1 & 4 \\ -2 & 7 \end{vmatrix} & -\begin{vmatrix} 2 & 4 \\ 5 & 7 \end{vmatrix} & +\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} \\ -\begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} & +\begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} \\ +\begin{vmatrix} 4 & 5 \\ -1 & 4 \end{vmatrix} & -\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} & +\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} \end{bmatrix}$$

$$Adj(A) = \begin{bmatrix} 1 & 6 & 1 \\ -38 & -4 & 26 \\ 21 & -2 & -11 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -38 & 21 \\ 6 & -4 & -2 \\ 1 & 26 & -11 \end{bmatrix}$$

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Now $A^{-1} = \frac{1}{|A|} \times Adj(A)$

Put The value In This formula

$$= \frac{1}{32} \times \begin{bmatrix} 1 & -38 & 21 \\ 6 & -4 & -2 \\ -1 & 26 & -11 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0312 & -1.1875 & 0.6562 \\ 0.1875 & -0.125 & -0.0625 \\ 0.0312 & 0.8125 & -0.3438 \end{bmatrix} = A^{-1}$$ Answer

So That A^{-1} is that

* ——— * ——— * ——— *

Q No 3 :> Solve The following — Guass-jordon Method.

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

SOLUTION :>

Given

$$2x + 2y + 4z = 18 \longrightarrow (1)$$
$$x + 3y + 2z = 13 \longrightarrow (2)$$
$$3x + 2y - 3z = 14 \longrightarrow (3)$$

There are 3 equation

converting The given equation into matrix form.

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

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$R_1 = R_1 + 2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$R_2 = R_2 - R_1 \longrightarrow = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$R_3 = R_3 - 3 \times R_1 \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & -13 \end{array}\right]$$

$$R_2 = R_2 \div 2 \longrightarrow = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right]$$

$$R_1 = R_1 - R_2 \longrightarrow = \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & 11 \end{array}\right]$$

$R_3 = R_3 \div (-9)$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right]$$

$$R_1 = R_1 - 2 \times R_3 \rightarrow = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 41/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right]$$

i-e $x = 41/9$

$y = 2$

$z = 11/9$

So That solution by

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Guass-Jordon Method Answer is

$x = 41/9, \quad y = 2 \quad \& \quad z = 11/9.$ → Answer.

Q-No. 4 = Show that this matrix is Diagonisable $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

Answer

We can be diagonalized if there exists an invertible matrix P & diagonal matrix D such that $A = PDP^{-1}$

Here $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

First we find eigen value of the matrix A.

$|A - \lambda| = 0$

$= \begin{vmatrix} (4-\lambda) & 2 & -2 \\ -5 & (3-\lambda) & 2 \\ -2 & 4 & (1-\lambda) \end{vmatrix} = 0$

$= (4-\lambda)((3-\lambda)\times(1-\lambda) - 2\times 4) - 2((-5)\times(1-\lambda) - 2\times(-5)) \times 4(3-$

$= (4-\lambda)((3-4\lambda+\lambda^2)-8) - 2((-5+5\lambda)-(-4)) - 2((-20)-(-6+2\lambda))$

$= (-20-11\lambda+18\lambda^2-\lambda^3) - (-2+10\lambda) - (-28-4\lambda) = 0$

$= (-\lambda^3+8\lambda^2-17\lambda+10) = 0$

$= (\lambda-1)(\lambda-2)(\lambda-5) = 0$

$= (\lambda-1)=0$ or $(\lambda-2)=0$ or $(\lambda-5)=0$

The eigen value of matrix given by $\lambda = 1, 2$

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1 - Eigen vectors for $\lambda = 1$ -

$$(A - \lambda) = \begin{bmatrix} 4 & 2 & -2 \\ 5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix}$$

Now Reduce This matrix Interchanging row $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} -5 & 2 & 2 \\ 3 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix}$$

$$R_1 \leftarrow R_1 \div (-5) = \begin{bmatrix} 1 & -0.4 & -0.4 \\ 3 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix}$$

$$R_1 \leftarrow R_1 + 0.4 \times R_2$$

$$= \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & 0.25 \\ 0 & 3.2 & -0.8 \end{bmatrix}$$

$$R_3 \leftarrow R_3 - 3.2 \times R_2$$

$$= \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.25 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with the eigenvalue $\lambda = 1$

$$v = \begin{pmatrix} 0.5 x_3 \\ 0.25 x_3 \\ x_3 \end{pmatrix} \quad \text{Let } x_3 = 1$$

$$v_1 = \begin{pmatrix} 0.5 \\ 0.25 \\ 1 \end{pmatrix}$$

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2- Eigenvector for $\lambda = 2$

$$(A - \lambda I) = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -2 \\ -5 & 1 & 2 \\ -8 & 4 & 1 \end{bmatrix}$$

$$R_3 = R_3 - 2.4 \times R_3$$

$$= \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with The eigen value $\lambda = 2$

$$= x_1 - 0.5x_1 = 0 \quad x_2 - 0.5x_3 = 0$$

$$= x_1 = 0.5x_3 \ , \ x_2 = 0.5x_3$$

$$= v = \begin{bmatrix} 0.5x_3 \\ 0.5x_3 \\ x_3 \end{bmatrix}$$

First we have find P^{-1}

1- The diagonal matrix D is Composed of The eigen Values

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Now find P^{-1}

$$|P| = \begin{vmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \times \begin{vmatrix} 1/2 & 1 \\ 1 & 1 \end{vmatrix} - \frac{1}{2} \times \begin{vmatrix} 1/4 & 1 \\ 1 & 1 \end{vmatrix} + 0 \times \begin{vmatrix} 1/4 & 1/2 \\ 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \times \left(\frac{1}{2} \times 1 - 1 \times 1\right) - \frac{1}{2}\left(\frac{1}{4} \times 1 - 1 \times 1\right) + 0 \times \left(\frac{1}{4} \times 1 - \frac{1}{2} \times 1\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} - 1\right) - \frac{1}{2} x \left(\frac{1}{4} - 1\right) + 0 \times \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= -\frac{1}{4} + \frac{3}{4} + 0$$

So we find Adj (A)

$$\text{Adj}(P) = \text{Adj} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & -1/2 & 1/2 \\ 3/4 & 1/2 & -1/2 \\ -1/4 & 0 & 1/8 \end{bmatrix} \rightarrow \text{Adj}(A)$$

Now $P^{-1} = \frac{1}{|P|} \times \text{Adj}(P)$

Put The Value in this formula

$$= \frac{1}{1/8} \times \begin{bmatrix} -1/2 & -1/2 & 1/2 \\ 3/4 & 1/2 & -1/2 \\ -1/4 & 0 & 1/8 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -4 & 4 \\ 6 & 4 & -4 \\ -2 & 0 & 1 \end{bmatrix}$$

$$= P^{-1} = \begin{pmatrix} -4 & -4 & 4 \\ 6 & 4 & -4 \\ -2 & 0 & 1 \end{pmatrix}$$

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Now Verify That $A = PDP^{-1}$

$$P \times D = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$P \times D = \begin{bmatrix} 1/2 \times 1 \times \frac{1}{2} \times 0 - 0 \times 0 & 1/2 \times 0 \times \frac{1}{2} \times 2 \times 0 \times 0 & \frac{1}{2} \times 0 \times \frac{1}{2} \times 0 - 0 \times 5 \\ 1/4 \times 1 \times \frac{1}{2} \times 1 \times 0 & 1/4 \times 0 \times \frac{1}{2} \times 2 \times 1 \times 0 & 1/4 \times 0 + \frac{1}{2} \times 0 \div 1 \times 5 \\ 1 \times 1 \times 1 \times 0 + 1 - 0 & 1 \times 0 + 1 \times 2 \times 1 \times 0 & 1 \times 0 + 1 \times 0 + 1 \times 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 + 0 + 0 & 0 + 1 + 0 & 0 + 0 + 0 \\ 1/4 + 0 + 0 & 0 + 1 + 0 & 0 + 0 + 5 \\ 1 + 0 + 0 & 0 + 2 + 0 & 0 + 0 + 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1 & 0 \\ 1/4 & 1 & 5 \\ 1 & 2 & 5 \end{bmatrix}$$

Now we find $(P \times D) \times (P^{-1}) =$

$$= \begin{bmatrix} 1/2 & 1 & 0 \\ 1/4 & 1 & 5 \\ 1 & 2 & 5 \end{bmatrix} \times \begin{bmatrix} -4 & -4 & 4 \\ 6 & 4 & -4 \\ -2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 6 + 0 & -2 + 4 + 0 & 2 - 4 + 0 \\ -1 + 6 - 10 & -1 + 4 + 0 & 1 - 4 + 5 \\ -4 + 12 - 10 & -4 + 8 + 0 & 4 - 8 + 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

So $P \cdot D \cdot P^{-1} =$

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

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$$P \cdot D \cdot P^{-1} = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Answer is That

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} \rightarrow \text{Matrix Diagonisable}$$

Q NO 5:⇒ Determine if The following homogenous

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 2x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

Solution:⇒

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x = \begin{bmatrix} 4/3 & 8 \\ 0 & \\ 8 & \end{bmatrix} = 8 \begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}$$

So Answer is $= 8 \begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}$

Q - No 6 :→ $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$

Solution :→ First we have Ruduce to Normal form

Given Matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$

Interchanging row $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$R_2 = R_2 - 0.3333 \times R_1$

$$= \begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$R_3 = R_3 - 0.3333 \times R_1$

$$= \begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$R_1 = R_1 - 1.5 \times R_2$

$$= \begin{bmatrix} 3 & 9 & 12 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$R_3 = R_3 + 0.5 \times R_2$

$$= \begin{bmatrix} 3 & 9 & 12 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so That Ruduce form to Normal form

$$\text{Normal form} = \begin{bmatrix} 3 & 9 & 12 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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~~Now Reduce This matrix interchanging~~

Now we find Rank of Matrix

$$\text{Rank}\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Now interchaging The matrix

$R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$R_1 = R_1 \div 3$

$$= \begin{bmatrix} 1 & 3 & 4 & 1 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$= \begin{bmatrix} 1 & 3 & 4 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$R_2 = R_2 \div 2$

$$= \begin{bmatrix} 1 & 3 & 4 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$R_3 = R_3 + R_2$

$$= \begin{bmatrix} 1 & 3 & 4 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So That

The Rank of a Matrix is the Number of all Non-zeros rows.

The Rank of Matrix = (2)

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End of Paper.

End of the Paper