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Subject Structural Dynamic & Earthquake

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Q # 1

Given data ;

distance = 1/2"
E = 29000 ksi
I = 150 in"
S = 7727 lb

Required ;

Natural time Period = ?
Equation of Motion = ?
Amplitude = ?
Graph of Vibration of displacement of with time = ?
Graph of Vibration of equalent Static force with time = ?

Solution ;

As we know that from general equation of motion for self degree of freedon system.

Now;

We also know that from general solution the EOM for undamped free vibration.

$$u(t) = u(0)\cos(w_n t) + \dot{u}(0)/w_n \sin(w_n t)$$

$$u(0) = 1/2'' = 1/24\,ft$$

$$\dot{u}(0) = 0$$

$$u(t) = (1/24) \times \cos(20.1t)$$

Equilant Static force at any time 't' is

$$f_s(t) = k \cdot u(t) = \frac{90625 \times \cos(54.9t)}{24}$$

$$= 3776.042 \cos 54.9t.$$

$$ku + cu + mu = P(t)$$

In this case system is undumped.

$c = 0$, So under going free libration.

$P(t) = 0$

Hence, general EOM become,

$$ku + mu = 0 \quad \text{— (1)}$$

We know that,

$$K = 3EI / L^3$$

$$= \frac{3 \times 2900 \times 150}{(10 \times 12)^3} = 7.55 \, k/N$$

$= 90625$ lb/ft.

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Now;

Mass (m)

$m = F/a = 7727/32.2$

$= 239.96$ slug.

Now we find Wn,

$W_n = \sqrt{k/m} = \sqrt{90625/239.96}$

$= 19.43$ rad/sec

Putting the values in eq (1)

$ku + mu = 0$

$90625u + 31.06u = 0$

Now find time Period;

$T = 2\pi/w_n = 2\pi/19.43$

$= 0.32$ sec

Now we find Amplitude of dynamic displacement u_0 for Undamped free vibration.

$$u_0 = \sqrt{(u(0))^2 + (\dot{u}(0)/\omega_n)^2}$$

$$= \sqrt{(1/24)^2 + 0} = \sqrt{(1/24)^2}$$

$$= 1/24 = 0.42 \text{ ft}$$

Now Amplitude of equilant static force.

$$ku = 90625 \times 1/24$$

$$= 3776.042 \text{ lb}$$

Q#2

Given data ;

distance = 1/2"
E = 29000 ksi
I = 150 in^2
S = 7727 lb
L = 10 ft
Damping ratio = S = 4 %

Required ;

(1) Develop & Solve the equation of motion for vibration at free End = ?

(2) Also develop equation showing vibration in the equilant of static force with time = ?

(3) Draw graph variation of displacement with time & the variation of equilant static force with time.

Solution;

We know that from equation of motion for damped vibration.

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{— (1)}$$

Now, We know that

$$k = \frac{3EI}{L^3} = \frac{3 \times 29000 \times 150}{(10)^3}$$

$$= 7.55 \text{ k/in}$$

$$= 90625 \text{ lb/ft}$$

Now Mass,

$$m = F/a = \frac{7727}{32.2} = 239.96$$

$$= 239.96 \text{ slug}$$

Now we find c

$c = \zeta \times 2m\omega_n$ — (ii)

$= \frac{4}{100} \times 2 \times 239.96 \times \omega_n$

First we find ω_n

$\omega_n = \sqrt{k/m} = \sqrt{90625/239.96}$

$= 19.433$ rad/sec

Now put in — (ii)

$c = \frac{4}{100} \times 2 \times 239.96 \times 19.433$

$= 373.051$

Now put these values in equation 1.

$ku + cu + mu = 0$

$90625u + 373.051u + 239.96u = 0$

Now solution to the EOM for damped free vibration.

$u(t) = e^{-\zeta w_n t}[u(0)\cos(w_D t) + \frac{1}{w_D}(u(0) + u(0)\zeta_n \times \sin w_D t$

$w_D = 19.433$ rad/sec

$w(t) = e^{-0.04 \times 19.433t}[1/24 \times \cos(19.433t)$
$[0 + 1/24 \times 0.04 \times 19.433 \times \sin(19.433t)]$

$e^{-0.804t}[0.042\cos(19.433t) + 0.0017\sin(t\omega)$

$= e^{-0.804}[0.042\cos(19.433t + 0.0017$
$\sin 19.433t)$

Now we know that.

$f(s) = K\,u(t)$

$f(s) = 90625\,(e^{-0.804} \times 0.042 \cos(19.433t) + 0.0017 \sin(19.433t)$

$= e^{-.804}(3806.25 \cos(19.433t) + 154.0625 \sin(19.433t))$

$e^{-.804}(3806.25 \cos(19.433t) + 154.0625 \sin(19.433t))$.

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Q# 3

Given data;

Force = 60 kips
U_1 = 7727/1000 = 7.727 in
After, J = 7 (cycles)
Completed = 3.57 sec

U_{J+1} = 2.286 cm = 0.9 in
Ignore the vertical vibration.

Required;

(a) Damping ratios.
(b) Natural Period of undamped Vibration.
(c) Stifness of structure.
(d) Weight of tank.
(e) Damping co-efficient
(f) Number of cycles to reduce the displacement amplitude to 0.5".

Solution

(a) Damping ratio = ζ = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

Putting values ;

$$7 = \frac{1}{2(3.14)\zeta} \ln\left[\frac{7.727}{0.9}\right]$$

$$\zeta\,(7 \times 2 \times 3.14) = 2.15$$

$$\zeta\,(43.96) = 2.15$$

$$\zeta = 2.15/43.96$$

$$= 0.048$$

$$\zeta = 0.048 = 4.8\ \%$$

(b) $T_n = ?$

As seven cycles are completed in '3.57' sec.

Thus time required to complete one cycle = 7/3.57 = 1.96 sec

$T_D = 1.96$ sec

Now; $\omega_D = \omega_n \sqrt{(1-\zeta^2)}$

$\Rightarrow 2\pi/\omega_D = 2\pi/\omega_n (\sqrt{1-\zeta^2})$

As;

$T_D = T_n / \sqrt{1-\zeta^2}$

$\Rightarrow T_n = T_D (\sqrt{1-\zeta^2})$

$= 1.96 (\sqrt{1-(0.048)^2}$

$T_n = 1.9578$ sec

(Natural Period of undamped vibration)

(c) Stifness of structure; $k = ?$

As; $k = F \cdot \cos\theta / 2$

$$k = \frac{60 \cos(60^\circ)}{2} = 15\ k/in$$

$k = 1800\ lb/ft.$

(d) Weight of tank; $W = ?$

As; $W_n = \sqrt{k/m} = \sqrt{k/(W/g)} = \sqrt{k \cdot g/W}$

$\Rightarrow W_n^2 = k \cdot g/W \Rightarrow (W = k \cdot g/W_n^2)$

Putting values of $W_n = 2\pi/T_n$

$W = k \cdot g/(4\pi^2/T_n^2) = k \cdot g\ (T_n^2/4\pi^2)$

$$W = 1800\ lb/ft \cdot 32.2\ ft/sec^2 \left(\frac{(1.957)^2}{4(3.14)^2}\right)$$

$W = 56284.75\ lb$

$= 56.284\ klb.$

(e) Damping Co-efficient;

It is known as;

$$\zeta = \frac{c}{2m\omega_n}$$

$$\Rightarrow c = \zeta(2m\omega_n) = \zeta\left(2m\left(\frac{2\pi}{T_n}\right)\right)$$

Putting the values;

$$c = \frac{0.048 \times \left(2\left(\frac{56284}{32.2}\right)\right)\left(2(3.14)\right)}{1.9578}$$

$c = 538.261$ lb sec/ft

(f) No of cycle reduce displacement altitude from 7.727 in to 0.5 in.

$J = ?$

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$$j = \frac{1}{2\pi G} \ln \left(\frac{u_1}{u_{j+1}} \right)$$

$$= \frac{1}{2(3.14)(0.048)} \ln \left[\frac{7.727}{0.9} \right]$$

$$= 7.13$$

$$j = 7 \text{ cycles}$$

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