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Sec :: B

Subject :: Differential Equations.

Submitted to :: Miss Shumaila.

Q1

i) The Order of Matrix AB is $\underline{m \times n}$.

ii) The number of non-zero rows in Echelon form is $\underline{One}$.

iii) if $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a Singular matrix Then $a = \underline{8}$.

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ $\because i^2 = -1$

$= -2i^2 - i^2$

$= -2(-1) - (-1)$

$= 2 + 1 \Rightarrow \underline{3}$.

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v) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is Scalar Matrix.

The given because the diagonal elements are Same and non-diagonal are - zero.

vi) Solution of $dy/dx + 2xy = y$?

Sol.

$dy/dx + 2xy = y$

$dy/dx = y - 2xy$

$dy/dx = y(1-2x)$ (y Taking common)

$dy/dx = y(1-2x)$

$dy/dx = (1-2x)\,dx$

take Integration

$\int 1/y \, dy = \int (1-2x)\,dx$

$\ln y = \int 1\,dx - \int 2x\,dx$

$\Rightarrow \ln y = x - 2x^2/2 + c$

$\ln y = x - x^2 + c$

$\ln y = x - x^2 + c$

$e^{\ln y} = e^{x - x^2 + c}$

$\boxed{y = e^{x(1-x)+c}}$ Ans.

vii) The Order and degree of differentinal equation.

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+(dy/dx)^3} \text{ is}$$

Sol:

Order = $\underline{1}$.

Degree = $\underline{3}$.

viii) The Order and degree of differential equation.

$$\frac{d^3y}{dx^2} - 4xy = \sin\left(\frac{dy}{dx^2}\right) \text{ is}$$

Sol:

Order = Two.

Degree = One.

ix) The differential equation $2\,dy/dx + x^2y = 2x+3,$

$y=(0)=5$ is —.

Sol:

$$2y' + x^2y = x^2+3,\ y(0) = 5$$

$$y' + (x^2/2)y = \frac{x^2+3}{2}$$

$$y' + (x^2/2)y = 1/2\,(x^2+3)$$

$$\mu = x^2/2$$

$$e^{\int x^2/2\,dx} = e^{x^3/6}$$

$$e^{x^3/6}y' + e^{x^3/6}(x^2/2)\,y = 1/2\,e^{x^3/6}(x^3+3)$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + c$$

$$y(0) = \frac{0+3}{2} = 3/2$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + 3/2 \text{ Ans.}$$

x)

$$\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

Expand by C_1

$$1 \begin{bmatrix} b & b^2 \\ c & c^2 \end{bmatrix} - 1 \begin{bmatrix} a & a^2 \\ c & c^2 \end{bmatrix} + 1 \begin{bmatrix} a & a^2 \\ b & b^2 \end{bmatrix}$$

$$\Rightarrow 1(bc^2 - cb^2) - 1(ac^2 + a^2c) + 1(ab^2 - a^2b)$$

$$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$$

$$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a) \text{ Ans.}$$

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Q2 Part A:

Express The Determine

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as The product of factors which are linear in a, b, c.

Sol.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 \\ a^3 \end{vmatrix}$$

$$= a\left(b^2c^3 - b^3c^2\right) - b\left(a^2c^3 - a^3c^2\right)$$
$$+ c\left(a^2b^3 - a^3b^3\right)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^3$$

$$+ a^2cb^3 - a^3b^2c$$

Common abc

$$\Rightarrow \quad abc\,(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b)$$

$$\Rightarrow \quad abc\left[bc(c-b) - ac(c-a) + ab(b-a)\right]$$

Ans

$$\Rightarrow \quad 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow 2$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} \quad \text{Expand by } R_1$$

$$\Rightarrow \quad 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big|\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1\big((-1)(2-\lambda) - (-1)(-1)\big)$$

$$- 1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big|$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

Q2 Part B

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}.$$

Sol :.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equ $\longrightarrow |A - \lambda I| = 0 \longrightarrow ①$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 5 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right)$

$= -(3-\lambda+\lambda^2-5\lambda+5)$

$= -\lambda^2+5\lambda-5-3+\lambda$

$= \boxed{-\lambda^2+6\lambda-8} \rightarrow$ (c)

put (a), (b) and (c) in (B)

$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$

$= -2\lambda^3+16\lambda^2-36\lambda+\lambda^4-8\lambda^3+16\lambda^2-8\lambda$

$-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2$

$-36\lambda-8\lambda+6\lambda+6\lambda+16-16$

$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

By Synthatic division

we known date

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$(\lambda=0)$

$\lambda-2=0$

$\Rightarrow \boxed{\lambda=2}$

$\lambda^2-8\lambda+16=0$

By Jaul-jadvan Method.

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)(\lambda-4)$

$\lambda=4\ ,\ \lambda=4$

$\boxed{\lambda_1=0, \lambda_2=2, \lambda_3=4, \lambda_4=4}$ Ans.

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \longrightarrow 1$$

$$\Rightarrow \quad +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow \quad -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow \quad -1\,(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + 1\,(-2 + \lambda - 1)$$

$$\Rightarrow \quad -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= \boxed{-\lambda^2 + 6\lambda - 8} \; \text{——} \; 2$$

$$\Rightarrow \quad -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_2

$$-\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0 \right]$$

①

Q3

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$x = 2,\ y = 6$$

Sol:-

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$\Rightarrow (x^2+3y)\,dx = 2xy\,dy$$

Dividing both Sides by $2xy\,dx$ we get

$$dy/dx = \frac{x^2+3y^2}{2xy}$$

$$dy/dx = x^{\cancel{2}}/2\cancel{x}y + 3y^{\cancel{2}}/2x\cancel{y}$$

$$dy/dx = 1/2\left[x/y + 3y/x\right] \to ①$$

Let

$$y = vx$$

Diff:

$$dy = v\,dx + x\,dv$$

Dividing by dx

Multip. both Sides by $\frac{v}{x(1+v^2)}$

we known That

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on both Sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both Sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

put $v = y/x$

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \longrightarrow ①$$

$dy/dx = v + \; dv/dx \longrightarrow (a)$

put (a) in (*)

$v + \; x\,dv/dx = 1/2$

$$\left[\frac{x^2}{x^2 v} + 3\,\frac{v x^2}{x^2}\right]$$

$$v + \frac{x\,dv}{dx} = 1/2\left[1/v + 3v\right]$$

Multip Both sides by "2"

We known That

$2v + \; 2x\,dv/dx = 1/v + 3v$

$2x\,dv/dx = 1/v + 3v - 2v$

$2x\,dv/dx = 1/v + v$

$$2x\,dv/dx = \frac{1+v^2}{v}$$

Multip... both sides by dx/dv

we known That

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Put $x=2,\ y=6$ in equ (1)

$(4)+(36)=8c$

$c = 40/8$

$\boxed{c=5} \longrightarrow$ put in (1)

So

$x^2+y^2=5x^3$

$y^2=5x^3-x^2$

$y^2=x^2(5x-1)$

Taking " $\sqrt{\ }$ " on both sides

$y=+x\sqrt{5x-1}$, $y=-x\sqrt{5x-1}$ or

$y=\pm x\sqrt{5x-1}$ Ans.