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Subject = DSP

Question No 1 (a).

Determine the response $y(n), n \geq 0$ of system.

Sol:

The characteristics equation is.

$$\lambda^2 - 4\lambda + 4 = 0.$$

$$\lambda = 2, 2 \text{ hence.}$$

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is.

$$y_p(n) = K(-1)^n u(n).$$

Subtituting the solution into difference equation we obtain.

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1).$$

For $n = 2$, $K(1+4+4) = 2 \Rightarrow K = 2/9$

The total solution is

$$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n \right] u(n).$$

From the initial condition we obtain.

$$y(-1) = y(-2) = 0.$$

$$c_1 + 2/9 = 0$$

$$c_1 = -2/9$$

$$2c_1 + 2c_2 - 2/9 = 0.$$

$$2c_2 = \frac{2}{9} - 2c_1.$$

$$\boxed{c_1 = -2/9}$$

$$2c_2 = 2/9 - 2(-2/9)$$

$$= 2/9 + 4/9 = 2+4/9$$

$$2c_2 = 6/9$$

$$c_2 = \frac{6}{9\times 2} = 3/9.$$

$$\boxed{c_2 = 1/3}$$

———x———

Question No 1 (b)
Determine the Impulse response and unit step response.

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2).$$

Sol:

$$\lambda^{n} - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0.$$

$$\lambda^{n-2}[\lambda^2 - 0.7\lambda + 0.1] = 0.$$

$$\lambda^2 - 0.5\lambda - 0.2\lambda + 0.1 = 0.$$

$$\lambda(\lambda - 0.5) - 0.1(\lambda - 0.5) = 0.$$

$$(\lambda - 0.5)(\lambda - 0.1) = 0.$$

$$\lambda = 0.5 \qquad \lambda = 0.1.$$

General form of the Solution to the Homogenous equation is.

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n.$$

$$y_h(n) = C_1 \tfrac{1}{2}^n + C_2 \tfrac{1}{2}^n.$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y_2(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4.$$

Hence $C_1 + C_2 = 2$ and

Impluse response.

$$\tfrac{1}{2}C_1 + \tfrac{1}{5} = 1.4 = \tfrac{7}{5}.$$

$c_1 + \frac{2}{5} c_2 = \frac{14}{5}$.

These equation yield

$c_1 = \frac{10}{3} \qquad c_2 = -\frac{4}{3}$

$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)$.

Step response is:

$s(n) = \sum_{k=0}^{n} h(n-k) \Rightarrow \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$

$= \frac{10}{3} \left[\frac{1}{2}\right]^n \sum_{k=0}^{n} 2^k - \frac{4}{3} \left[\frac{1}{5}\right]^n \sum_{k=0}^{n} 5^k$.

$= \frac{10}{3} \left[\frac{1}{2}\right]^n (2^{n+1} - 1) u(n) - \frac{1}{3} \left[\frac{1}{5}\right]^n (5^{n+1} - 1) u(n)$

—— x ———— x ———— x ——

Question No 2 (a).

Determine the casual signal $x(n)$

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol

$$\frac{X(z)}{z} = \frac{z^2}{(2z-1)(z-1)}$$

$$\frac{X(z)}{z} = \frac{A}{2z-1} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

Find A, B and C.

$A = 4$

$B = 3$

$C = -1$

Hence $x(n) = [4(2)^n - 3 - n]u(n)$.

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Question no 2 (b).

Solution:

we have

$$x(n) = \frac{1}{2\pi j} \oint \frac{z^{n-1}}{(1-az^{-1})} dz$$

$$= \frac{1}{2\pi j} \oint \frac{z^n dz}{(z-a)}.$$

where c is a circle at radius greater than $|a|$ we shall evaluate this target.

1) if $n \geq 0$ $f(z)$ has only zeros and have no poles inside c. The only pole inside c is $z = a$

Hence

$$x(n) = f(z_0) = a^n \quad n = 0.$$

If $n < 0$ $f(z) = z^n$ has an nth-order pole at zero which is also inside c. Thus there are contribution from both poles. For $n = -1$ we have.

$$x(-1) = \frac{1}{2\pi j} \oint \frac{1}{c\, z^2(z-a)} dz.$$

$$\frac{d}{dz}\left(\frac{1}{z-a}\right)\Bigg|_{z=0} + \frac{1}{z^2}\Bigg|_{z=a} = 0.$$

By continuting in the same way. we can show that:

$$x(n) = 0.$$

for $n < 0$ thus

$$x(n) = a^n u(n).$$

———— x ————

Question No 3 (a).

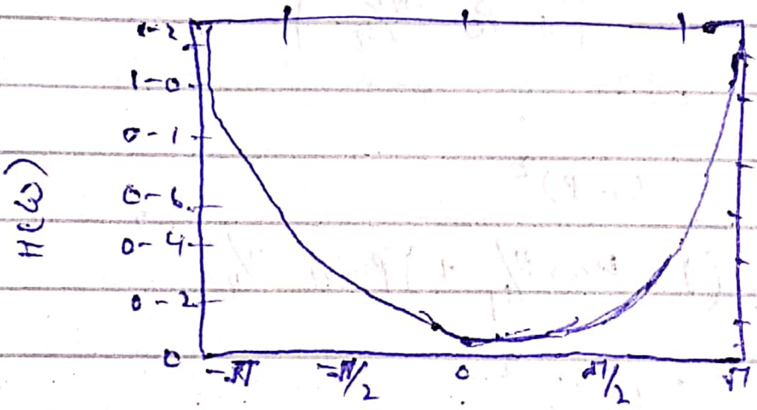
A two-pole low pass has the System response ---. $H(z) = \frac{b_0}{(1-pz^{-1})^2}$ ---

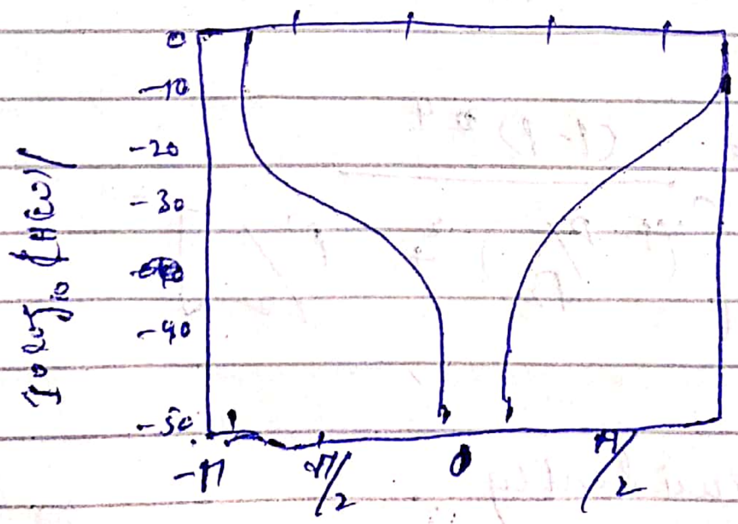
Sol.

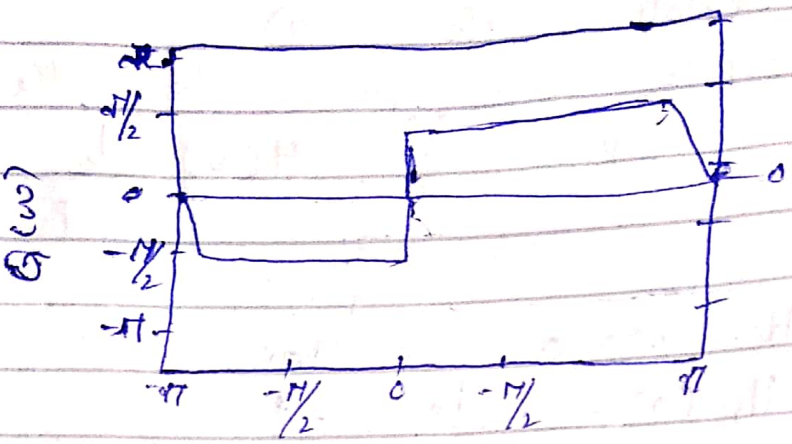
At $w = 0$ we have

$$H_2(0) = \frac{b_0}{(1-p)^2} = 1$$

$$b_0 = (1-p)^2$$







At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{\left(1-Pe^{-j\pi/4}\right)^2}$$

$$= \frac{(1-P)^2}{\left(1 - P\cos\pi/4 + jP\sin\pi/4)\right)^2}$$

$$= \frac{(1-P)^2}{\left(1-P/\sqrt{2} + jP/\sqrt{2}\right)^2}$$

Hence $= \dfrac{(1-P)^4}{\left[\left(1-P/\sqrt{2}\right)^2 + P^2/2\right]^2} = 1/2$

or equilently

$$\sqrt{2}\,(1-P)^2 = 1 + P^2 - \sqrt{2}P$$

The value of P = 0.32 satisfy this equation the system f/n for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

The same priciple can be applied for Design of band pass filters.

x ——————— x

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Question 3(b):

Design a Two-Pole band pass filter.

Solution:

clearly, The filter Must have poles at $P_{1,2} = r e^{\pm j\pi/2}$ and

zeros at $z=1$ and $z=-1$

consequently the system f/n is:

$$H(z) = \frac{G(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{(z^2-1)}{(z^2+r^2)}$$

The Gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$ ... Thus we have.

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$ Thus we have.

$$\left[H\left(\frac{4\pi}{9}\right)\right]^2 = \frac{(1-r^2)^2}{4}\,\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

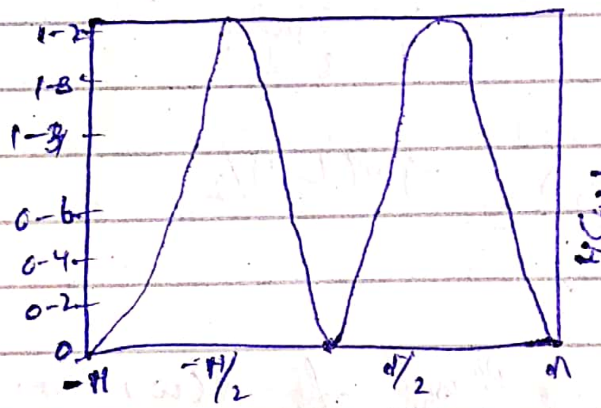
$$= \frac{1}{2}.$$

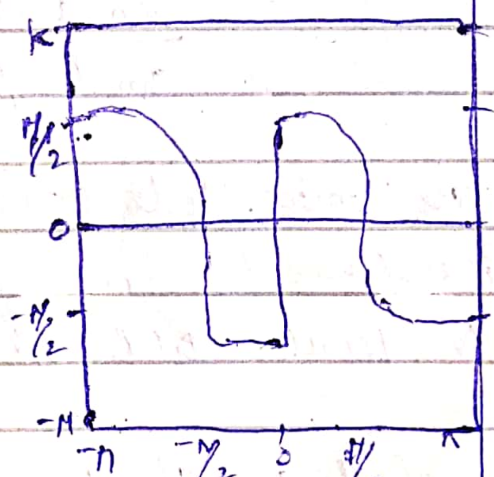
or $1.94(1-r^2)^2 = 1-1.08\,r^2 + r^4$

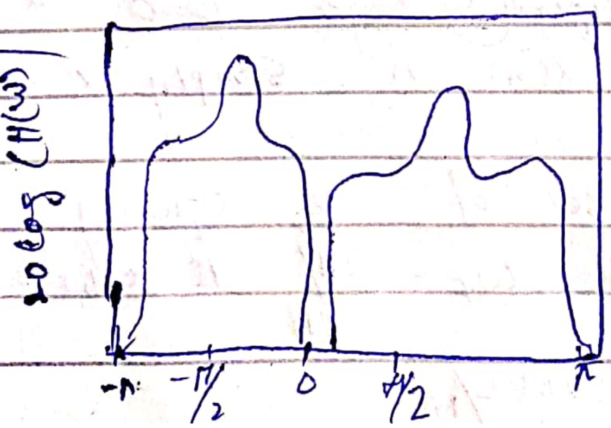
The value of $r^2 = 0.7$ satisfy this equation.

The system f/n for the design filter is.

$$H(z) = \frac{0.15\,(1-z^{-2})}{(1+0.7\,z^{-2})}$$







Question no 4 (a).

A finite duration sequence of length L is given as.

$$x(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{therwise} \end{cases}$$

Determine N point DFT for this sequence for $N \ge L$.

Sol:-

The fourier Transform of this sequence is.

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} \Rightarrow$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}.$$

The magnitude and Phase of $X(\omega)$ are illustrated in figure 1 for L=0 The N-point DFT of $x(n)$ is simply $X(\omega)$.

evaluted at the set of N equally spaced frequencies $\omega_k = \frac{2\pi k}{N}$ $k = 0, 1, 2 \ldots N-1$

$$X(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}} \qquad k = 0, 1, 2 \ldots N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}.$$

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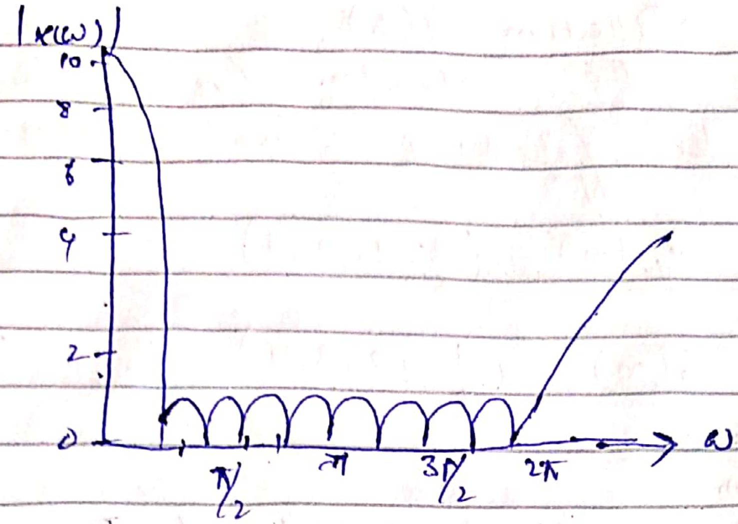


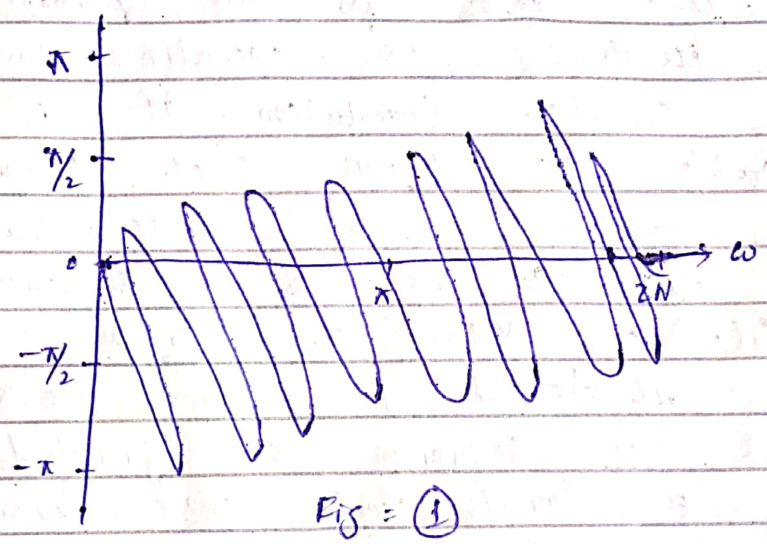


Fig = ①

If N is selected such that N = L. then DFT become,

$$X(K) = \begin{cases} L & k = 0 \\ 0 & k = 1, 2, 3 - L-1 \end{cases}$$

Thus there is only one non zero value in the DFT. This is apparent from observation of $x(\omega) = 0$ at frequencies $\omega_k = 2\pi k/L$ $k \neq 0$ then reader should verify that $x(n)$ can be ~~cat~~ recovered from $x(k)$ by performing an L-Point IDFT.

Question 4(b).

Perform the circular Convolution of the following two Sequences.

$$x_1(n) = (\underset{\uparrow}{2}, 1, 2, 1)$$

$$X_2(n) = (\underset{\uparrow}{1}, 2, 3, 4)$$

Solution..

Each equation consist of four non-zero points for this purpose of illustrating the operation involved in Circular Convolution. It is desirable to graph each Sequence as point as a circle. Thus as illustrated in as the sequence $X_1(n)$ and $X_2(n)$ are graph as illustrated in Figure. we Note that The sequence are graphed in a Counter clock wise direction on a circle. Now $X_3(m)$ is obtained by circularly convoling $X_1(n)$ and $X_2(n)$ as specified by begining with $m=0$ we have.

$$X_3(0) = \sum_{n=0}^{3} X_1(n)\, X_2((-n))_N$$

$X_2((-n))_4$ is simply the sequence $X_2(m)$ folded and graphed on circle illustrated in Fig:

The product sequence is obtained by multiplying $X_1(n)$ with $X_2((1-n))_4$ - Point by Point. This sequence is illustrated in Fig.--

Finally we sum the value in the product sequence to obtain.

$X_3(0) = 14$.

for $m = 1$ we have

$$X_3(1) = \sum_{n=0}^{3} X_1(n) X_2((1-n))_4.$$

It is easily verified that $X_2((1-n))_4$ is simply the sequence $X_2((-n))_4$ rotated counter clockwise by one unit in time as illustrated. This illustrated sequence multiplies $X_1(n)$ to yield the product sequence.

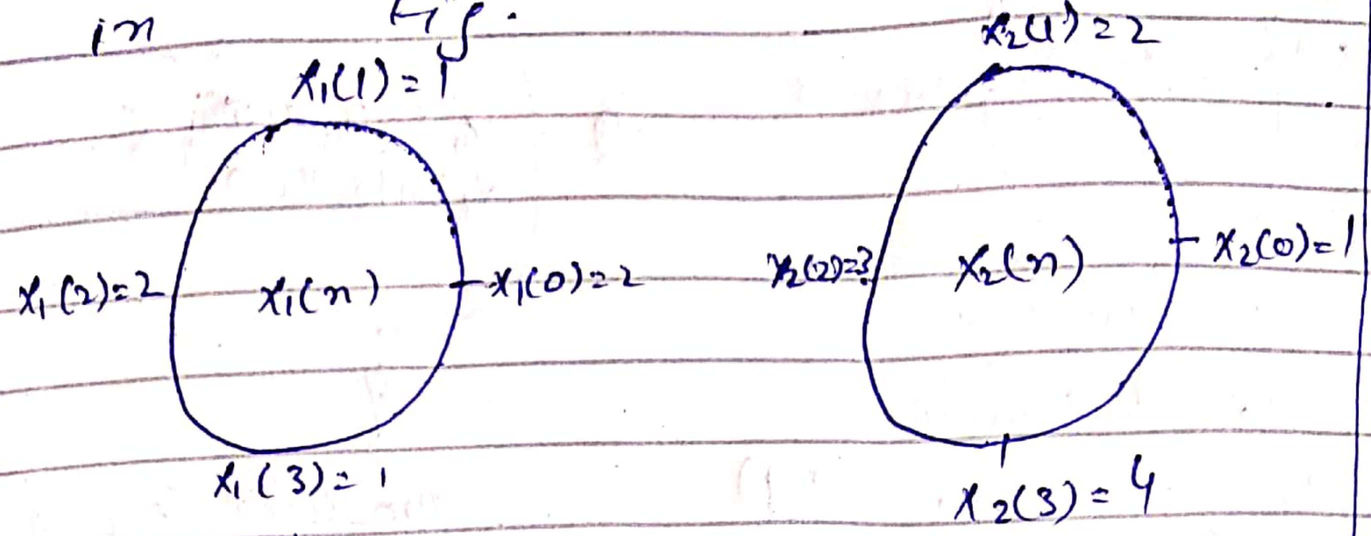
$X_3(1) = 16$

$m = 2$ we have

$$X_3(2) = \sum_{n=0}^{3} X_1(n) X_2((2-n))_4.$$

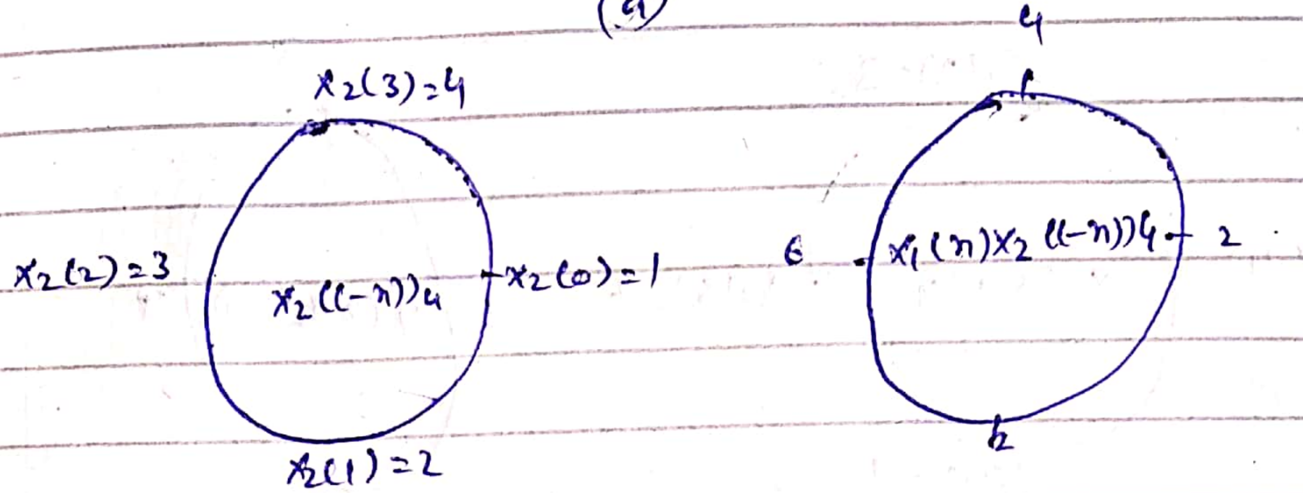
Now $X_2((2-n))_4$ is the folded sequence in Fig. rotated two units of time in the countreclock wise direction.

The resultant sequence is illustrated in Fig.



Folded sequence.

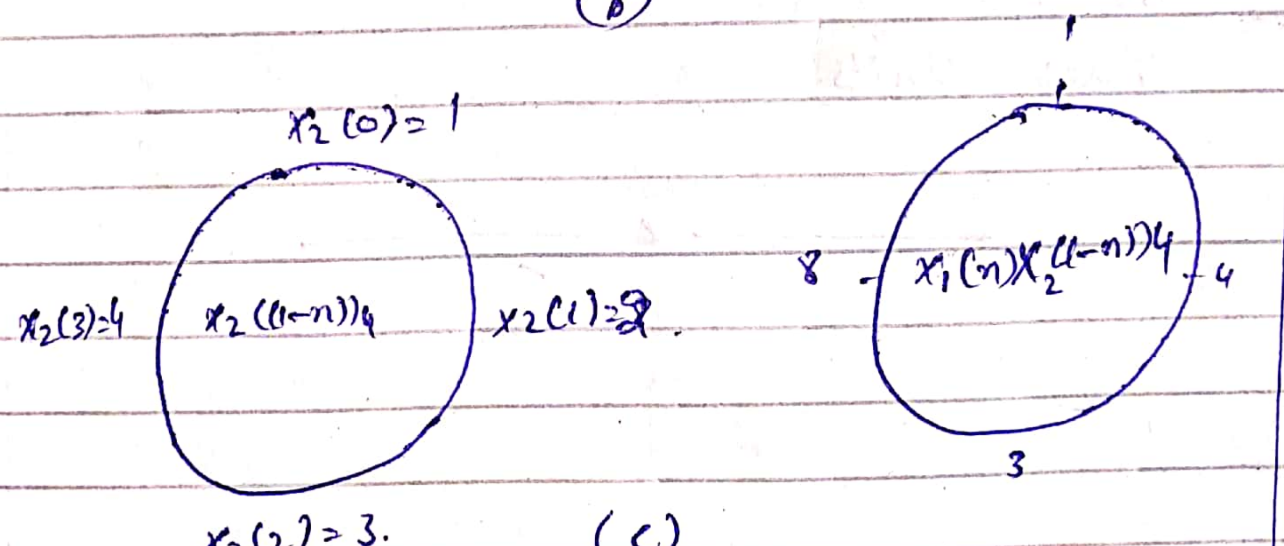
(a)



Folded Sequence

Product Sequence

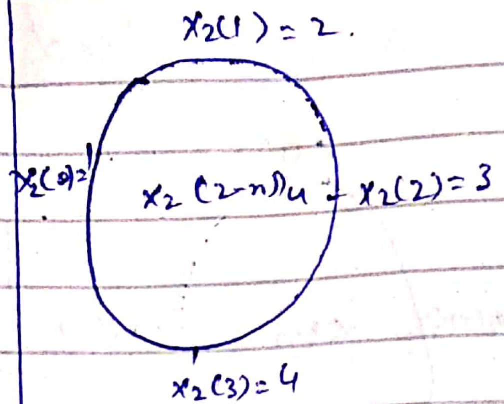
(b)

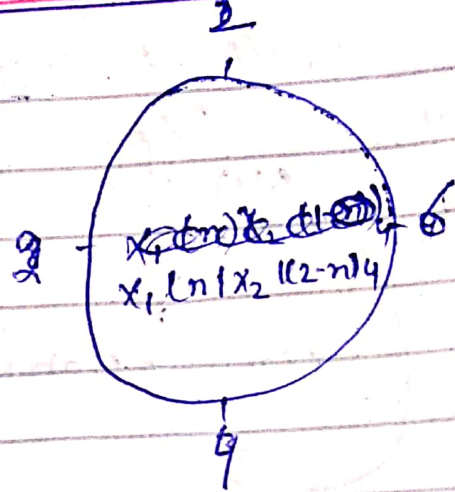


(c)

Folded sequence

Product sequence

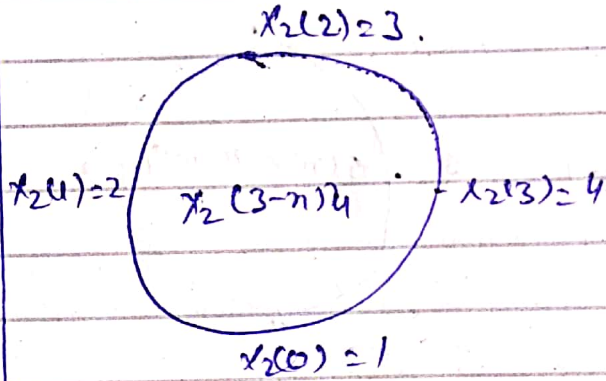


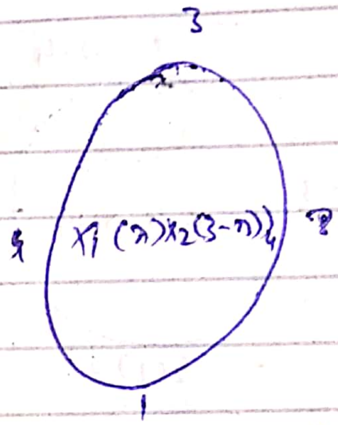


Folded sequence by two unit in Time

(d)

product sequence





Folded sequence rotated by 3 times Units.

product sequence

——— x ———