Q 1)

Ans By Euler's Method :~

Given Data :~

$y(0)=1$, $h=0.1$ $x_0=0$

By Formula

$$y_{n+1} = y_n = hf(x_n, y_n)$$

$$y_{n+1} = y_n + h[2x_n]$$

"I st Iteration :~"

$n=0$

$$y_1 = y_0 + h(2x_0)$$

$$y_1 = 1 + 0.1(2(0))$$

$$y_1 = 1 + 0.1$$

$$\boxed{y_1 = 1.0}$$

$\rightarrow x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$x_1 = 0.1$

"2nd Iteration :~"

$n = 1$

$y_2 = y_1 + h(2x_1)$

$y_2 = 1.0 + 0.1(2(0.1))$

$\boxed{y_2 = 1.02}$

$x_{n+1} = x_n + h$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

"3rd Iteration :~"

$n = 2$

$y_3 = y_2 + h(2x_2)$

$y_3 = 1.02 + 0.1(2(0.2))$

$\boxed{y_3 = 1.06}$

$x_{n+1} = x_n + h$

$x_3 = x_2 + 0.1$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

b) By Modified Euler Method

$$\frac{dy}{dx} = 2x$$

Given data

$y_0 = 1, x_0 = 0, h = 0.1$

Formula

$$y^*_{n+1} = y_n + h[f(x_n)]$$

$$y^*_{n+1} = y_n + h(2x_n) - (1)$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right]$$

$$= y_n + \frac{h}{2}\left[2x_n + 2x_n\right]$$

$$= y_n + \frac{h}{2}\left[4x_n\right]$$

1st Iteration	2nd Iteration	3rd Iteration
$n = 0$	$n = 1$	$n = 2$
$x_{n+1} = x_n + h$	$x_2 = x_1 + h$	$x_3 = x_2 + h$
$x_1 = x_0 + h$	$x_2 = 0.1 + 0.1$	$x_3 = 0.2 + 0.1$
$x_1 = 0 + 0.1$	$\boxed{x_2 = 0.2}$	$\boxed{x_3 = 0.3}$
$\boxed{x_1 = 0.1}$	$y_2 = y_1 + \frac{h}{2}(4x_1)$	$y_3 = y_2 + \frac{h}{2}(4x_2)$
$y_1 = y_0 + \frac{h}{2}(4x_0)$	$y_2 = 1 + \frac{0.1}{2}(4(0.1))$	$= 1.02 + \frac{0.1}{2}(4(0.2))$
$y_1 = 1 + \frac{0.1}{2}(4(0))$	$\boxed{y_2 = 1.02}$	$\boxed{y_3 = 1.06}$
$y_1 = 1$		

Q2)

Given Data:~

$$y=0 \;,\; x=0 \;,\; h=0.2 \qquad 0 \le x \le 0.6$$

$$y_{n+1} = y_n + K$$

"1st Iteration:~"

$$n=0$$

$$y_1 = y_0 + K \;,\; K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$K_1 = hf(x_n, y_n)$$

$$K_1 = h(x_0^2 - x_0 \cdot y_0)$$

$$K_1 = 0.2(0^2 - 0 - 0)$$

$$\boxed{K_1 = 0}$$

$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\right)$$

$$= 0.2 f(x_0 + h/2, y_0 + h/2)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$$

$$= 0.2 f(0.1, 0.1)$$

$$= 0.2(0.1^2 + 0.1 - 0.1)$$

$$\boxed{K_2 = 0.0020}$$

$$K_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{K_2}{2}\right)$$

$$= 0.2f\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$$

$$= 0.2f(0.1, 0.001)$$

$$= 0.2\left(0.1^2 + 0.1 - 0.001\right)$$

$$K_3 = 0.0218$$

$$K_4 = hf(x_n + h, y_n + K_3)$$

$$= 0.2f(0 + 0.2, 0 + 0.0218)$$

$$= 0.2f(0.2, 0.0218)$$

$$= 0.2\left(0.2^2 + 0.2 - 0.0218\right)$$

$$\boxed{K_4 = 0.0436}$$

$$K = \frac{1}{6}\left(0 + 2(0.002) + 2(0.0218) + 0.0436\right)$$

$$K = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$\boxed{y_1 = 0.0152}$$

Q3)

Given Data :~

$$a = 0,\ b = 10,\ n = 10$$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Solution :~

x	0	1	2	3	4	5	6	7	8	9	10
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

using formula

$$\int f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4) \ldots f(x_{10})\right)\right]$$

$$= \frac{1}{2}\left[10.1 + 2\left(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 60.3 + 61.2\right) + 62.1\right]$$

$$= \boxed{412.9 \text{ Ans}}$$

Q4)

Solution:~

$n = 10$

$h = \frac{3-2}{10} = 0.1$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.628	1.628	1.777	1.922	2.062

Now using Formula

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(x_0)+4(f(x_1)+f(x_3)\cdots)+2[f(x_2\cdots)+f(x_n)\right]$$

$$= \frac{0.1}{3}\left[0.693+4(0.846+1.162+1.476+1.777)+2(1.003+1.320 + 1.628+1.922)+2.062\right]$$

$$= \boxed{1.184} \text{ Ans}$$