SUBMITED TO

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SUBMITED BY

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STUDENT ID

7700

SECTION "B"

SUBJECT:

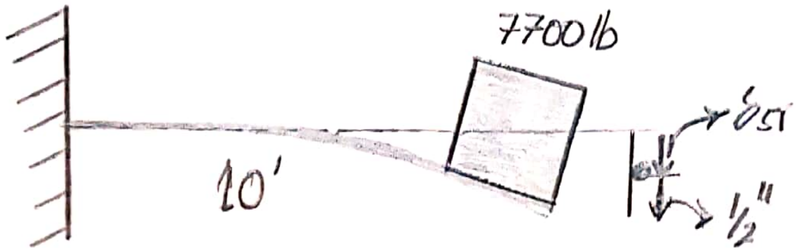
INTRO, STRUCTURAL DYANMICS & EARTH QUAKE.

ANSWER TO QUESTION #01

GIVEN DATA:

$E = 29{,}000\,ksi$, $I = 150\,in^4$

δ_{ST} = deflection due to 7700lb static Load Beam is pulled ½" downwards



REQUIRED DATA:

→ Natural time period of System
→ Develop and Solve the Equation of motion.
→ Draw graph to show the variation of Equivalent static forces with time.

Sol: General Equation of Motion for SDOF System is;

$$ku + c\dot{u} + m\ddot{u} = p(t).$$

Since System is Undamped $(c=0)$ under going for vibration $p(t)=0$

Hence general EOM become

$ku + m\ddot{u} = 0 \rightarrow$ (A)

$$k = \frac{3EI}{L^3} = \frac{3 \times 29000\,\frac{k}{in^2} \times 150}{(10 \times 12in)^3}.$$

$k = 7.55\, k/in.$

→ In Order to Eliminate chances of mistake during calculation, it is more appropriate to use fundamental units. Like lb, ft, sec or kg, m, sec.

$k = 7.55\, k/in$

$k = 90625\, lb/ft.$

$$m = \frac{W}{g} = \frac{7700}{32.2} = 239.13 \text{ slugs.}$$

$$W_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{239.13}}$$

$W_n = 19.46$ rad/sec.

$$T_n = \frac{2\pi}{W_n} \Rightarrow \frac{2\pi}{19.46} = 0.322 \text{ Sec.}$$

put the values of m and k in Eq (A).

$$90625u + 239.13\ddot{u} = 0$$

where k is in lb/ft and m is in.

lb.sec/ft² = slugs.

→ General Sol to EOM for Undamped free vibration is

$$u(t) = u(0)\cos(w_n t) + \frac{\ddot{u}(0)}{w_n \sin(w_n t)}.$$

$$u_m(0) = \frac{1}{2''} = \frac{1}{24}\text{ ft and } \ddot{u}(0) = 0$$

$$u(t) = 1/24 \times \cos(19.46t) + 0$$

$$= \frac{1}{24} \times \cos(19.46t).$$

Equivalent static force at any Time "t" is

$$f_s(t) = k.u(t)$$

$$= \frac{90625 \times \cos(19.46t)}{24}.$$

$$= 3776\cos(19.46t).$$

Amplitude of dynamics displacement, u_0 for undamped free vibration is

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\frac{v(0)}{w_n}\right)^2\right]}.$$

$$= \sqrt{\left((1/24)^2 + 0\right)}$$

$$= \sqrt{(1/24)^2}$$

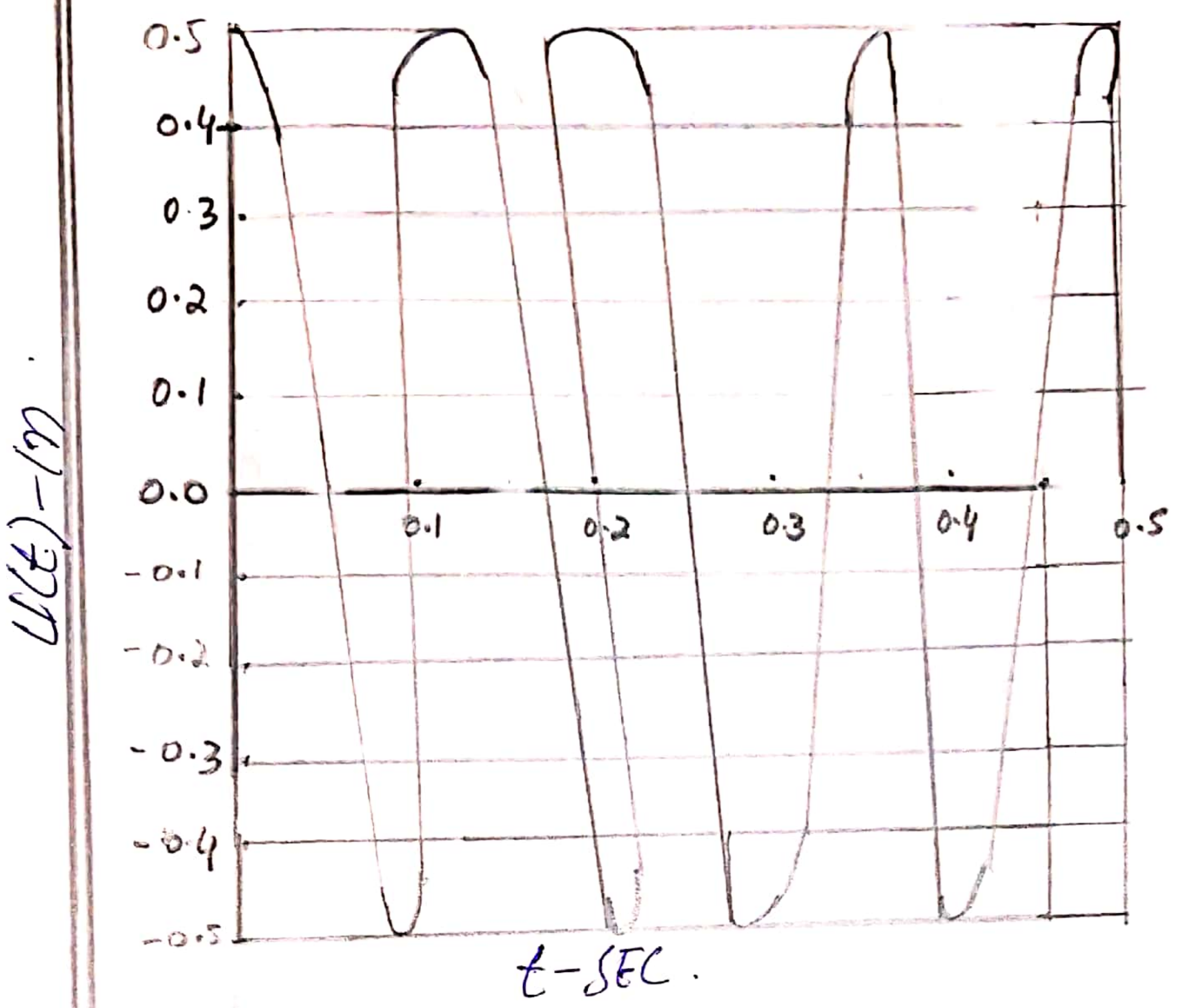
$$= 1/24 \text{ ft}$$

Amplitude of Equilent Static forces.

P_{s0}

$$k_{u0} = 90625 \times 1/24$$

$$k_{u0} = 3776 \text{ lb.}$$


0.5
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-0.1
-0.2
-0.3
-0.4
-0.5
0.1
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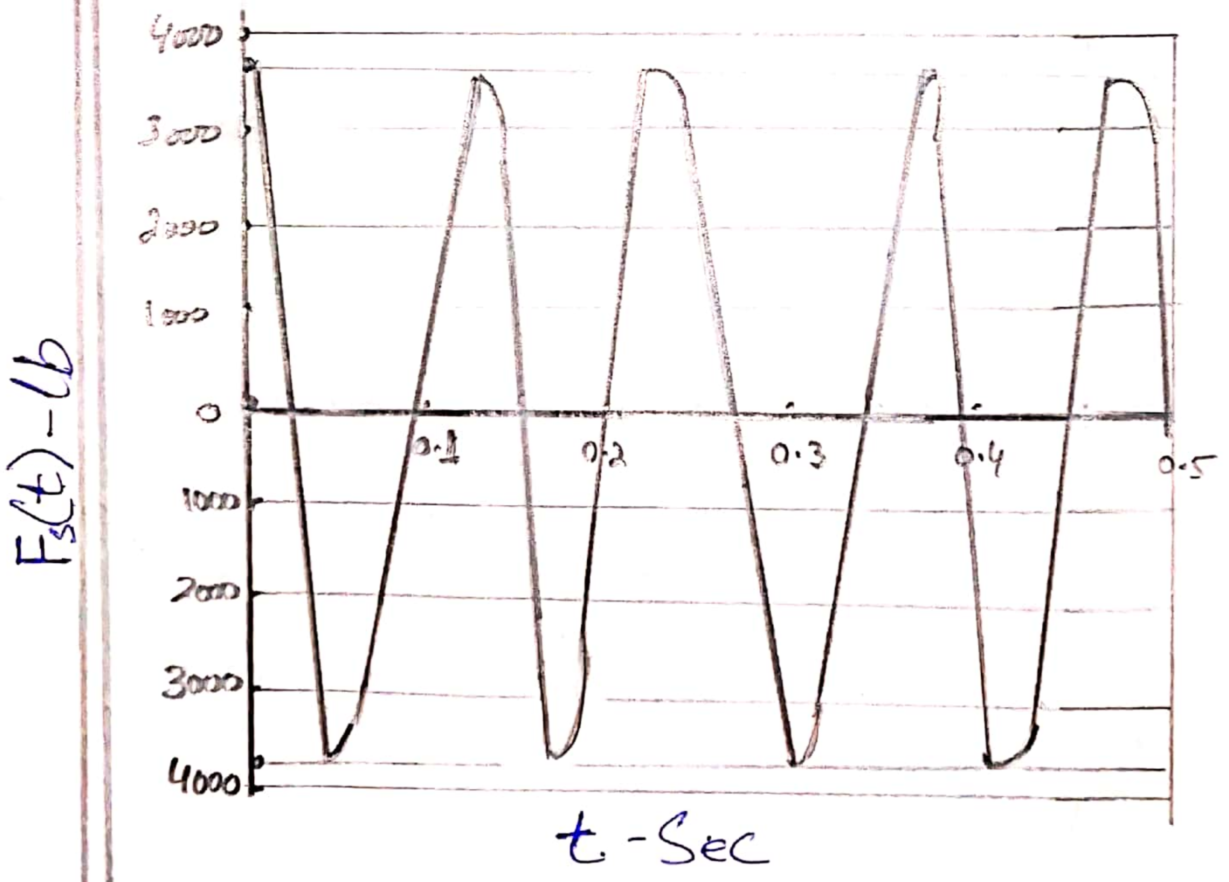
0.3
0.4
0.5
U(t) - (?)
t - SEC.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0.3
0.4
0.5
$F_s(t)$ - lb
t - Sec

ANSWER TO QUESTION #02

GIVEN DATA:

ξ (damping ratio) of Reinforced Concrete with Consideration Cracking
$= 3-5\%$
$= 3\%$

Data of Beam from Question #1.

REQUIRED:

→ Develope and solve the Equation showing variation in Equilant static force with time.

→ Draw graph to show the variation of displacement with time and the variation of Equalent static force with time.

Sol: EOM for damped free vibration is

$$ku + c\dot{u} + m\ddot{u} = 0 \rightarrow (A)$$

As from Question 1.

$k = 90625\ lb/ft$, $M = 239.13\ lb\ sec^2/ft$

$\omega_n = 19.46\ rad/sec$

$c = \zeta \times 2 m \omega_n$

$c = 3\% \times 2(239.13)(19.46)$

$c = 0.03 \times 2(239.13)(19.46)$ 0.000416

$c = 279.20 \ lb \cdot sec/ft$

→ put the value in Eq (A)

$90625 + 279.20 + 239.13 = 0$

Solution to the EOM for damped free vibration is

$u(t) = e^{-\zeta \omega_n t} \left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\zeta\omega_n\right]\sin(\omega_D t)\right]$

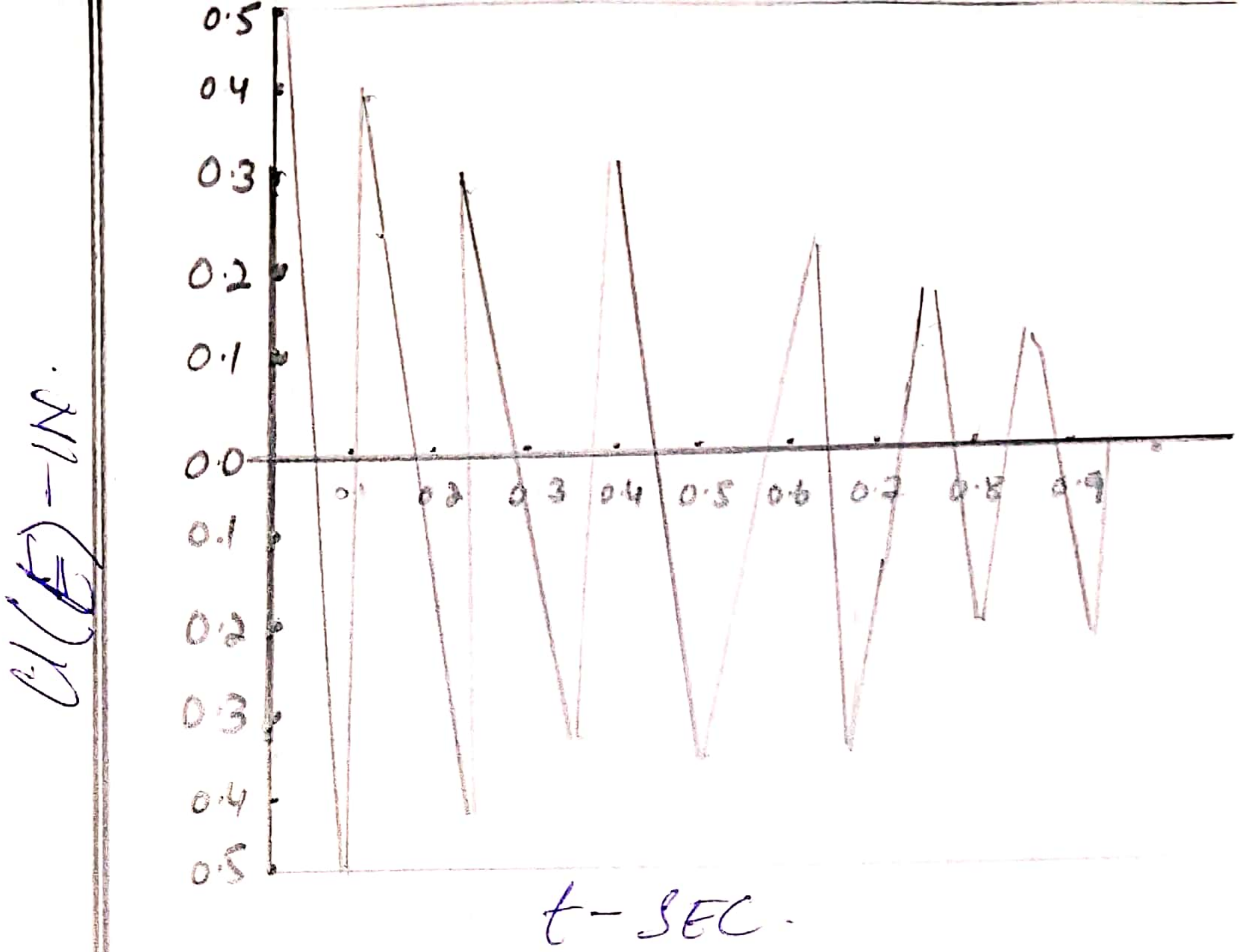
$\omega_D = 19.46 \ rad/sec$

$u(t) = e^{-0.583t}\left[0.041 \times \cos(19.46t) + 0.00125 \times \sin(19.46t)\right]$

$F_s(t) = k \cdot u(t) = 90625 \times u(t)$

$F_s(t) = e^{-0.583t}\left[(90625 \times 0.041)\cos(19.46t) + 90625 \times 0.00125 \sin(19.46)\right]$

$F_s(t) = e^{-0.583t}\left[3715.62 \cdot \cos(19.46t) + 113.28 \times \sin(19.46)\right]$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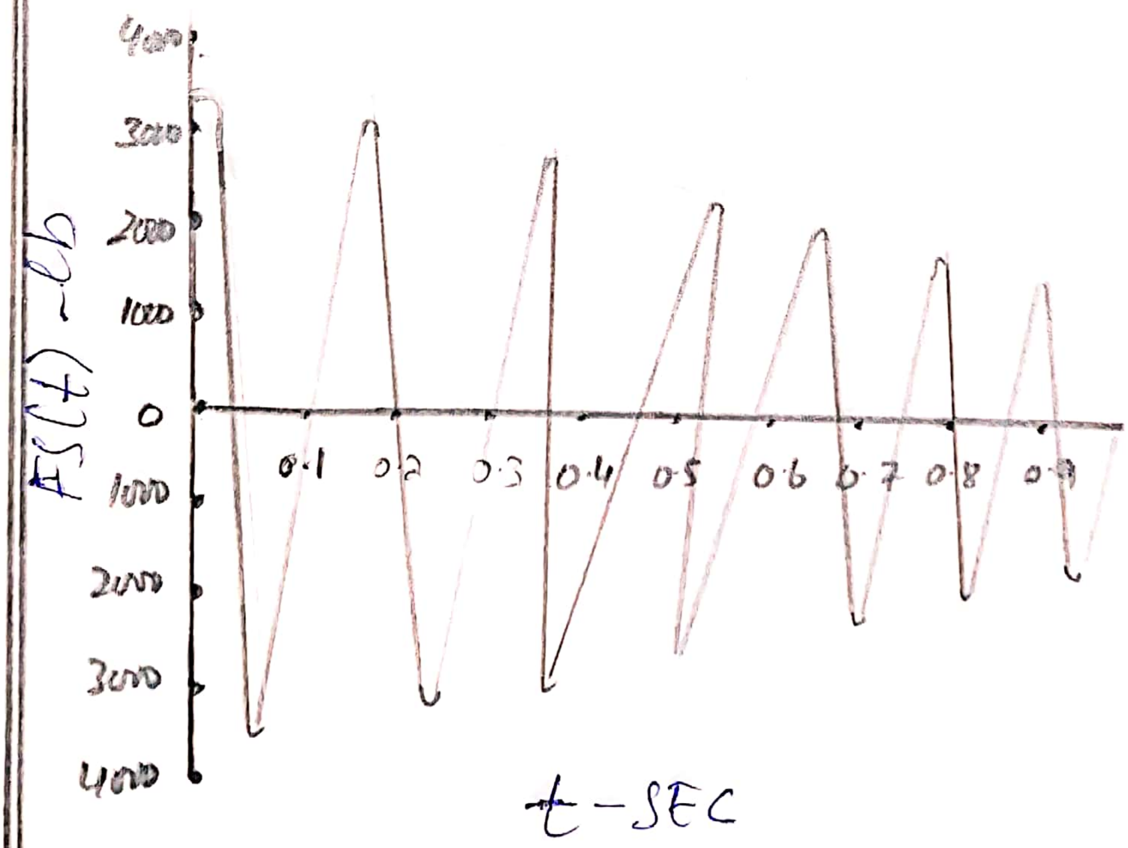
0.3
0.4
0.5
0.6
0.7
0.8
0.9
U(t) - IN.
t - SEC.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2000
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4000
0.1
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0.3
0.4
0.5
0.6
0.7
0.8
0.9
FS(t) - lb
t - SEC

ANSWER TO QUESTION # 03

GIVEN DATA :

Force = 60 kips.

Displacement of tank = $\frac{ID}{1000} = \frac{7700}{1000} = 7.70''$

Time taken to complete 7 cycles = 3.57 sec

amplitude of displacement = 2.286 cm.
= 0.9"

REQ :

(a) Damping Ratio

(b) Natural period of Un-damped vibration.

(c) Stiffness of Structure

(d) weight of tank

(e) damping Co-efficient

(f) The displacement amplitude to 0.5" Number of cycle to Reduced.

Sol:

Displacement of tentle, $u_1 = 7.70$

After, 7 cycles

$J = 7$, $u_{J+1} = u_8 = 0.9''$

(a) Damping Ratio:

$$J = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_1+1}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left(\frac{7.70}{0.9}\right)$$

$$\zeta = 0.0488$$

$$\zeta = 4.88\%$$

(b) Natural period of Undamped Vibration = T_n = ?

As, the 7 cycle of vibration are completed in $\frac{3.57}{7}$ ~~sec~~ sec.

→ Time Req to complete one cycle, $T_D = \frac{3.57}{7} = 0.51$ sec.

$T_D = 0.51$ sec.

Now,

$$W_D = W_n \sqrt{(1-\xi^2)}$$

$$\frac{2\pi}{W_D} = \frac{2\pi}{(W_n\sqrt{1-\xi^2})}$$

$$T_D = \frac{T_n}{(1-\xi)^2}$$

$$T_n = T_D \times \sqrt{1-\xi^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$$T_n = 0.5094 = 0.51 sec.$$

$$T_n = 0.51 sec$$

(c) Stiffness of Structure, K = ?

$$K = \frac{60 \times \cos 60^\circ}{7.70} = 3.896 \, k/in.$$

$$K = 3.9 \, k/in.$$

$$K = 46920 \, lb/ft.$$

(d) Weight of tank, W = ?

$$W_n = \sqrt{\frac{K}{m}} = \frac{K}{W/g} \quad \because W = mg$$

$$\Rightarrow \sqrt{\frac{K \cdot g}{W}}$$

$$\therefore \quad W_n = \sqrt{\frac{k \times g}{w}}$$

Taking Square

$$W_n^2 = \frac{k \times g}{W}$$

$$W = \frac{k \times g}{W_n^2}.$$

also;

$$W_n = \frac{2\pi}{T_n}.$$

$$W = \frac{k \times g}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$= k \times g \frac{T_n^2}{4\pi^2}.$$

$$= \left[\frac{46920}{ft} \times \frac{32.2\ ft}{sec^2}\right] \times \left[\frac{(0.51\ sec)^2}{4\pi}\right]$$

$$= 9953.93\ lb$$

$$= 9.95\ k.$$

ⓔ DAMPING Co-efficiant , C = ?

$$\zeta = \frac{c}{2m\omega_n}$$

$$c = \zeta \times 2m\omega_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$= \frac{0.0488 \times 4 \times \pi \times \frac{9953.9}{32.2}}{0.51}$$

$$c = 371.71 \text{ lb.sec/ft}$$

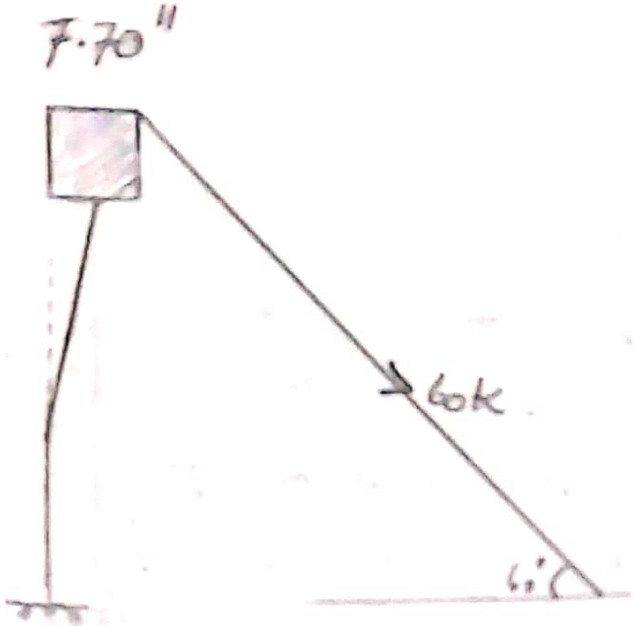
(F) Number of cycles to Reduce the displacement amplitude to 0.5", J = ?

$$J = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$J = \frac{1}{2\pi \times 0.0488} \ln\left|\frac{7.70}{0.5}\right|.$$

$$J = 8.91$$

$\simeq$ 9 cycles.


7.70"
60k