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SECTION "A"

SUBJECT APPLIED CALCULUS.

DATE 21 AUGUST 2020.

Question No 1.

The function $g(t)$ is defined by $g(t) = 0$ $t < 0$

t^2 "	$0 \leq t \leq 3$
$2t+3$	$3 < t \leq 4$
12	$t > 4$

a- State any point of discontinuity.

b- Find, if they exist.

i. $\lim_{t \to 1} g$

Answer:-

Solution:

a- To check possibility of the discontinity of the function is at $t = 0$ & 4.

→ First at $t = 0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

For R.H.L

$$\lim g(1+h) = \lim (1+h)^2$$

$$= \lim_{h \to 0} 1 + h^2 + 2h$$

Apply limits

$= 1 + 0^2 + 2(0)$

for L.H.L.

$\lim_{h \to 0} g(1-h) = 2t+3$

$= \lim_{h \to 0} 2(1-h) = 2t+3$

$= \lim_{h \to 0} 2-2h+3$

Apply limit

$= 2-2(0)+3$

$= 5$

$R.H.L \neq L.H.L = g(t) = 5$

Now at $t=4$

$g(4) = 2(4)+3$

$= 8+3$

$= 11$

for R.H.L

$\lim_{h \to 0} g(1+h) = \lim 2(1+h)+3$

$= \lim_{h \to 0} 2+2h+3$

Apply limits.

$= 2 + 2\,(0) + 3$

$= 2 + 0 + 3$

$= 2 + 3$

$= 5$

For L.H.L.

$\lim_{h \to 0} g(1-h) = 12.$

~~Xxxx~~ $g(4) = R.H.L \neq L.H.L$

Point of ~~cont~~ discontinuity is at $t = 4$

b:- Find, if they exist.

i:- $\lim_{t \to 1} g$

For $g(t) = t^2$

R.H.L

$\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Apply limits

$= 1 + 3^2 + 2\,(3)$

$= 1 + 9 + 6$

$= 10 + 6$

$= 16.$

L.H.L

$$\lim g(1-h) = \lim_{h \to 3} 2t + 3$$

$$= \lim_{h \to 3} 2(1-h) + 3$$

$$= \lim_{h \to 3} 2 - 2h + 3$$

Apply limit

$$= 2 - 2(3) + 3$$

$$= 2 - 6 + 3$$

$$= -4 + 3$$

$$= -1$$

R.H.L $\neq$ L.H.L (do not exist since L.H.L is -ve)

Question No 2.

i:- Find the Maclaurin's series For

$Y(x) = x^2 + \sin x.$

Answer:-

$$Y(x) = x^2 + \sin x$$

Since we know that the Maclaurin series is

$$Y(x) = Y(x_0) + Y'(x_0)(x - x_0 + \frac{Y''(x_0)(x-x_0)^2}{2!} + \cdots$$

Put $x_0 = 0$

$$Y(x) = Y(0) + (x-0)Y'(0) + \frac{(x-0)^2 Y''(0)}{2!} + \cdots$$

$$Y(x) = Y(0) + xY'(0) + \frac{x^2 Y''(0)}{2!} + \cdots \longrightarrow (1)$$

Now find

$Y(0) = ?$

$Y(x) = x^2 + \sin x$

$Y(0) = 0 + \sin 0$

$= 0 + 0$

$= 0$

$$\boxed{Y(0) = 0}$$

$y(x) = x^2 + \sin x$

$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$

$y'(x) = 2x + \cos x$

$y'(0) = 2(0) + \cos 0$

$y'(0) = 0 + 1$.

$y'(0) = 1$

Since $y'(x) = 2x + \cos x$

$\frac{d}{dx} y'(x) = 2 \frac{d}{dx} x + \frac{d}{dx} \cos x$

$= 2 + \sin x$

$y''(x) = 2 - \sin x$

$y''(0) = 2 - \sin 0$

$= 2 - 0$

$y''(0) = 2$

Now

$y''(x) = 2 - \sin x$

$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$

$$= 0 - \cos x$$

$$y''(x) = 0 - \cos x$$

$$y''(0) = -\cos 0$$

$$\boxed{y''(0) = -1}$$

Put in equation 1

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \text{------}$$

$$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \text{------}$$

$$= x + x^2 - \frac{x^3}{3!} + \text{------}$$

So

$$\boxed{y(x) = x + x^2 + \frac{x^3}{3!} + \text{------}}$$

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Question 3.

i:- Find y given

$$1 + xy = x^2 + y^2$$

ii:- Find y by using logarithmic differentiation.

$$y = x^3 (1+x)^9 e^{6x}.$$

Part (1).

$1 + xy = x^2 + y^2$, Find $y'' = ?$

Given;

$$1 + xy = x^2 + y^2$$

taking $\frac{d}{dx}$ on both sides

$$1 + \cancel{\frac{d}{dx}} \cdot x \frac{d}{dx} y = \frac{d}{dx} x^2 + \frac{d}{dx} y^2$$

$$\Rightarrow 1 + (1)\left(\frac{dy}{dx}\right) = 2x + 2y \frac{dy}{dx}$$

$$1 + \frac{dy}{dx} = 2x + 2y \frac{dy}{dx}.$$

$$\frac{dy}{dx} - 2y \frac{dy}{dx} = 2x - 1.$$

$$\frac{dy}{dx}(1-2y) = 2x-1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x-1}{1-2y}$$

$$\Rightarrow \boxed{y' = \frac{2x-1}{1-2y}} \longrightarrow 1$$

Diff again

$$\frac{d}{dx}y' = \frac{d}{dx}\left(\frac{2x-1}{1-2y}\right)$$

$$y'' = \frac{(1-2y)\frac{d}{dx}(2x-1) - (2x-1)\frac{d}{dx}(1-2y)}{(1-2y)^2}$$ use Quaitent rule

$$\Rightarrow \frac{1-2y\,(2) - (2x-1)(-2y')}{(1-2y)^2}$$

$$= \frac{(2-4y) - (2x-1)(-2y')}{(1-2y)^2}$$

from equation 1

$$y' = \frac{2x-1}{1-2y}.$$

Put in above.

$$y'' = \frac{(2\,4y) - (2x-1)\left(-2\left(\frac{2x-1}{1-2y}\right)\right)}{(1-2y)^2}$$

$$= \frac{\cancel{2(1-2y)}}{(1-2y)^2} - \frac{(2x-1)(2x-1)(-2)}{(1-2y)^3}$$

$$\Rightarrow \quad y'' = \frac{2}{1-2y} - \frac{-2(2x-1)^2}{(1-2y)^3}$$

(Part b)

ii:- **Find y by using logarithmic differentiation.**

$$y = x^3(1+x)^9 e^{6x}.$$

Solution:-

$$\ln(y) = \ln\left(x^3(1+x)^9 e^{6x}\right)$$

$$\cancel{\frac{d\ln y}{dx}} = \ln\left(x^3(1+x)^9 + \ln e^{(6x)}\right)$$

$$= \ln x^3 + \ln(1+x)^9 + 6x.$$

$$= 3\ln x + 9\ln(1+x) + 6x$$

(18)

Now

$$\frac{d\ln(y)}{dx} = \frac{d}{dx}(3\ln x + 9\ln(1+x) + 6x)$$

$$= \frac{3d}{dx}\ln x + 9\frac{d}{dx}\ln(1+x) + 6\frac{dx}{dx}.$$

$$= 3\frac{1}{x} + 9\cdot\frac{1}{1+x} + 6.$$

$$\boxed{\frac{d\ln(y)}{dx} = \frac{3}{x} + \frac{9}{x+1} + 6}$$