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**Section:-** A

**Semester:-** 4th

**Subject:-** Differential Equations

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# ASSIGNMENT

## Q NO 1

Solve the following objective type questions.

i) The order of matrix AB is **mxn.**

ii) The numbers of non-zero rows in an Echelon form is **Rank of matrix.**

iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a =$ **8.**

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= -2i^2 - i^2 \qquad \star\, i^2 = -1$$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$= \mathbf{3}\ \textbf{Ans}$$

v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is **Scalar Matrix.**

**REASON:**

As the diagonal elements are same (9) and non diagonal elements are zero that's why it is a scalar matrix.

vi) Solution of $\frac{dy}{du} + 2uy = y$ ?

**SOLUTION:**

$$\frac{dy}{du} + 2uy = y$$

$$\frac{dy}{du} = y - 2uy$$

Taking "y" common

$$\frac{dy}{du} = y(1-2u)$$

$$\frac{dy}{y} = (1-2u)\,du$$

Take integration

$$\int \frac{1}{y}\,dy = \int (1-2u)\,du$$

$$\ln y = \int 1\,du - \int 2u\,du$$

$$\ln y = u - \frac{\cancel{2}u^2}{\cancel{2}} + c$$

$$\ln y = u - u^2 + c$$

$$\ln y = u - u^2 + c$$

$$e^{\ln y} = e^{u-u^2+c}$$

$$\boxed{y = e^{u}(1-u) + c}$$ Ans

vii) The order and degree of differential equation $\left(\frac{dy}{du}\right)^3 = \sqrt{1+\left(\frac{dy}{du}\right)^2}$ is ?

**SOLUTION:**

Order $= \underline{1}$

Degree $= \underline{3}$

viii) The order and degree of differential equation $\frac{d^2y}{du^2} - 4uy = \sin\left(\frac{d^2y}{du^2}\right)$ is ?

**SOLUTION:**

Order $=$ **Two**

Degree $=$ **One**

ix) The differential equation $2\frac{dy}{du} + u^2y = 2u+3, y(0) = 5$ is?

**SOLUTION:**

$$2\frac{dy}{du} + u^2y = 2u+3$$

$$2y' + u^2y = u^2+3, \quad y(0) = 5$$

$$y' + \left(\frac{u^2}{2}\right)y = \frac{u^2+3}{2}$$

$$y' + \left(\frac{u^2}{2}\right)y = \frac{1}{2}(u^2+3)$$

$$\mu = \frac{u^2}{2}$$

$$e^{\int \frac{u^2}{2} du} = e^{u^3/6}$$

$$e^{u^3/6} y' + e^{u^3/6}\left(\frac{u^2}{2}\right)y = \frac{1}{2} e^{u^3/6}(u^2+3)$$

$$y(u) = \frac{e^{u^3/6}u^2 + 3e^{u^3/6} + c}{2e^{u^3/6}}$$

$$y(0) = \frac{0+3}{2} = 3/2$$

$$\boxed{y(u) = \frac{e^{u^3/6} u^2 + 3e^{u^3/6}}{2e^{u^3/6}} + \frac{3}{2}}$$ Ans

x. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is?

**SOLUTION:-**

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by $C_1$

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$$= 1(bc^2 - cb^2) - 1(ac^2 + ca^2) + 1(ab^2 - a^2b)$$

$$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$$

$$= ab^2 - cb^2 + ac^2 - a^2b - ac^2 + bc^2$$

$$= a^2c - a^2b + ab^2 - cb^2 - ac^2 - bc^2$$

$$\boxed{= a^2(c-b) + b^2(a-c) + c^2(b-a)}$$ Ans

# Q. No 2

i) Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c.

## SOLUTION:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by $R_1$

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3a^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

Taking common "a b c"

$$= abc(bc^2 - b^2c - ac^2 + ab^2 - a^2b)$$

$$\boxed{\Rightarrow abc\,[bc(c-b) - ac(c+a) + ab(b-a)]}$$ Ans

ii) Find the Eigen value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

## SOLUTION:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equ $\rightarrow |A - \cancel{\lambda}^{\lambda} I| = 0 \longrightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinent

$$|A - \cancel{\ } \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by $R_1$

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$- 1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow \text{(B)}$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by $R_1$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + \big((-1)(2-\lambda) - (-1)(-1)\big)$$

$$-1\big((-1)(-) - (-1)(3-\lambda)\big)\Big]$$

$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-(+1+3-\lambda)$

$= (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$

$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$

$\boxed{= -\lambda^3+8\lambda^2-18\lambda+8} \rightarrow$ (a)

$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$

Expand by $C_1$

$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$

$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$

$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$

$\boxed{= -\lambda^2+6\lambda-8} \rightarrow$ (b)

$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$

Expand by $C_1$

$-\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$

$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]$

$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5)$

$= -\lambda^2+5\lambda-5-3+\lambda$

$\boxed{= -\lambda^2+6\lambda-8} \rightarrow$ (c)

Put eq a, b and c in B

$$= (2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8$$
$$-\lambda^2+16\lambda-8$$

$$= \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda$$
$$+6\lambda+6\lambda+16-16$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By synthetic division

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0 \Rightarrow \boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By factorization method

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)$$

$$\lambda=4, \quad \lambda=4$$

$$\boxed{\lambda_1=0,\ \lambda_2=2,\ \lambda_3=4,\ \lambda_4=4}$$

## Q No 3

The rate of change in the form of differential equation is given by

$(x^2+3y^2)dx-2xydy=0$. find the general solution at $x=2$ and $y=6$.

## SOLUTION:-

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$\Rightarrow (x^2+3y^2)\,dx = 2xy\,dy$$

Dividing both side by $2xydy$ we get:-

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{\cancel{x}^2}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow (*)$$

Diff:-

$$dy = v\,dx + x\,dv$$

Dividing by $dx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (a)$$

Put (a) in (*)

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying both side by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2u \frac{dv}{du} = \frac{1}{v} + 3v - 2v$$

$$2u \frac{dv}{du} = \frac{1}{v} + v$$

$$2u \frac{dv}{du} = \frac{1+v^2}{v}$$

Multiplying both sides by $\frac{du}{dv}$

$$2u\,dv = \frac{1+v^2}{v} du$$

Multiplying both side by $\frac{v}{u(1+v^2)}$

$$\frac{v}{1+v^2} dv = \frac{1}{u} du$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2} dv = \int \frac{1}{u} du + c$$

$$\ln |1+v^2| = \ln u + \ln c$$

Take "e" on both side

$$e^{\ln|1+v^2|} = e^{\ln|uc|}$$

$$1+v^2 = uc$$

Put $v = y/u$

$$1 + (y/u)^2 = uc$$

$$\frac{u^2+y^2}{u^2} = uc$$

$$u^2+y^2 = u^3 c \rightarrow (ii)$$

⑪

Put $x=2$ , $y=6$ in eq (ii)

$$4+36=8c$$

$$c=\frac{\cancel{40}}{\cancel{8}}$$

$$\boxed{c=5} \rightarrow \text{Put in ii}$$

$$x^2+y^2+5x^3$$

$$y^2=5x^3-x^2$$

$$y^2=x^2(5x-1)$$

Taking square root on both side

$$\boxed{y=+x\sqrt{5x-1}},\ \boxed{y=-x\sqrt{5x-1}}$$

$$\boxed{y=\pm x\sqrt{5x-1}}$$

THE

END