

Final Term

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Q1. Answer the following short questions briefly.

PART. 1). Write down the Maxwell's Equations?  
Maxwell's equations comprise the fundamental tenets of electromagnetic theory.

Answer:-  
Maxwell's equations describe how electric charges and electric currents create electric and magnetic fields.

Further, they describe how an electric field can generate a magnetic field, and vice versa. The first equation allows you to calculate the electric field created by a charge.

⇒ Maxwell's Equations  
Differential form

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



## Maxwell's Equations :

Integral form

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{l} = - \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

$$\oint \vec{B} \cdot d\vec{a} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a}$$

⇒ With the Publication of "A Dynamical Theory of the Electromagnetic field" in 1865, Maxwell demonstrated that electric and magnetic fields travel through space as waves moving at the speed of light.

He proposed that light is an undulation in the same medium that is the cause of electric and magnetic phenomena.



PART. 2). Explain the Coulombs Law?

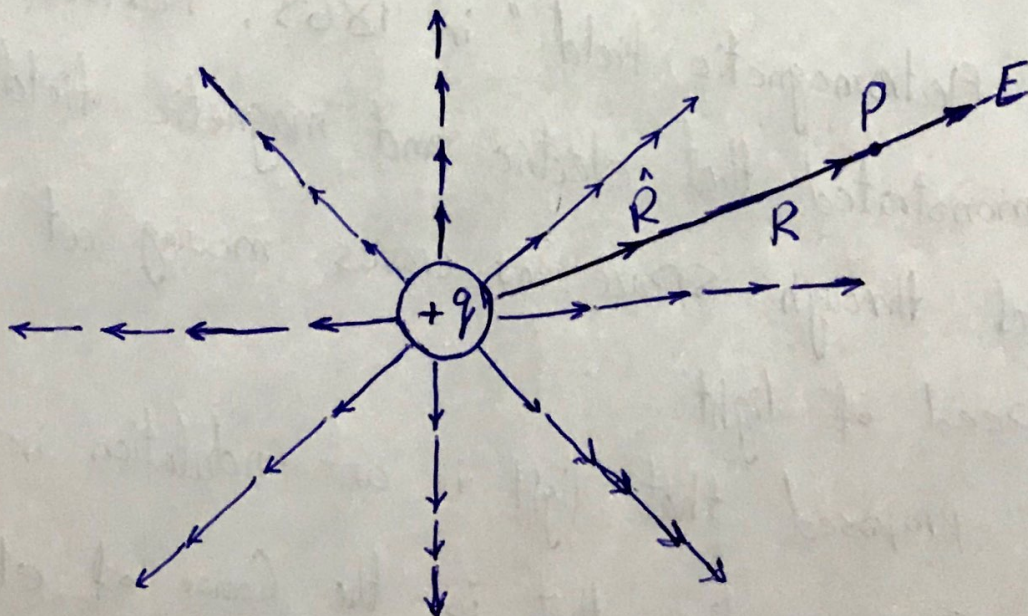
Pg# 04

Also state it with the help of expressions.

(Ans). Coulomb's Law states that

1. An isolated charge  $q$  induces an electric field  $E$  at every point in space, and at any specific point  $P$ ,  $E$  is given by.

$$E = \hat{R} \frac{q}{4\pi\epsilon R^2} \text{ V/m}$$





2. In the presence of electric field  $E$ , at a given point in space, which may be due to a single charge or a distribution of many charges  $q'$ , when the charge is placed at that point, is given by

$$F = q' E \quad (N)$$

- With  $F$  measured in newtons (N) and  $q'$  in coulombs (C), the unit of  $E$  is (N/C), which is same as volt per meter (V/m).
- For a material with electrical permittivity  $\epsilon$ , the electric field quantities  $D$  and  $E$  are related by

$$\vec{D} = \epsilon \vec{E}$$



PART.3). What is the difference between Convection and Conduction Currents?

(Ans). Convection and Conduction  
Difference :-

⇒ The difference between conduction and convection is that in convection heat is actually transferred by moving particles as when a fan is used to move heat from one place to another by blowing air.

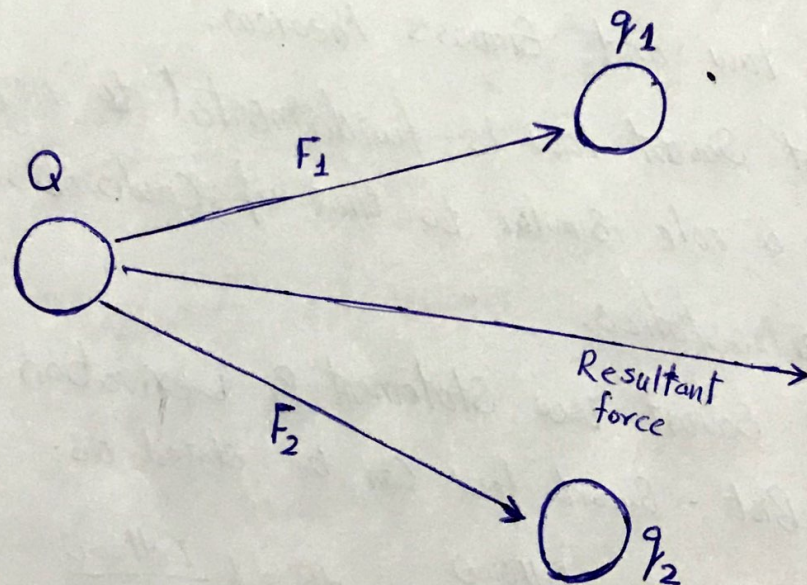
⇒ In conduction heat is transferred through solids by means of the vibrating molecules in a substance.



PART. 4). State the principle of linear Superposition as it applies to the electric field due to a distribution of electric charge?

(Ans). The principle of Superposition states that every charge in space creates an electric field at point independent of the presence of other charges in that medium.

The resultant electric field is a vector sum of the electric field due to individual charges.





PART. 5). What is Biot - Savart Law?

Pg # 08

Also state it with the help of expression?

(Ans).  $\Rightarrow$  Biot - Savart law :

The Biot - Savart law is an equation describing the magnetic field generated by a constant electric current. It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current.

Biot - Savart law is consistent with both Ampere's Circuital law and Gauss's theorem.

The Biot Savart law is fundamental to magnetostatics, playing a role similar to that of Coulomb's law in electrostatics.

$\Rightarrow$  Biot Savart law Statement & Derivation :

The Biot - Savart law can be stated as:

$$\text{Hence, } dB \propto \frac{Idl \sin \theta}{r^2} \text{ or } dB = k \frac{Idl \sin \theta}{r^2}$$

Where,  $k$  is a constant, depending upon the magnetic properties of the medium and system of the units employed. In the SI system of unit,

$$k = \frac{\mu_0 \mu_r}{4\pi}$$



Qd (part A) A 2mm diameter copper wire  
----- ( $m^2/vs$ ) ----- the free  
electron.

(Ans) Given data

The density of diameter copper wire  
conductivity of  $58 \times 10^7 S/m$   
and mobility of  $0.00032 (m^2/vs)$   
we know carrier  
concentration  $(n)$

$$\frac{\text{Avogadro number} \times \text{Density}}{\text{Atomic weight}}$$

The conductivity of copper

$$n = 84.6 \times 10^{25} m^{-3}$$

The electrical conductivity

$$\sigma = \frac{1}{\rho} = \frac{1}{1.73 \times 10^{-8}}$$

We know

$$\sigma = \frac{n \cdot e^2}{m}$$

Average time collision

$$\tau = \frac{em}{ne^2}$$



Q<sub>2</sub>. PART. b). The x-y Plane is a charge-free boundary

Separating two dielectric media with permittivities  $\epsilon_1$  and  $\epsilon_2$  as show in Fig. 4-19.

If the electric field in medium 1 is  $E_1 = xE_{1x} +$

$yE_{1y} + zE_{1z}$ . find

(a). the electric field  $E_2$  in medium 2 and

(b). the angles  $\theta_1$  and  $\theta_2$ .

(Ans).

Free interface, the tangential components of  $E$  and the normal components of  $D$  are continuous. Consequently,

$$E_{2x} = E_{1x}, \quad E_{2y} = E_{1y},$$

and

$$D_{2z} = D_{1z} \quad \text{or} \quad \epsilon_2 E_{2z} = \epsilon_1 E_{1z}.$$

Hence,

$$E_2 = \hat{x} E_{1x} + \hat{y} E_{1y} + \hat{z} \frac{\epsilon_1}{\epsilon_2} E_{1z}.$$



(b) The tangential components of  $E_1$  and  $E_2$

are  $E_{1t} = \sqrt{E_{1x}^2 + E_{1y}^2}$  and  $E_{2t} = \sqrt{E_{2x}^2 + E_{2y}^2}$ .

The angles  $\theta_1$  and  $\theta_2$  are then given by

$$\tan \theta_1 = \frac{E_{1t}}{E_{1z}} = \frac{\sqrt{E_{1x}^2 + E_{1y}^2}}{E_{1z}}$$

$$\tan \theta_2 = \frac{E_{2t}}{E_{2z}} = \frac{\sqrt{E_{2x}^2 + E_{2y}^2}}{E_{2z}} = \frac{\sqrt{E_{1x}^2 + E_{1y}^2}}{(\epsilon_1 / \epsilon_2) E_{1z}}$$

and the two angles are related by

$$\frac{\tan \theta_2}{\tan \theta_1} = \frac{\epsilon_2}{\epsilon_1}$$



∴ PART (a):-

Q3 - (a) The Semi-circular Conductor shown in fig 5-4 lies in the  $x-y$  plane. Current  $I$ .

The closed loop is in the  $xy$  plane.  $B = \hat{y} B_0$ .

Determine (a)  $F_1$  on the straight section and

(b)  $F_2$  on the curved section?

(Ans). Solution: a) To evaluate  $F_1$ , consider that the straight section of the circuit is of length  $2r$  and its current flows along the  $+x$  direction. Application of  $\vec{F} = I \vec{l} \times \vec{B}$  with  $l = \hat{x} 2r$  gives

$$F_1 = \hat{x} (2Ir) \times \hat{y} B_0 = \hat{z} 2Ir B_0 \text{ (N)}$$

The  $\hat{z}$  direction in Fig. 5-4 is out of the Page.

(b) To evaluate  $F_2$ , consider a segment of differential length  $d\vec{l}$  on the curved part of the circle.

The direction of  $d\vec{l}$  is chosen to coincide with the direction of the current.



Since  $d\vec{l}$  and  $B$  are both in the  $x$ - $y$  plane, their cross product  $d\vec{l} \times B$  points in the negative  $z$  direction, and the magnitude of  $d\vec{l} \times B$  is proportional to  $\sin \phi$ , where  $\phi$  is the angle between  $d\vec{l}$  and  $B$ . Moreover, the magnitude of  $d\vec{l}$  is  $d\vec{l} = r d\phi$ . Hence,

$$F_2 = I \int_{\phi=0}^{\pi} d\vec{l} \times B$$

$$= -\hat{z} I \int_{\phi=0}^{\pi} r B_0 \sin \phi d\phi = -\hat{z} 2I r B_0 \quad (N)$$

The  $-\hat{z}$  direction of the force acting on the curved part of the conductor is into the page.

We note that  $F_2 = -F_1$ , implying that no net force acts on the closed loop, although opposing forces act on its two sections.



Q3 PART. (b). A free-standing linear conductor of length  $l$  carries a current  $I$  along the  $z$  axis as shown in Fig 5-10.

Determine the magnetic flux density  $B$  at a point  $P$  located at a distance  $r$  in the  $x-y$  plane.

(Ans) Solution: From the differential length vector  $d\mathbf{l} = \hat{z} dz$ . Hence,  $d\mathbf{l} \times \hat{R} = dz (\hat{z} \times \hat{R}) = \hat{\phi} \sin \theta dz$ , where  $\hat{\phi}$  is the azimuth direction and  $\theta$  is the angle between  $d\mathbf{l}$  and  $\hat{R}$ .

Application

$$H = \frac{1}{4\pi} \int_{z=-l/2}^{z=l/2} \frac{d\mathbf{l} \times \hat{R}}{R^2} = \hat{\phi} \frac{I}{4\pi} \int_{-l/2}^{l/2} \frac{\sin \theta}{R^2} dz$$

Both  $R$  and  $\theta$  are dependent on the integration variable  $z$ , but the radial distance  $r$  is not.

For convenience, we convert the integration variable from  $z$  to  $\theta$  by using the transformations

$$R = r \cos \theta,$$

$$z = -r \sin \theta,$$

$$dz = r \cos^2 \theta d\theta.$$



Pg# 15

Where  $\theta_1$  and  $\theta_2$  are the limiting angles at  $z =$   
 $z = -l/2$  and  $z = l/2$ , respectively.

From the right triangle in Fig. 5-10 (b), it follows that

$$\cos \theta_1 = \frac{l/2}{\sqrt{r^2 + (l/2)^2}}$$

$$\cos \theta_2 = -\cos \theta_1 = \frac{-l/2}{\sqrt{r^2 + (l/2)^2}}$$

Hence

$$B = \mu_0 H = \hat{\phi} \frac{\mu_0 I l}{2\pi r \sqrt{4r^2 + l^2}} \quad (7)$$

$$B = \mu_0 H = \hat{\phi} \frac{\mu_0 I l}{2\pi r \sqrt{4r^2 + l^2}} \quad (8)$$

For an infinitely long wire with  $l \gg r$

reduces to

$$B = \hat{\phi} \frac{\mu_0 I}{2\pi r} \quad (\text{infinitely long wire}).$$



Q4. PART. a). Explain the Faraday's law?

Also explain in brief its Differential and Integral Forms.

(Ans). Faraday's law states that the absolute value or magnitude of the circulation of the electric field  $E$  around a closed loop is equal to the rate of change of the magnetic flux through the area enclosed by the loop.

The equation below expresses Faraday's law in mathematical form.

Starting with the differential form of Faraday's law

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

It is a local statement. We first integrate on both sides about an arbitrary surface

$$\sum \int_{\Sigma} \nabla \times E \cdot d\mathbf{a} = - \int_{\Sigma} \frac{\partial B}{\partial t} \cdot d\mathbf{a}$$

on the left hand side of the above equation, we use Stokes theorem,



$$\int_{\Sigma} \nabla \times \mathbf{E} \cdot d\mathbf{a} = \oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{l}, \text{ where } \partial\Sigma \text{ is}$$

the boundary of the surface.

on the right hand side, We argue that the surface doesn't change with time, therefore the derivative sign can be moved outside of the integral sign; in addition, the integral is now only a function of time, therefore it is justified to use the total derivative symbol.

So we obtain

$$\oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{l} = - \frac{d}{dt} \int_{\Sigma} \mathbf{B} \cdot d\mathbf{a}$$

Which is the integral form of the Faraday's law.



Q.4 PART. (b). Determine voltage  $V_1$  and  $V_2$  across the  $2\Omega$  and  $4\Omega$  resistor

The loop is in the  $x-y$  Plane. The area is  $4m^2$ .  $B = -20.3t(T)$  wire may be ignored.

(Ans). Solution :- The flux flowing through the loop is

$$\phi = \int_S B \cdot ds = \int_S (-\hat{z} 0.3t) \cdot \hat{z} ds$$

$$= -0.3t \times 4 = -1.2t \text{ (Wb)}$$

and the corresponding transformer emf is

$$V_{emf}^{tr} = -\frac{d\phi}{dt} = 1.2 \text{ (V)}$$

