-: COURSE DETAILS:-

Course Tittle :- E.M.F

Instructor Name :- Sir Mansoor Rifaq

Module :- 4th.

-: STUDENT DETAIL :-

Name :- Naveed Alam

Student ID. No. :- 14965

Student Sign :- Naveed

Q.No. (1) Solve the following short Question.

(A) Transform the vector $B = yi(x+z)j$ located at point $(-2, 6, 3)$ into cylindrical coordinates.

Sol:- $B = yi(x+z)j$

Given points are $(-2, 6, 3)$ then

$$B = yi(xj + zj)$$

$$B = yxij + yzij$$

$\Rightarrow$ $P = \sqrt{x^2 + y^2}$

$$P = \sqrt{(-2)^2 + (6)^2}$$

$$P = \sqrt{40}$$

$$\boxed{P = 6.32}$$

$\Rightarrow$ As we know that

$$z = z$$

So $\boxed{z = 3}$

⇒ As we know that

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\phi = \tan^{-1}\left(\frac{6}{-2}\right)$$

$$\phi = \tan^{-1}(-3)$$

$$\boxed{\phi = -71.56}$$

so

$$\boxed{B = 6.32,\ -71.56,\ 3}$$ Ans.

(B) Convert the point (3, 4, 5) from Cartesion to spherical coodinates.

Sol:- $P(3,4,5)$

$x = 3$, $y = 4$, $z = 5$

In spherical coordinates system

r, θ, ϕ.

$r = \sqrt{x^2 + y^2 + z^2}$

$r = \sqrt{(3)^2 + (4)^2 + (5)^2}$

$r = \sqrt{9 + 16 + 25}$

$r = \sqrt{50}$

$\boxed{r = 7.07}$

As

$\theta = \tan^{-1}(y/x)$

$\theta = \tan^{-1}(4/3)$

$\theta = \tan^{-1}(1.33)$

$$\boxed{\theta = 53.1^\circ}$$

As

$$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{(3)^2+(4)^2}}{5}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{9+16}}{5}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{25}}{5}\right)$$

$$\phi = \tan^{-1}\left(\frac{5}{5}\right)$$

$$\phi = \tan^{-1}(1)$$

$$\boxed{\phi = 45}$$

$$r = 7.07, \quad \theta = 53.1^\circ, \quad \phi = 45$$

$$r, \theta, \phi = 7.07, 53.1^\circ, 45$$

Qc:- Find the spherical coordinates of A(2, 3, -1)

Sol:-C:- r, θ, ϕ

As

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{(2)^2 + (3)^2 + (-1)^2}$$

$$r = \sqrt{14}$$

$$\boxed{r = 3.74}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{3}{2}\right)$$

$$\theta = \tan^{-1}(1.5)$$

$$\boxed{\theta = 56.3^0}$$

As

$$\phi = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{(2)^2 + (3)^2}}{-1}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{4 + 9}}{-1}\right)$$

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$$\phi = \tan^{-1}\left(\frac{\sqrt{13}}{-1}\right)$$

$$\phi = \tan^{-1}(-3.60)$$

$$\boxed{\phi = 74.4}$$

$r = 3.74$, $\theta = 56.3^{\circ}$, $\phi = 74.4$

$(r, \theta, \phi) = (3.74, 56.3^{\circ}, 74.4)$

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Q4 Find the cartesion coordinates of $B(4, 25, 120)$.

Sol:- The point $B(4,25,120)$ is given in spherical (r, θ, ϕ). so we have to find (x, y, z).

Now

$$x = r\sin\theta \cdot \cos\phi$$
$$x = 4\sin(25) \cdot \cos(120)$$
$$x = 4(0.42)(-0.5)$$
$$\boxed{x = -0.84}$$

As

$$y = r\sin\theta \cdot \sin\phi$$
$$y = 4\sin(25) \cdot \sin(120)$$
$$y = 4(0.42)(0.86)$$
$$\boxed{y = 1.45}$$

As

$$z = r\cos\theta$$
$$z = 4\cos(25)$$
$$z = 4(0.90)$$
$$\boxed{z = 3.62}$$

$(x, y, z) = (-0.84, 1.45, 3.62)$.

(e) Find the force between two Charges when they are brought in 4cm apart, charges are 2nC and -1nC, in μN

Given data:-

$q_1 = 2nC$, $q_2 = -1nC$

$d = 4cm$

Required :-

$F = ?$

Sol:- where

$$F = K \frac{q_1 q_2}{r^2}$$

As $K = \frac{1}{4\pi \epsilon_0}$

$$F = \frac{2\times10^{-9} \times -1 \times 10^{-9}}{4(3.14)\times8.85\times10^{-12}\times(4\times10^{-2})^2}$$

$F = -1.124\times10^{-5}$

$$\boxed{F = -11.24\ \mu N}$$

Q 8:- Find the electric field intensity of two charges $-2C$ and $-1C$ separated by a distance $1m$ in air-

Given data:-

$q_1 = -2C$, $q_2 = -1C$

$d = 1m$

Required:-

$E = ?$

Sol:-

$$E_1 = \frac{K q_1}{d^2}$$

$K = 9 \times 10^9$

$$E_1 = \frac{9 \times 10^9 \times -2}{(1)^2}$$

$$E_1 = -18 \times 10^9 \ V/m$$

Now

$$E_2 = \frac{K q_2}{d^2}$$

$$E_2 = \frac{9 \times 10^9 \times (-1)}{(1)^2}$$

$$E_2 = -9 \times 10^9 \ V/m$$

As

$$E_T = E_1 + E_2$$

$$E_T = -18\times10^9 + (-9\times10^9)$$

$$E_T = -18\times10^9 - 9\times10^9$$

$$\boxed{E_T = -27\times10^9\ V/m}$$

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Q9:- Determine the charge that produce an electric field strength of 40 V/cm at a distance of 30 cm in vacuum (in 10^{-8} C) -

Given data:-

$E = 40\ V/cm$, $d = 30\ cm$

Required:-

$Q = ?$

Solution:-

$$E = \frac{KQ}{d^2}$$

$$Ed^2 = KQ$$

$$\frac{Ed^2}{K} = Q \longrightarrow (*)$$

Now putting value in (*)

$$Q = \frac{Ed^2}{K}$$

$$Q = \frac{40 \times (30)^2}{9 \times 10^9}$$

$$\theta = \frac{40 \times 900}{9 \times 10^9}$$

$$\theta = 4 \times 10^{-6} C$$

OR

$$\boxed{\theta = 4 \mu C}$$

Qn :- A Charge of 2×10^{-7} is acted upon by a force of 0.1N. Determine the distance to the other charge of $4.5\times10^{-7}C$, both the charges are in Vacuum.

Given data:-

$q_1 = 2\times10^{-9}C$ $\quad$ $F = 0.1N$

$q_2 = 4.5\times10^{-9}C$ $\quad$ $K = 9\times10^{9}$

Required

$d = ?$

Solution:-

By using formula

$$F = K\frac{q_1 q_2}{d^2}$$

$$d^2 = K\frac{q_1 q_2}{F}$$

Now putting values

$$d^2 = 9\times10^{9}\cdot\frac{(2\times10^{-7})(4.5\times10^{-7})}{0.1}$$

$$d^2 = 8.1\times10^{-3}$$

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As

$d^2 = 0.0081$

Now taking under root on both sides.

$\sqrt{d^2} = \sqrt{0.0081}$

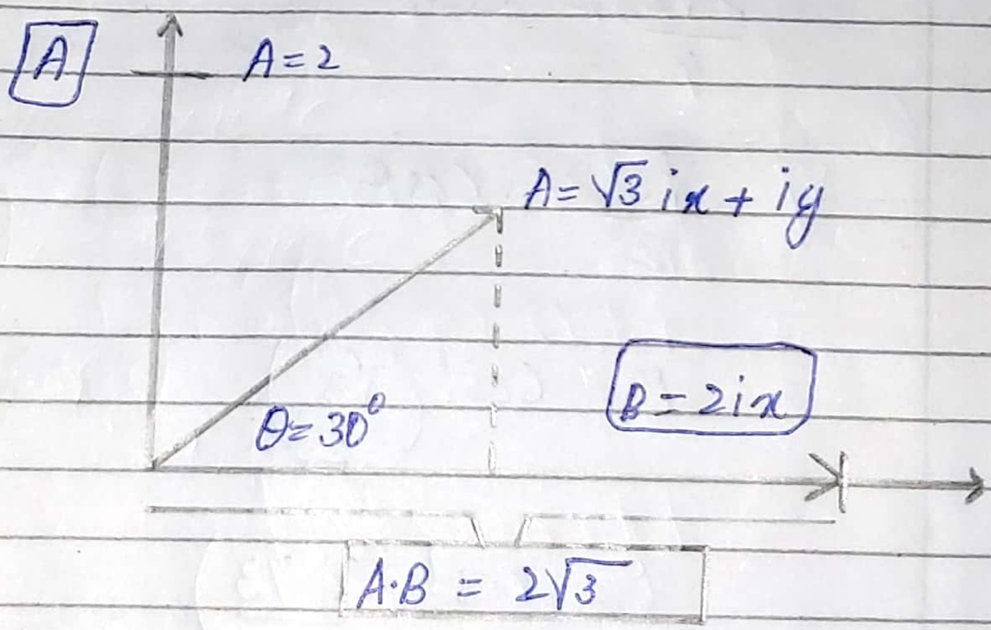
$d = 0.09\ m$

$d = 9 \times 10^{-9}\ m$

OR

$\boxed{d = 9 cm}$

Q No. 2 :- (A):- Find the angle between the vector shown in figure:-



Solution:-

As

$$A \cdot B = |A||B| \cos\theta \longrightarrow (*)$$

$$A \cdot B = 2\sqrt{3}$$

$$|A| = \sqrt{2^2} \quad , \quad |B| = \sqrt{2^2}$$

$$|A| = 2 \quad , \quad |B| = 2$$

put in e.q (*).

So ($\star$) becomes

$$2\sqrt{3} = 2 \times 2 \cos\theta$$

$$2\sqrt{3} = 4\cos\theta.$$

$$\frac{2\sqrt{3}}{4} = \cos\theta$$

$$\frac{\sqrt{3}}{2} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = \cos^{-1}\left(\frac{1.73}{2}\right)$$

$$\theta = \cos^{-1}(0.866)$$

$$\boxed{\theta = 30^\circ}$$

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(B) Find the gradient of each of the following functions where "a" and "b" are constant.

(i) $f = ax^2 + by^3z$.

Sol:-

$$f = ax^2 + by^2z$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(ax^2 + by^2z)$$

$$\frac{\partial f}{\partial x} = 2ax$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(ax^2 + by^3z)$$

$$\frac{\partial f}{\partial z} = 3by^2z$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(ax^2 + by^3z)$$

$$\frac{\partial f}{\partial z} = by^3$$

$$\nabla f(x, y, z) = (2ax + 3by^2z, by^3).$$

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(ii) $f = ar^2 \sin\phi + brz\cos 2\phi$.

Sol- gradient for spherical.

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}.$$

So

$$\nabla f = \frac{\partial}{\partial r}(ar^2\sin\phi + brz\cos 2\phi)\hat{r} + \left(\frac{1}{r}\frac{\partial}{\partial\theta}(ar^2\sin\phi + brz\cos 2\phi)\right)\hat{\theta} + \left(\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(ar^2\sin\phi + brz\cos 2\phi)\right)\hat{\phi}.$$

taking partial derivative:-

$$\nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} + \frac{1}{r}(0) + \frac{1}{r\sin\theta}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}.$$

So

$$\nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} + \frac{1}{r\sin\theta}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}$$

P.T.O

Now gradient for cylindrical

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla f = \frac{\partial}{\partial \rho}(ar^2 \sin\phi + brz\cos 2\phi)\hat{\rho} + \frac{1}{\rho}\frac{\partial}{\partial \phi}(ar^2 \sin\phi + brz\cos 2\phi)\hat{\phi} + \frac{\partial}{\partial z}(ar^2\sin\phi + brz\cos 2\phi)\hat{z}.$$

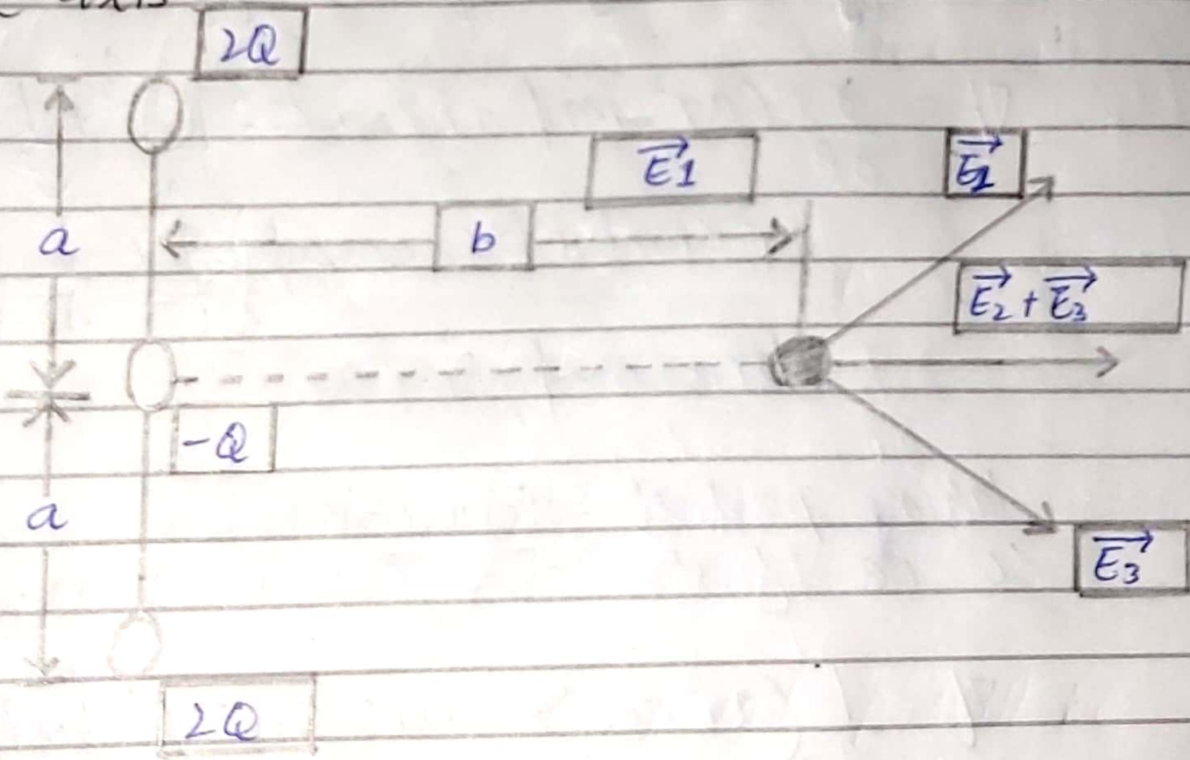
taking partial derivatives:.
so the first term becomes zero.

$$\nabla f = \frac{1}{\rho}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi} + (br\cos 2\phi)\hat{z}.$$

So

$$\nabla f = \frac{1}{\rho}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi} + (br\cos 2\phi)\hat{z}$$

Q3:- Three pointer charges are placed on the Y-axis as shown - Find the electric field at point "P" on the x-axis -



Sol:- The distance between charge 2Q and point "P". is

$$r^2 = b^2 + a^2$$

so

$$r = \sqrt{b^2 + a^2}$$

Let assume that charges 2Q makes angle (α) and $(-\alpha)$ with x-axis -

Magnitude of $|\vec{E_1}| = |\vec{E_2}| = \frac{Kq}{r^2}$

$$= \frac{K(2Q)}{r^2}$$

$$= \frac{K(2Q)}{b^2+a^2}$$

So Resultant of $\vec{E_1}$ and $\vec{E_2}$ is

$$\vec{E}_{1+2} = \vec{E}_1 + \vec{E}_2 = \vec{E}_{1x} + \vec{E}_{2x}$$

(y- component will be cancel).

$$\vec{E}_{1+2} = \frac{K(2Q)}{b^2+a^2} (\cos(\alpha) + \cos(-\alpha))$$

$$\vec{E}_{1+2} = \frac{K(2Q)}{b^2+a^2} (2\cos(\alpha))$$

So

$$\cos(\alpha) = \cos(-\alpha)$$

$$\vec{E}_{1+2} = \frac{K(2Q)}{b^2+a^2} (2\cos(\alpha)) \longrightarrow (i)$$

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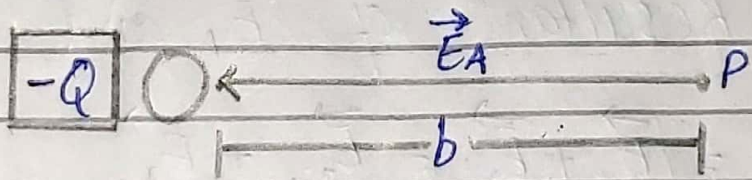
↳ Now electric field at point "p" due to charge "-Q"-

↳ As charge is Negative Electric field at point will be directed towards charge "-Q"-



So

$$\vec{E}_A = \frac{-K(Q)}{b^2}$$

Net electric field at point "p" will be

$$\vec{E}_{net} = \vec{E}_A + (\vec{E}_1 + \vec{E}_2)$$

$$\vec{E}_{net} = \frac{-K(Q)}{b^2} + \frac{4K\,Q\cos\alpha}{b^2+a^2}$$

$$\vec{E}_{net} = \frac{-KQ(a^2+b^2) + 4KQb^2\cos\alpha}{b^2(a^2+b^2)}$$

$$\vec{E}_{net} = \frac{KQ}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

where $K = 9\times10^9\ Nm^2/C^2$

$$\vec{E}_{Net} = \frac{9\times10^{9}Q}{b^{2}(a^{2}+b^{2})}\left[4b^{2}\cos\alpha-(a^{2}+b^{2})\right]$$

Now

$$\alpha = \tan^{-1}\left(\frac{a}{b}\right)$$

So

$$\vec{E}_{Net} = \frac{9\times10^{9}Q}{b^{2}(a^{2}+b^{2})}\left[4b^{2}\cos\left[\tan^{-1}\left(\frac{a}{b}\right)-(a^{2}+b^{2}\right]\right]$$