Date: / /

Q#01

(1) The order of matrix A is $m \times p$ & the order of B is $p \times n$ then the order of matrix AB is ?

Sol

The order of matrix is equal to the number of its row multiply by no of column.

Similarly.

$$B = p \times n$$

then its "p" no of rows and n has no of column.

Also the number of column in A is equal to the no of rows in B so there matrix are conformable for multiplication and there order will be

$$\boxed{AB = m \times n}$$

Date: / /

2

(ii) The number of non-zero rows in Echelon form?

Number of non-zero Rows in Echelon form is called Rank of the matrix -

Date: / /

3

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Sol :- For singular matrix $|B| = 0$

So $|B| = 1 \times a - 4 \times 2 = 0$

$= a - 8 = 0$

So value of $a = 8$

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ Then $|A| = ?$

Solution $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$|A| = (2i)(-i) - (i)(i)$

$|A| = -2i^2 - (i^2)$

$|A| = -2(-1) - (-1)$

$|A| = 2 + 1$

$\boxed{|A| = 3}$

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(9)

(v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

Solution

⇒ If each element of a principal diagonal of a matrix is some non-zero scalar and all other elements are zero then it is a scalar matrix

So,

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a scalar matrix

(vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Ans

⇒ $\frac{dy}{dx} + 2xy = y$

$\frac{dy}{dx} = y - 2xy$

$\frac{dy}{dx} = y(1-2x)$

$y\,dy = (1-2x)\,dx$

$\int y\,dy = \int (1-2x)\,dx$

$\frac{y^2}{2} = x - \frac{2x^2}{2} + c$

$\boxed{y^2 = 2x - 4x^2 + 2c}$

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Date: / /

(5)

(vii) The order & degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Sol :- The order of differential equation is the highest derivative known as differential co-efficient and degree is the power of highest derivative so,

order = 1
degree = 3

(viii) The order & degree of differential equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is?}$$

Order = 2
Degree = undefine b/c The derivative of dependent variable is appear in the domain of transcendental function.

Date: / /

(ix) The differential equation $2\frac{dy}{dx} + x^2 y = 2x + 3,\ y(0) = 5$ is ?

Sol $2\frac{dy}{dx} + x^2 y = 2x + 3$

$\int 2\,dy = \int (2x + 3 - x^2 y)\,dx$

$2y = \frac{\cancel{2}x^2}{\cancel{2}} + 3x - y\frac{x^3}{3} + C$

$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + C$

put $x = 0,\ y = 5$

$5 = 0 + 0 - 0 + C$

$5 = C$

Then

$$\boxed{y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + 5}$$

Date: / /

(7)

(1) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol.

Expand by R_1

$$|A| = +1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= 1(bc^2 + b^2c) - a(c^2 - b^2) + a^2(c - b)$$

$$|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$$

Q#02(i)
Express The Determinant?

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

As the product of factors which are linear in a, b, c -

Sol :-
$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

Common abc

$$= abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

ⓐ

$$= abc\left[bc(c-b) - ac(c+a) + ab(b-a)\right]$$

Ans

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Date: / /

Q No 2 part B

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol :-

$$A - \lambda I = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$= A - \lambda I = \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

$$= (2-\lambda)\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$+(-1)\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow$$

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Date: / /

Again

$$\begin{bmatrix} 3-\lambda & -1 & -1 \\ 1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix} = 0$$

Expand by R_1

$$= 3-\lambda \begin{bmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} -(-1)\begin{bmatrix} -1 & -1 \\ -1 & 2-\lambda \end{bmatrix} -1 \begin{bmatrix} -1 & 3-\lambda \\ -1 & -1 \end{bmatrix}$$

$$= 3-\lambda \left[(3-\lambda)(2-\lambda) - (-1)(-1) + 1(-1)(2-\lambda) - (-1)(-1) - 1(-1) - (-1)(3-\lambda) \right]$$

$$= (3-\lambda)(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + (-2 + \lambda - 1) - (1 + 3 - \lambda)$$

$$= (3-\lambda)(\lambda^2 - 5\lambda + 5) + (-3 + \lambda) - (4 - \lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \rightarrow (1)$$

$$\Rightarrow \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ 1 & 2-\lambda \end{vmatrix} + 0$$

$$= -1\left(6 - 3\lambda - 2\lambda + \lambda^2 - 1\right) + \left(-2 + \lambda - 1\right)$$

$$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= \boxed{-\lambda^2 + 6\lambda - 8} \rightarrow \text{equation}(2)$$

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$= \left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow \text{equation } 3$$

Put 1 2 & 3 in eq (A)

$$= (2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8)-\lambda^2+6\lambda-8-\lambda^2 + 6\lambda-8 = 0$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda - \lambda^2+6\lambda-8-\lambda^2+6\lambda-8 = 0$$

Date: / /

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

Now
We solve the above equation By synthetic division

	1	10	32	32
2	↓		16	32
	1	-8	16	0

So,
we get

$= (\lambda - 2)(\lambda^3 - 8\cancel{\lambda} + 16\lambda) = 0$

$= \lambda(\lambda - 2)(\lambda^2 - 8\lambda + 16) = 0$

$\lambda = 0$ | $\lambda = 2$ | $\lambda^2 - 8\lambda + 16 = 0$

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$(\lambda - 4)(\lambda - 4)$

$\lambda = 4$ | $\lambda = 4$

So,
$\lambda_1 = 0$
$\lambda_2 = 2$
$\lambda_3 = 4$
$\lambda_4 = 4$ Ans

Babar Regis...

Q#3

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$x = 2$, $y = 6$

Sol

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$\Rightarrow (x^2+3y^2)dx = 2xy\,dy$$

Dividing both sides by "$2xy\,dx$"

$$\frac{\cancel{2xy}\,dy}{\cancel{2xy}\,dx} = \frac{(x^2+3y^2)\,\cancel{dx}}{2xy\,\cancel{dx}}$$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2\cancel{x}y} + \frac{3y^2}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \longrightarrow \textcircled{*}$$

Date: / /

2

let $y = vx$

Diff: $dy = v\,dx + x\,dv$

Dividing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (a)$$

put (a) in (A)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying Both Sides by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1 + v^2}{v}$$

Multiplying both sides by $\frac{dx}{dv}$
we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiplying both sides by $\frac{v}{x(1+v^2)}$
we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

put $v = y/x$

$$1+(y/x)^2 = xc$$

4

Date: / /

$$\frac{x^2+y^2}{x^2} = xC$$

$$x^2+y^2 = x^3 C \rightarrow (**)$$

put $x=2$, $y=6$ in eq (**)

$$4+36 = 8C$$

$$C = \frac{40}{8}$$

$$\boxed{C=5}$$

So $x^2+y^2 = 5x^3$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking "$\sqrt{\ }$" on both sides

$$y = +x\sqrt{5x-1}, \quad y = -x\sqrt{5x-1}$$

$$\boxed{y = \pm x\sqrt{5x-1}}$$ Ans