Final Exam Summer

Course Name: Linear Algebra

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BS (SE-8) Section: A

Submitted To:

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Q1

Answer:

Q1) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$

Solution :-

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} (-\lambda) & 1 & 0 \\ 0 & (-\lambda) & 1 \\ 4 & -17 & (8-\lambda) \end{vmatrix} = 0$$

$\therefore (-\lambda)((-\lambda)\times(8-\lambda) - 1\times(-17)) - 1(0\times(8-\lambda) - 1\times 4) + 0(0\times(-17) - (-\lambda)\times 4) = 0$

$\therefore (-\lambda)((-8\lambda+\lambda^2) - (-17)) - 1(0-4) + 0(0-(-4\lambda)) =$

$\therefore (-\lambda)(17 - 8\lambda + \lambda^2) - 1(-4) + 0(4\lambda) = 0$

$\therefore (-17\lambda + 8\lambda^2 - \lambda^3) - (-4) + 0 = 0$

$\therefore (-\lambda^3 + 8\lambda^2 - 17\lambda + 4) = 0$

$\therefore -(\lambda-4)(\lambda-0.26794919)(\lambda-3.73205081) =$

$\therefore (\lambda-4) = 0$ or $(\lambda-0.26794919) = 0$ or $(\lambda - 3.73205081) = 0$

The eigenvalues of the matrix A are given by $\lambda = 0.26794919, 3.73205081, 4$.

Q2

Answer:

Q2)

Solution:

a) A can be diagonalized if there exists an invertible matrix P and diagonal matrix D such that $A = PDP^{-1}$

Hence $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

Find the eigenvalues of the matrix A.

$|A - \lambda I| = 0$

$$\begin{vmatrix} (-\lambda) & 0 & -2 \\ 1 & (2-\lambda) & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$(-\lambda)((2-\lambda) \times (3-\lambda) - 1 \times 0) - 0(1 \times (3-\lambda) - 1 \times 1) + (-2)(1 \times 0 - (2-\lambda) \times 1) = 0$$

$$\therefore (-\lambda)((6 - 5\lambda + \lambda^2) - 0) - 0((3-\lambda) - 1) - 2(0 - (2-\lambda)) = 0$$

$$\therefore (-\lambda)(6 - 5\lambda + \lambda^2) - 0(2-\lambda) - 2(-2+\lambda) = 0$$

$$\therefore \cancel{(2-\lambda)(6\lambda + 5\lambda^2 - \lambda^3) + 0(2-\lambda) - 2(-2+\lambda)=}$$

$$\therefore (-6\lambda + 5\lambda^2 - \lambda^3) - 0 - (-4 + 2\lambda) = 0$$

$\therefore \ (-\lambda^3 + 5\lambda^2 - 8\lambda + 4) = 0$

$\therefore \ -(\lambda - 1)(\lambda - 2)(\lambda - 2) = 0$

$\therefore \ (\lambda - 1) = 0$ or $(\lambda - 2) = 0$ or $(\lambda - 2) = 0$

The eigenvalues of the matrix A are given by $\lambda = 1, 2$

2) Eigenvectors for $\lambda = 2$

$$v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

2. The eigenvectors compose the columns of matrix P

$$\therefore P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

1. The diagonal matrix D is composed of the eigenvalues.

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

3 Now find P^{-1}

$$P = \begin{vmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= -2 \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0 \times \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 1 \times \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$= -2 \times (1 \times 1 - 0 \times 0) + 0 \times (1 \times 1 - 0 \times 1) - 1 \times (1 \times 0 - 1 \times 1)$

$= -2 \times (1 + 0) + 0 \times (1 + 0) - 1 \times (0 - 1)$

$= -2 \times (1) + 0 \times (1) - 1 \times (-1)$

$= -2 + 0 + 1$

$= -1$

Q3

Answer:

Q3)

Solution

Here $A = (1, -2, 3)$, $B = (5, 6, -1)$

$C = (3, 2, 1)$

The vectors A, B, C are Linerally dependent if their determinant is zero i.e $|D| = 0$

$$|D| = \begin{vmatrix} 1 & -2 & 3 \\ 5 & 6 & -1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} 6 & -1 \\ 2 & 1 \end{vmatrix} + 2 \times \begin{vmatrix} 5 & -1 \\ 3 & 1 \end{vmatrix} + 3 \times \begin{vmatrix} 5 & 6 \\ 3 & 2 \end{vmatrix}$$

$$= 1 \times (6 \times 1 - (-1) \times 2) + 2 \times (5 \times 1 - (-1) \times 3) + 3 \times (5 \times 2 - 6 \times 3)$$

$$= 1 \times (6 + 2) + 2 \times (5 + 3) + 3 \times (10 - 18)$$

$$= 1 \times (8) + 2 \times (8) + 3 \times (-8)$$

$$= 8 + 16 - 24$$

$$= 0$$

Since $|D| = 0$,

So vectors A, B, C

are linearly dependent.

Q4

Part (A)

Answer:

A vector is a set V on which two operation + and - are defined, called vector addition and scalar multiplication.

The operation + (vector addition) must satisfy the following conditions:

Closure: if u and v are any vectors in V, then the sum u + v belongs to V.

1. Commutative law: For all vectors u and v in V, u + v=v + u.

2. Associative law: For all vectors u, v, w in V, u + (v + w) = (u + v) + w.

3. Additive identity: The set V contains an additive identity element, denote by 0, such that for any vector v in V, 0 + v=v and v + 0=v.

4. Additive inverses: for each vector v in V, the equations v + x = 0 and x + v = 0 have a solution x in V, called an additive inverse of v, and denoted by – v.

The set Pn(x) of all the polynomials over R in variable x of degree <= n forms a vector space over R.

If Fn(x) = a0 + a1x +…… + anxn

And gn(x) = b0 + b1x +……. +bn xn, ai, bi E R

Then Fn(x) + gn (x) = (a0 + b0) + (a1 + b1) x +…… (an + bn) xn

The associative additive property is included from the additive associative property of R.

The zero polynomial f(x) = 0 of degree 0 acts as the additive identity of P(x) and –f(x) = -a0 + (-a1) x +…. + (-an) xn additive inverse of fn(x).

Commutative property follows from the commutative property of R. Hence Pn(x) is an additive abelian group.

The scalar multiplication of a E R by fn (x) = aa0 + (aa1) x + (aa2) x2…. + (aan) xn E Pn(x).

it observes properties of scalar multiplication which can easily be verified. So that pn(x) forms a vector space over R.

Part(B)

Answer:

Question #14(b)

Let F be a field, A non-empty set V together with two binary operations (+) and (·) form the algebraic structure of a vector space over the field F, if

(a) V forms an additive (+) abelian group

(b) The scalar multiplication (·) as a function from $F \times V$ into V observes the following properties:

(i) $\forall \alpha \in F, x, y \in V, \alpha(x+y) = \alpha x + \alpha y$

(ii) $\forall \alpha, \beta \in F, (\alpha+\beta)x = \alpha x + \beta x \quad \forall x \in V$

(iii) $\forall \alpha, \beta \in F, \forall x \in V; \alpha(\beta x) = (\alpha$

(iv) For e_F, the identity of F, $e_F x = x$

$\forall x \in V.$