Assignment No# 2

Teachear name
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Sub:
Linear algebra

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Q 1:- Compute adjoin of :

(i) $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

Sol:-

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 6 - 1 = \boxed{5}$$

$$A_{12} = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = \boxed{1}$$

$$A_{13} = \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = 2 - 9 = \boxed{-7}$$

$$A_{21} = \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} = 4 - 5 = \boxed{-1}$$

$$A_{22} = \begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} = 2 - 15 = \boxed{-13}$$

$$A_{23} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1 - 6 = \boxed{-5}$$

$$A_{31} = \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} = 2 - 15 = \boxed{-13}$$

$$A_{32} = \begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = 1 - 10 = \boxed{-9}$$

$$A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = \boxed{-1}$$

Co-factor of matrix A

$$\begin{bmatrix} +5 & -1 & -7 \\ +1 & -13 & +5 \\ -13 & +9 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -1 & -7 \\ 1 & -13 & 5 \\ -13 & 9 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 5 & 1 & -13 \\ -1 & -13 & 9 \\ -7 & 5 & -1 \end{bmatrix}$$ Ans.

(ii) Sol:- $B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$

$B_{11} = \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = -8-(-2)8 = \boxed{8}$

$B_{12} = \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = 16-40 = \boxed{-24}$

$B_{13} = \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -4-(-5) = \boxed{1}$

$B_{21} = \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = 32-(-10) = \boxed{42}$

$B_{22} = \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = 24-25 = \boxed{-1}$

$B_{23} = \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -6-20 = \boxed{-26}$

$B_{31} = \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = 32-(-5) = \boxed{37}$

$B_{32} = \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = 24-10 = \boxed{14}$

$B_{33} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3-8 = \boxed{-11}$

Co-factor of matrix B

$$\begin{bmatrix} 8 & -24 & 1 \\ 42 & -1 & -26 \\ 37 & 14 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 8 & -42 & 37 \\ 24 & -1 & -14 \\ 1 & 26 & -11 \end{bmatrix} \text{ Ans}$$

Q2:- Find the co-factor of A_{21}, A_{31}, A_{33} if $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$

Solution:-

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix} \quad (-1)^{i+j}$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = -(-2)(2)-(-3)(3)$$
$$= 13$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = (-2)(1)-(3)(3)$$
$$= -11$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = (1)(3)-(-2)(-2)$$
$$= -1$$

So

Co-factor of A_{21}, A_{31}, A_{33}

$$= \begin{bmatrix} 1 & -2 & 3 \\ 13 & 3 & 1 \\ -11 & -3 & -1 \end{bmatrix}$$

Ans ✓

Q3:- Find Eigen values and Eigen vectors.

Solution:-

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$A - \lambda I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$2-\lambda((3-\lambda)(2-\lambda) - 2) - 1((1)(2-\lambda)+2) + 1$$
$$(1 - (3-\lambda)(-1)$$

$$2-\lambda(6 - 3\lambda - 2\lambda + \lambda^2 - 2) - 1(4-\lambda) +$$
$$4-\lambda$$

$$2-\lambda(4 - 5\lambda + \lambda^2) \cancel{-4} + \cancel{\lambda} + \cancel{4} - \cancel{\lambda} = 0$$

$$2-\lambda(\lambda^2 - 5\lambda + 4) = 0$$

$$2-\lambda = 0$$

$2 - \cancel{x} + \cancel{x} = 0 + x$

$2 = x$

or

$\boxed{x = 2}$

$x^2 - 5x + 4 = 0$

$x^2 - 4x - x + 4 = 0$

$x(x - 4) - 1(x - 4) = 0$

$(x - 1)(x - 4) = 0$

$x - 1 = 0$

$x - \cancel{1} + \cancel{1} = 0 + 1$

$\boxed{x = 1}$

$x - 4 = 0$

$x - \cancel{4} + \cancel{4} = 0 + 4$

$\boxed{x = 4}$

Eigen vector:

$[A - \lambda I][x] = 0$

$$\begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ 1 & 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

For $\lambda = 1$

$$\begin{bmatrix} 2-1 & 1 & 1 \\ 1 & 3-1 & 2 \\ 1 & 1 & 2-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + y + z = 0 \longrightarrow (1)$

$x + 2y + 2z = 0 \longrightarrow (2)$

$x + y + z = 0 \longrightarrow (3)$

$$\frac{x}{0} = \frac{\cancel{-}y}{\cancel{-}1} = \frac{z}{-1}$$

$$\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \rightarrow \text{Eigen vectors for } \lambda = 1$$

R.W

~~For x~~

$x = \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix}$

$x = 2 - 2$

$\boxed{x = 0}$

$y = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$

$y = 1 - 2$

$\boxed{y = -1}$

$z = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$

$z = 1 - 2$

$\boxed{z = -1}$