Q#1

$$\frac{dy}{dt} = e^{y-t}\sec(y)(1+t^2) \qquad y(0)=0$$

Solution

$$\frac{dy}{dt} = \frac{e^y}{e^t}\frac{1}{\cos y}(1+t^2)$$

or

$$e^{-y}\cos y\,dy = e^{-t}(1+t^2)dt$$

integrating b/s

$$\int e^{-y}\cos y\,dy = \int e^{-t}(1+t^2)dt \quad \text{——} (i)$$

By parts

$$\cos y\cdot\frac{e^{-y}}{-1} - \int + \sin y\cdot\frac{e^{-y}}{+1}dy = \int e^{-t}(1+t^2)dt$$

$$-e^{-y}\cdot\cos y - [-\sin y\cdot e^{-y} + \int \cos y\cdot e^{-y}dy = \int e^{-t}(1+t^2)dt$$

$$-e^{-y}\cdot\cos y + e^{-y}\sin y - \int \cos y\cdot e^{-}dy = \int e^{-t}(1+t)dt$$

$$e^{-y}(\sin y - \cos y) - \int \cos y\cdot e^{-y}dy = \int e^{-t}(1+t^2)dt$$

From equation ①.

$$\int e^{-t}(1+t^2)dt = \int e^{-y}\cos dy$$

So

$$e^{-y}(\sin y-\cos y)-\int e^{-t}(1+t^2)dt=\int e^{-t}(1+t^2)dt$$

$$e^{-y}(\sin y-\cos y)=\int e^{-t}(1+t^2)dt+\int e^{-t}(1+t^2)dt$$

$$e^{-y}(\sin y-\cos y)=2\int e^{-t}(1+t^2)dt.$$

$$e^{-y}(\sin y-\cos y)=2\left[(1+t^2)\frac{e^{-t}}{-1}+\int 2t\cdot e^{-t}dt\right]$$

$$e^{-y}(\sin y-\cos y)=2\left[e^{-t}(1+t^2)+2\left(\frac{te^{-t}}{-1}+\int e^{-t}dt\right)\right]$$

$$e^{-y}(\sin y-\cos y)=2\left[e^{-t}(1+t^2)+2\left[te^{-t}-e^{-t}\right]\right]+c$$

$$e^{-y}(\sin y-\cos y)=2\left[-e^{-t}-e^{-t}\cdot t^2-2t\cdot e^{-t}-2e^{-t}\right]+c$$

$$e^{-y}(\sin y-\cos y)=2\left[-3e^{-t}-t^2\cdot e^{-t}-2t\cdot e^{-t}\right]+c$$

$$\boxed{e^{-y}(\sin y-\cos y)=-2e^{-t}\left[t^2+2t+3\right]+C}$$

Scanned with CamScanner

Q#2: Solve

$$(\sqrt{x+y}+\sqrt{x-y})\,dx-(\sqrt{x+y}-\sqrt{x-y})\,dy=0$$

Solution:-

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \longrightarrow (1)$$

This is homogeneous differential equation in x and y to solve this put

$$y=vx$$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

This eq (1) becomes.

$$v+x\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{1+v+\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}\left(1+\sqrt{1-v^2}\right)}{\cancel{2}v}$$

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}$$

taking integral on b/s

$$\int \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \int \frac{dx}{x}$$

put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int \frac{-dt}{t} = \int \frac{du}{u}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln\left(1+\sqrt{1-u^2}\right) = \ln cx$$

$$\ln\left(1+\sqrt{1-v^2}\right) = -\ln cx$$

$$\cancel{\ln}\left(1+\sqrt{1-v^2}\right) = \cancel{\ln}(cx)^{-1}$$

$$\left(1+\sqrt{1-v^2}\right) = \frac{1}{cx}$$

$$1+\sqrt{1^2-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$$x+\sqrt{x^2-y^2} = C_1 \qquad \because \ 1/c = C_1$$

which is a required solution

7879

Q No:3:-

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

Solution:-

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

$$\Rightarrow F(D)y = f(x)$$

As it is non-homogeneous linear equation

So solution will be.

$$y = y_c + y_p \longrightarrow (i)$$

Complementary solution y_c

$$D^4 + D^2 = 0 \Rightarrow D^2(D^2+1) = 0$$

Either $D^2 = 0 \Rightarrow \boxed{D = 0}$

$$D^2 + 1 = 0 \Rightarrow D^2 = -1$$

$$D = \sqrt{-1} \Rightarrow D = i \text{ or } D = \boxed{0 + i}$$

Roots are real complex.

$$y_c = c_1 e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$$

$$y_c = c_1 + c_2 \cos x + c_3 \sin x$$

$$y_p = \frac{1}{F(D)} f(x)$$

Scanned with CamScanner

$$\therefore y_P = \frac{1}{D^4+D^2}\left(3x^2 + 4\sin x - 2\cos x\right)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$$F(D) = D^4 + D^2$$

at $D=0 \Rightarrow F(D)=0$

So $f'(D) = 4D^3 + 2D$

Now also for $D=0 \Rightarrow f'(D)=0$

again differentiating

$$f''(D) = 12D + 2$$

So for $D=0$

$$f''(0) = 12(0) + 2 = 2$$

So replacing $\frac{1}{F(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_p = \frac{x^2\, 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2}\, 2\cos x$$

putting $D=0$ in all

So,

$$y_P = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3x^4}{2} + 2x^2 \sin x - x^2 \cos x$$

So putting in eq (i)

$$y = C_1 + C_2 \cos x + C_3 \cos x + 3/2\, x^4 + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = C_1 + (C_2 - x^2) \cos x + (C_3 + 2x^2) \sin x + 3/2\, x^4}$$