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Paper	Probability & Statistics
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QUESTION - No - 1

A man throws two fair dice, what is the conditional probility that the sum of the two sum dice will be 7, given that

1- The sum is even
2- The sum is greater
3- The two dice had the same outcome.

Sol:-

s = { (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }

Let A = { The sum is 7 }
B = { The sum is even }
C = { The sum is greater than 8 }
D = { The two dice lead the same outcom }

Then

A = { (1,6) (2,5) (3,4) (4,3) (5,2) (6,1) }
B = { (1,1) (1,3) (1,5) (2,2) (2,4) (2,6) ..... (6,6) }
C = { (3,6) (4,5) (4,6) (4,6) (5,4) (5,5) (5,6) (6,3) ...... (6,6) }
D = { (1,1) (2,2) (3,3) (4,4) (5,5) (6,6) }

$(A \cap B) = \{ \}$
$(A \cap C) = \{ \}$
$(A \cap D) = \phi$

$P(A) = \frac{6}{36}$, $P(B) = \frac{18}{86}$, $P(C) = \frac{10}{36}$, $P(D) = \frac{6}{36}$

$P(A \cap B) = \frac{6}{36}$, $P(A \cap C) = \frac{6}{36}$ and $P(A \cap D) = 0$

$$P(A/B) = P\left(\frac{A \cap B}{P(B)}\right) = \frac{6}{36} \times \frac{36}{18} = \frac{1}{3}$$

$$P(A/C) = P\left(\frac{A \cap C}{P(C)}\right) = \frac{6}{36} \times \frac{36}{18} = \frac{3}{5}$$

$$P(A/D) = P\left(\frac{A \cap D}{P(D)}\right) = 0 \times \frac{36}{6} = 0$$

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QUESTION - No- 2

ANS:- sum of 2 has 1 way 1, 1
sum of 3 has 2 ways 1,2 and 2,1
sum of 4 has 3 ways (1,3) and (2,2), (3,1)
5 has 4 ways
6 has 4 ways
8 has 5 ways
9 has 4 ways
10 has 3 ways
11 has 2 ways
12 has 1 ways

=> 15/36 for each side with a sum of 30/36

=> That leaves a 6/36 = 1/6 probility for sum of 7

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QUESTION- No- 3

A and B play a game in which A's probability of winning is 2/3 In a series of 8 games. what is the probability that A will win.

1- Exactly 4 games
2- Atleast 4 games
3- From 3 to 6 games

Sol:

According to the given condition we have:

$$p = \frac{2}{3} \quad , \quad n = 8$$

$$q = 1 - p$$

$$q = 1 - \frac{2}{3}$$

$$q = \frac{1}{3}$$

Let 'x' be the number of games won by "A"

1- Exactly 4 games $\longrightarrow P(x=4)$

$$\Rightarrow \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561}$$

$$\Rightarrow 0.1707$$

2 Atleast 4 games $\longrightarrow P(x \geq 4)$

$= 1 - P(x < 4)$

$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$

$= 1 - \left\{\left(\frac{1}{3}\right)^8 + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^7 + 28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 + 56\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5\right\}$

$\Rightarrow 1 - \frac{1}{6561}\{1 + 16 + 112 + 448\}$

$\Rightarrow 1 - \frac{577}{6561}$

$\Rightarrow \frac{6561 - 577}{6561}$

$\Rightarrow \frac{5984}{6561}$

$\Rightarrow 0.9121$

3. 3 to 6 games $\longrightarrow P(3 \leq x \leq 6)$

$\Rightarrow \sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$

$= \binom{8}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5 + \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4 + \binom{8}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^3$
$\binom{8}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^2$

$\Rightarrow \frac{8}{3^8}\{56 + 140 + 224 + 224\}$

$\Rightarrow \frac{8 \times 644}{6561}$

$\Rightarrow \frac{5152}{6561} \Rightarrow 0.7852$

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QUESTION - No - 4

ANS:-

The C_i's from a partition of the sample space we can apply the Law of total probability for $A \cap B$

$$P(A \cap B) = \sum_{i=1}^{m} P(A \cap B / c_i)\, P(c_i)$$

$$P(A \cap B) = \sum_{i=1}^{m} P(A/c_i)\, P(B/c_i)\, P(c_i)$$

=> A & B are conditionally independent.

$$P(A \cap B) = \sum_{i=1}^{m} P(A/c_i)\, P(B)\, P(c_i)$$

=> B is independent of all c_i's

$$P(A \cap B) = P(B) \sum_{i=1}^{m} P(A/c_i) P(c_i)$$

$$P(A \cap B) = P(B)\, P(A)$$

Law of total probability

So therefore, A & B are independent.

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QUESTION-No-5

Derive the binomial distribution and find its mean and variance.

Ans:-

Proof:- $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

for $x = 0, 1, 2 \ldots n$

$\mu = np$

$\sigma^2 = np(1-p)$

A binomial random variable can be thought of as the sum of n independent Bernoulli random variables each with mean p and variance $p(1-p)$

Let $u_1 \ldots u_n$ be independent Bernoulli random variable

$E(u_i) = p$ and $Var(u_i) = p(1-p)$

$X = u_1 + \ldots + u_n$

$$var(x) = var(U_1) + \dots + var(U_n)$$

The Bionomial theorm

$$(a+b)^m = \sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y}$$

$$E(x) = \sum_{x} x p(x)$$

$$E(x) = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

$$E(x) = \sum_{x=0}^{n} x \frac{n}{x!(n-x)(n-1)-(x-1)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-1)-(x-1)!}$$

$$m = (n-1), \quad y = (x-1)$$

$$= np \sum_{y=0}^{m} \frac{m}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}$$

Now

$$var(x) = E((x-u)^2)$$

$$= \sum_{x} (x-u)^2 p(x)$$

$$E((x-u)^2) = E(x^2) - (E(x))^2$$

$$E(x(x-1)) = \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)} p^{x-2} (1-p)^{n-x}$$

By Bionomial theorem

$E(x(x-1)) = n(n-1)p^2$

$E(x^2 - x) = n(n-1)p^2$

$E(x^2) - E(x) = n(n-1)p^2$

since $E(x) = np$ which is mean of Bionomial distribution

$E(x^2) = n(n-1)p^2$

$E(x^2) = n(n-1)p^2 + np$

$var(x) = E(x^2) - (E(x))^2$

$var(x) = n(n-1)p^2 + np - (np)^2$

$var(x) = np[(n-1)p + 1 - np]$

which variance of Bionomial distribution.

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QUESTION- No-6

ANS:-

Bi-nominal distribution:-

The Bi-nominal distribution is denoted and formulated by

$f(x) = P(X=x) = \binom{n}{x} p^x q^{n-x}$

where is $x = 0, 1, 2, 3, \ldots, n$

It shows only the probability of an individual.

Bi-nomial Frequency Distribution:-

If the bi-nomial probability distribution is multiplied by N, the number of experiments or sets, the resulting distribution is known as the binomial frequency distribution. Thus the expected frequency of x successes in N experiments is $N.\binom{n}{x} p^x q^{n-x}$.

It should be noted that the n independent trials constitute one experiment or one set.

x——x——x——x

QUESTION - No - 7

ANSWER:-

Measure	Data set 'A'	B	C	D
Cofficient of variation	$CV = \frac{3}{45} \times 100$	$CV = \frac{11}{60} \times 100$	$CV = \frac{5}{50} \times 100$	$CV = \frac{15}{25} \times 100$
	CV = 6.7	CV = 18.3	CV = 10	CV = 60

x——x——x