Course Title:

EMF

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Module:

4th

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16th April, 2020

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Question No: 1 (a);

Solve the following;

Transform the vector $B = yi(x+z)j$ located at point $(-2, 6, 3)$ into cylindrical coordinates.

Given:

$$B = yi(x+z)j$$
$$(-2, 6, 3)$$

Required:

Transform the vector into cylindrical co-ordinates.

Solution:

we have to find
ρ, z, ϕ

for ρ:

As

$$\rho = \sqrt{x^2+y^2}$$
$$\rho = \sqrt{(-2)^2+(6)^2}$$
$$\rho = \sqrt{40}$$
$$\rho = 6.32$$

for z :

As

$$Z = Z$$

So

$$z = 3$$

for ϕ :

As

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\phi = \tan^{-1}\left(\frac{6}{-2}\right)$$

$$\phi = \tan^{-1}(-3)$$

$$\phi = -71.56$$

Result:

$B = 6.32, -71.56, 3$ Ans

Question No: 1 (b) ;

Convert the point (3,4,5) from cartesian to spherical co-ordinates

Given:

point (3,4,5)

Required:

Convert from Cartesian to Spherical co-ordinates

Solution:

$P(3,4,5)$

$x = 3, \quad y = 4, \quad z = 5$

we have to find

r, θ, ϕ

For r:

As

$$r = \sqrt{x^2+y^2+z^2}$$

$$r = \sqrt{3^2+4^2+5^2}$$

$$r = \sqrt{9+16+25}$$

$$r = \sqrt{50}$$

$$r = 7.07$$

For θ :

As

$\theta = \tan^{-1}$

$\theta = \tan^{-1}(4/3)$

$\theta = \tan^{-1}(1.33)$

$\theta = 53.1^\circ$

For ϕ :

As

$$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{3^2+4^2}}{5}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{9+16}}{5}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{25}}{5}\right)$$

$$\phi = \tan^{-1}\left(\frac{5}{5}\right)$$

$$\phi = \tan^{-1}(1)$$

$$\phi = 45^\circ$$

Result:

$r = 7.07$, $\theta = 53.1$, $\phi = 45^\circ$

Question No: 1 (c) ;

Find the spherical co-ordinates of A (2, 3, -1)

Given:

A (2, 3, -1)

Required:

Spherical co-ordinates

Solution:

we have to find r, θ, ϕ

for r:

As

$$r = \sqrt{x^2+y^2+z^2}$$

$$r = \sqrt{2^2+3^2+(-1)^2}$$

$r = \sqrt{14}$

$r = 3.74$

For θ:

As

$\theta = \tan^{-1}(y/x)$

$\theta = \tan^{-1}(3/2)$

$\theta = \tan^{-1}(1.5)$

$\theta = 56.3^\circ$

for ϕ:

As

$$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{2^2+3^2}}{-1}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{13}}{-1}\right)$$

$\phi = \tan^{-1}(-3.60)$

$\phi = 74.4$

Result:

$r = 3.74$, $\theta = 56.3^\circ$, $\phi = 74.4$

Question No: 1 (f);

Find the electric field intensity of two charges $-2C$ & $-1C$ seperated by a distance 1m in air

Given:

$q_1 = -2C$

$q_2 = -1C$

$d = 1m$

Required:

$E = ?$

Solution:

For E_1:

$$E_1 = \frac{kq_1}{d}$$

$$E_1 = \frac{9 \times 10^9 \times -2}{(1)^2}$$

$$E_1 = -18 \times 10^9 \ V/m$$

For E_2:

$$E_2 = \frac{kq_2}{d}$$

$$E_2 = \frac{9\times10^9 \times(-1)}{(1)^2}$$

$$E_2 = -9\times10^9 \ V/m$$

For E_T:

As

$$E_T = E_1 + E_2$$

$$E_T = -18\times10^9 + (-9\times10^9)$$

$$E_T = -18\times10^9 - 9\times10^9$$

$$E_T = -27\times10^9 \ V/m$$

Question No:1 (g);

Deteomine the charge that produce an electric field strength of 40 V/cm at a distance of 30cm in Vacuum (in 10^{-8} C)

Given:

$$E = 40 \ V/cm$$

$$d = 30 \ cm$$

Required

$q = ?$

Solution:

As

$$E = \frac{kq}{d^2}$$

$$Ed^2 = kq$$

$$\frac{Ed^2}{k} = q$$

$$q = \frac{Ed^2}{k}$$

putting the values

$$q = \frac{40 \times (30)^2}{9 \times 10^9}$$

$$q = \frac{40 \times 900}{9 \times 10^9}$$

$$q = 4 \times 10^{-6}\ C$$

OR

$$q = 4\ \mu C$$

Question No: 1 (h);

A charge of 2×10^{-7} is acted upon by a force of 0.1 N. Determine the distance to the other charge of 4.5×10^{-7} C. Both the charges are in vacuum.

Given:

$q_1 = 2 \times 10^{-7}$

$q_2 = 4.5 \times 10^{-7}$

$F = 0.1 N$

Required:

$d = ?$

Solution:

As

$$F = k \frac{q_1 q_2}{d^2}$$

$$d^2 = k \frac{q_1 q_2}{F}$$

putting the values

$$d^2 = \frac{9 \times 10^9 (2 \times 10^{-7})(4.5 \times 10^{-7})}{0.1}$$

$d^2 = 8.1 \times 10^{-3}$

OR

$d^2 = 0.0081$

Taking under root on b/s

$\sqrt{d^2} = \sqrt{0.0081}$

$d = 0.09\,m$

OR

$d = 9cm$

Result:

$d = 9cm$ Ans

Question No: 1 (e);

Find the force between two charges when they are brought in contact and seperated by 4cm apart, charges are 2nC & -1 nC in air

Given:

$q_1 = 2nC$

$q_2 = -1nC$

$d = 4cm$

Required:

$F = ?$

Solution:

As

$$F = k \frac{q_1 q_2}{r^2}$$

where

$$k = \frac{1}{4\pi \varepsilon_0}$$

putting the values

$$F = \frac{2\times10^{-9} \times -1\times10^{-9}}{4(3.14)\times8.85\times10^{-12}\times(4\times10^{-2})^2}$$

$$F = -1.124 \times 10^{-5}$$

$$F = -11.24\ \mu N$$

Question No : 1 (d) ;

Find the cartesian co-ordinates of $B(4, 25, 120)$

Given:

B(4, 25, 120) it's in spherical co-ordinates. $r = 4$, $\theta = 25$, $\phi = 120$

Required:

Cartesian co-ordinates (x, y, z)

Solution:

for x:

As

$x = r \sin\theta \cdot \cos\phi$

$x = 4 \sin 25 \cdot \cos 120$

$x = 4(0.42)(-0.5)$

$x = -0.84$

For y:

As

$y = r \sin\theta \cdot \sin\phi$

$y = 4 \sin 25 \cdot \sin(120)$

$y = 4(0.42)(0.86)$

$y = 1.45$

For z:

As

$z = r \cos\theta$

$z = 4 \cos(25)$

$Z = 4(0.90)$

$Z = 3.62$

Result:

$(x, y, z) = (-0.84, 1.45, 3.62)$

Question 2: (a)

Find the angle between the vector shown in figure.

Given:

$A = \sqrt{3}\,ix + iy$

$|A| = 2$

$B = 2ix$

$|B| = 2$

$A.B = 2\sqrt{3}$

Required:

Find the angle between the vector

Solution:

As

$$A \cdot B = |A||B| \cos\theta \rightarrow (i)$$

$$A \cdot B = 2\sqrt{3}$$

$$|A| = \sqrt{2^2} = 2$$

$$|B| = \sqrt{2^2} = 2$$

putting values in (i)

we get,

$$2\sqrt{3} = 2 \times 2 \cos\theta$$

$$2\sqrt{3} = 4 \cos\theta$$

$$\frac{\cancel{2}\sqrt{3}}{\cancel{4}_{2}} = \cos\theta$$

$$\frac{\sqrt{3}}{2} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = 30^\circ$$

Question No: 2 (b):

· Find the gradient of each of the following functions where "a" & "b" are constant

(i) $f = ax^2 + by^3 z$

Solution:

$$f = ax^2 + by^3 z$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(ax^2 + by^3 z)$$

$$\frac{\partial f}{\partial x} = 2ax$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(ax^2 + by^3 z)$$

$$\frac{\partial f}{\partial y} = \cancel{\frac{\partial}{\partial y}}\ 3by^2 z$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(ax^2 + by^3 z)$$

$$\frac{\partial f}{\partial z} = by^3$$

Result:

$$\nabla f(x,y,z) = (2ax + 3by^2 z, by^3)$$

(ii) $f = ar^2 \sin\phi + brz \cos 2\phi$

Solution:

As Gradient for Spherical are;

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

So

$$\nabla f = \frac{\partial}{\partial r}(ar^2 \sin\phi + brz \cos 2\phi)\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}(ar^2 \sin\phi + brz \cos 2\phi)\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(ar^2 \sin\phi + brz \cos 2\phi)\hat{\phi}.$$

Taking Partial Derivative.

$$\nabla f = (2ar \sin\phi + bz \cos 2\phi)\hat{r} + \frac{1}{r}(0) + \frac{1}{r\sin\theta}(ar^2 \cos\phi - 2brz \sin 2\phi)\hat{\phi}.$$

So

$$\nabla f = (2ar \sin\phi + bz \cos 2\phi)\hat{r} + \frac{1}{r\sin\theta}(ar^2 \cos\phi - 2brz \sin 2\phi)\hat{\phi}.$$

Gradient for Cylindrical axe;

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla f = \frac{\partial}{\partial \rho}(ar^2 \sin\phi + brz\cos 2\phi)\hat{\rho} + \frac{1}{\rho}\frac{\partial}{\partial \phi}(ar^2 \sin\phi + brz\cos 2\phi)\hat{\phi} + \frac{\partial}{\partial z}(ar^2 \sin\phi + brz\cos 2\phi)\hat{z}.$$

Taking Partial Derivative.

The first term become zero.

$$\nabla f = \frac{1}{\rho}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi} + (br\cos 2\phi)\hat{z}.$$

So

$$\nabla f = \frac{1}{\rho}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi} + (br\cos 2\phi)\hat{z}.$$

Question No: 3

Three pointer charges are placed on the y-axis as shown. Find the electric field at point P on the x-axis.

Solution:

The distance between charges 2Q & point "p" is

$$r^2 = b^2 + a^2$$

$$r = \sqrt{b^2+a^2}$$

Let assume that charge 2Q make angle (θ) & ($-\theta$) with x-axis

magnitude of $|\vec{E_1}| = |\vec{E_2}| = \frac{kq}{r^2}$

$$= \frac{k(2Q)}{r^2}$$

$$= \cancel{k2(Q)}$$

$$= \frac{k(2Q)}{b^2+a^2}$$

So
Resultant of $\vec{E_1}$ & $\vec{E_2}$ is

$$\vec{E}_{1+2} = \vec{E_1} + \vec{E_2} = \vec{E}_{1x} + \vec{E}_{2x}$$

The Y-components will be cancelled

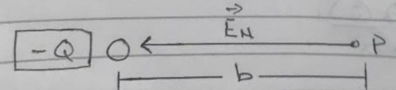
$$= \frac{k(2Q)}{b^2+a^2} (\cos(\theta) + \cos(-\theta))$$

$$= \frac{k(2Q)}{b^2+a^2} (2\cos(\theta)) \quad \because \cos(\theta) = \cos(-\theta)$$

$$\vec{E}_{1+2} = \frac{4kQ\cos(\theta)}{b^2+a^2} \rightarrow (a)$$

The electric field at point "P" due to charge "-Q"

As the charge is negative, Electric field at point will be directed towards charge "-Q".



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$$\vec{E}_N = \frac{-k(Q)}{b^2}$$

Net eletric field at point "p" will be

$$\vec{E}_{Net} = \vec{E}_N + (\vec{E}_1 + \vec{E}_2)$$

$$= \frac{-k(Q)}{b^2} + \frac{4kQ\cos(\theta)}{b^2 + a^2}$$

$$= \frac{-kQ}{b^2(a^2+b^2)}\left[4b^2\cos\theta - (a^2-b^2)\right]$$

where $k = 9\times10^9\ Nm^2/C^2$

$$\vec{E}_{Net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\theta - (a^2-b^2)\right]$$

Now

$$\theta = \tan^{-1}\left(\frac{a}{b}\right)$$

Result:

$$\vec{E}_{Net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\left[\tan^{-1}\left(\frac{a}{b}\right)\right] - (a^2+b^2)\right]$$