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SEMESTER	10^{th}
SECTION	B
PAPER	DIFFERENTIAL EQUATION
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Q NO 1 :-

(1) The order of matrix A is $m \times p$ and the order of B is $p \times n$. Then the order of matrix AB is?

Ans: The order of matrix is equal to the number of its row multiply by no. of column. So,

A = $m \times p$ has "m" no. of rows and p no. of column. Similarly,

$$B = p \times n$$

Then its "p" no. of rows and n has no. of column. Also the number of column in A is equal to the no. of row in B. So there matrix are comformable for multiplication and there order w be.

$$\boxed{AB = m \times n}$$

(2) The number of non zero rows in Echelon form?

Ans: Above the diagonal.

(3) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a Singular matrix then

$a = ?$

Ans: For Singular matrix $|B| = 0$

So $|B| = 1 \times a - 4 \times 2 = 0$

$= a - 8 = 0$

So value of $a = 8$.

(4) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Ans: Now,

$$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= (2i)(-i) - (i)(i)$$

$$= -2i - i^2$$

As we know that $i = -1$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$\boxed{|A| = 3}.$$

⑤ The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Ans: If each element of a principle diagnal of a matrix is some non zero scalar and all other elements are zero then it is a scalar matrix so;

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a scalar matrix.

⑥ Solution of $\frac{dy}{dx} + 2xy = y$?

Ans:

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

$$y\,dy = (1-2x)\,dx$$

$$\int y\,dy = \int (1-2x)\,dx$$

$$\frac{y^2}{2} = x - \frac{2x^2}{2} + c$$

$$y^2 = 2x - 4x^2 + 2C.$$

7: The order and degree of differential equation:

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad \text{is ?}$$

Ans: The order of differential equation is the order of highest derivative known as differential co-efficient and the degree is the power of highest derivative so;

order = 1

degree = 3.

8: The order and degree of differential equation is $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) = ?$

Ans: The order of a differential equation is the order of the highest order derivative occuring in the equation.

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \quad \text{is } 2.$$

2. **Degree:** The degree of a differential equation is the power of the highest order derivative occuring in the equation.

$$\left(\frac{d^2y}{dx^2}\right)^1 - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \quad \text{is } 1.$$

(5)

9: The differential equation $2\frac{dy}{dx} + x^2 y = 2x+3$, $y(0) = 5$ is ?

Ans:

$$2\frac{dy}{dx} + x^2 y = 2x+3$$

$$\int 2\,dy = \int (2x+3-x^2y)\,dx$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + c$$

$$y = \frac{\cancel{2}x^2}{\cancel{2}\times 2} + \frac{3x}{2} - \frac{yx^3}{3\times 2} + c$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + c$$

put $x=0$, $y=5$

$$5 = 0 + 0 - 0 + c$$

$$5 = c$$

then $$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + 5.$$

10:
$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Ans: As we know that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by Row 1

$$|A| = 1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$|A| = 1(bc^2 - b^2c) - a(c^2 - b^2) + a^2(c - b)$$

$$|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b.$$

QNo 2:

i - Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factor's which are linear in a, b, c.

ns: As

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$\Rightarrow a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$\Rightarrow a(b^2c^3 - c^2b^3) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$\Rightarrow ab^2c^3 - ac^2b^3 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$$

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$|A| = a_{13} A_{13} + a_{23} A_{23} + a_{33} A_{33}$

$$|A| = c \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix} - c^2 \begin{vmatrix} a & b \\ a^3 & b^3 \end{vmatrix} + c^3 \begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix}$$

$$= c(a^2b^3 - a^3b^2) - c^2(ab^3 - ba^3) + c^3(ab^2 - ba^2)$$

$$= a^2b^3c - a^3b^2c - ab^3c^2 + a^3bc^2 + ab^2c^3 - a^2bc^3$$

Now re-arrange above equation

$$= ab^2c^3 - ac^2b^3 - a^2bc^3 + a^2b^3c + a^3bc^2$$

$$- a^3b^2c$$

NOTE:

We get the same result, no matter which row or column is used to expand a 3 X 3 detarminant

P.T.O)

ii- Find the Eigen value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Ans: Solution: As we know that

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic equation $|A - \lambda I| = 0 \longrightarrow$ eq (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$\Rightarrow 2-\lambda\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow \text{(B)}$$

Again $\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$ Expand by R_1

$$\Rightarrow 3-\lambda\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\Big[(3-\lambda)(2-\lambda)-(-1)(-1)+1(-1)(2-\lambda)-(-1)(-1)$$
$$-1((-1)(-1)-(-1)(3-\lambda))\Big]$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-(1+3-\lambda)$$

$$\Rightarrow (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$\Rightarrow 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$\Rightarrow \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \longrightarrow \text{eq (a)}$$

$$\Rightarrow \quad +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1\,(6-3\lambda-2\lambda+\lambda^2-1) + 1\,(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \longrightarrow \text{eq (b)}$$

$$\Rightarrow \quad -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$= -\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$= -\left[-1\,(-2+\lambda-1) + 1\,(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \longrightarrow \text{eq (c)}$$

put ⓐ, ⓑ & ⓒ in ⓑ

$$\Rightarrow (2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda$$

$$-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda+6\lambda$$

$$+6\lambda+16-16$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By Synthatic division we get;

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0$$

$$\boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By Factorization method

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)$$

$$\lambda=4,\ \lambda=4$$

$$\boxed{\lambda_1=0,\ \lambda_2=2,\ \lambda_3=4,\ \lambda_4=4}$$

Q3: The rate of change in the form of differential equation is given by $(x^2+3y^2)\,dx - 2xy\,dy = 0$. Find the general solution at $x = 2$ and $y = 6$.

Ans: **Solution:** As we know that

$$(x^2 + 3y^2)\,dx - 2xy\,dy = 0$$

$$\Rightarrow \quad (x^2 + 3y^2)\,dx = 2xy\,dy$$

÷ivde both sides by $2xy\,dx$, we get:

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \longrightarrow \otimes$$

Let $y = vx$

Diff:

$$dy = v\,dx + x\,dv$$

÷iving by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow (a)$$

put eq (a) in (x)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{x}{xv} + 3\frac{vx}{x}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

multiplying both sides by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

multiplying both sides by $\frac{dx}{dv}$ we get;

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

xing both sides by $\frac{v}{x(1+v^2)}$, we get:

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2}\,dv = \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln(1+v^2)} = e^{\ln(xc)}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

put $v = y/x$

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \longrightarrow (y)$$

put $x=2$, $y=6$ in eq (y)

$$(4)+(36) = 8c$$

$$c = 40/8$$

$$\boxed{c = 5} \longrightarrow \text{put in eq } (y)$$

So

$$x^2 + y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x - 1)$$

Taking Square root on both sides;

$$y = +x\sqrt{5x-1}, \quad y = -x\sqrt{5x-1}$$

or;

$$y = \pm x\sqrt{5x-1}.$$