Q/1 $F(t) = 1+t \quad -\pi \leq t \leq \pi$ 1

Here we use the formula.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt \rightarrow eq 1$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi^2}{2}\right)\right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2}\right)$$

$$\boxed{a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2\right)}$$

2 $$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)dt$$

$$a_n = \frac{1}{\pi} \left((1+t) \frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{\sin nt\, d}{n\, dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi} \left((1+t) \frac{\sin nt^0}{n} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$a_n = \frac{-1}{n^2\pi} (\cos n\pi - \cos n(-\pi))$$

$$a_n = \frac{-1}{n^2\pi} (-1 - (-1))$$

$$\boxed{a_n = 0}$$

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②

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\,dt$$

$$b_n = \frac{1}{\pi}(1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\sin nt - \frac{d}{dt}(1+t)dt\right)$$

$$b_n = \frac{1}{\pi}\left(\frac{((1+t)(-\cos nt)}{n}\Big|_{-\pi}^{\pi} + \int\left(-\frac{\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\frac{-(1+t)((\cos nt)}{n}\Big|_{-\pi}^{\pi}\left(\cancel{\frac{\sin nx}{n^2}}\Big|_{-\pi}^{\pi}\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left(-(1+\pi)((\cos n\pi) - ((1+(x))(\cos n(dt))\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi \cancel{- \cos n\pi} + \pi\cos n\pi\right)$$

$$b_n = \frac{-1}{n\cancel{\pi}}(2\cancel{\pi}\cos n\pi)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

So eq become

$$\boxed{f(x) = \frac{1}{2\pi}(2\pi+\pi^2) + 0 + \sum_{n=1}^{\infty}\frac{2}{n}(-1)^{n+1}\sin nt}$$

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Ans, 02

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values = ?

Sol:

$\boxed{\text{Step} = 01}$

We have

$(A - \lambda I)X = 0$ A = Given Matrix.

We have, The Characteristic equation is given by

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step # 2

$$\lambda^3 - \left|\begin{matrix}\text{Sum of} \\ \text{Diagnol element}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of} \\ \text{Diagnol minors}\end{matrix}\right| \lambda - |A| = 0 \quad \text{— (B)}$$

Sum of Diagnol elements $= 1 + 1 + 2 = 4$

Sum of Diagnol Minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= -6 + 2 + 1$

$= -3$

By putting value in (B)

$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0$ (C)

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

By putting value in (B) ;

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

5

Using Quadratic formula.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -4$
$c = -3$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2}$$

$$\frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 \pm \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

We have eigenvalues

$$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2}\right)$$ Required Solution.

Q No=3 Solve the following System of Linear equations

$$5x + 0 + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2x + 0 = 1$$
$$x + y + z + m = 0$$

Solution:

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad R_4 - R_2$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1+4/5 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad -1/5 + R_3$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \quad \frac{1}{5} \times R_3 \text{ and } 5 \times R_4$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 6 & 7 & 6 & 1 \end{array}\right] \xrightarrow{5\times R_3 \text{ and } 5\times R_4}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 6 & 7 & 8 & 1 \end{array}\right] \quad 1/5 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \xrightarrow{R_2 \times 5}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \xrightarrow{R_3 - R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \begin{array}{l} \xrightarrow{R_3 \Leftrightarrow R_4} \\ \xrightarrow{1/7 R_3} \\ \xrightarrow{1/3 R_4} \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \xrightarrow{(2 \times (-5)}$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/2 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \xrightarrow{\frac{5}{4} \times R_1}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = (3/4, 31/21, -11/21, 1/3)$$

$x = \frac{3}{4}$

$y = 31/21$

$z = -11/21$

$m = 1/3$

① No # 4 Verify that (10)

$u(x,t) = \sin(x+2t)$

is a solution of one-dimensional equation.

Solution:

Given that:

$u(x,t) = \sin(x+2t)$

Differentiate w.r.t x partialy

$$\frac{\partial u}{\partial x} = \frac{d}{dx}\sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)\frac{d}{dx}(x+2t)$$

$$\frac{du}{dx} = \cos(x+2t)(1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{d^2u}{dx^2} = \frac{d}{dx}\cos(x+2t)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t)\frac{d}{dx}(x+2t)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t)(1+0)$$

$$\boxed{\frac{d^2u}{dx^2} = -\sin(x+2t)}$$

and

$$u(x,t) = \sin(x+2t)$$

Differentiate w.r.t 't'

$$\frac{du}{dt} = \frac{d}{dt} \sin(x+2t)$$

$$\frac{du}{dt} = \cos(x+2t)(0+2)$$

$$\frac{du}{dt} = 2\cos(x+2t)$$

$$\frac{d^2u}{dt^2} = 2 - \sin(x+2t)(0+2)$$

$$\boxed{\frac{d^2u}{dt^2} = -4\sin(x+2t)}$$

We know that one-dimensional wave equation is

$$\frac{d^2u}{dt^2} = c^2\frac{d^2u}{dx^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

for the arbitrary constt $c = \pm 2$

$$-4\sin(x+2t)+(\pm 2)^2\sin(x+2t)=0$$

$$-4\sin(x+2t)+4\sin(x+2t)=0$$

$$0=0$$

Then it will be verified for the arbitrary Constt $C=2$.

THE END