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SAADAT

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Subject:- Differential Equation

Program:- BS(CS)

INU

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Q1:- Define differential equation along with 2 examples?

Definition:-

Differential equation:-

A differential equation is any equation which contains derivatives that may be ordinary derivatives or partial derivatives.

E.g:-

1) $\frac{dy}{dx} = 2x$

2) $\frac{dy}{dx} = 5$

$\Rightarrow dy = 5\,dx$ Integrating both sides

$\Rightarrow y = 5x + k$

$= 0\ y = 2$, we have $k = 2$ so:

$\Rightarrow y = 5x + 2$

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"Question no 1"

"Part B"

Define the Separable Differential Equation DE?

Definition:-

Separable equation:-

A separable differential equation is any differential equation that we can write in the following for

$$N(y) \frac{dy}{dx} = M(x)$$

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1) Solve the following (IVP) using Separable DE and find the interval of the Solution.

a) $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $y(0) = -1$

Solution:-

$$y' = \frac{xy^3}{\sqrt{1+x^2}} \quad y(0) = -1$$

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

By separating variables and Integrating

$$\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}} dx$$

$$\int y^{-3} dy = \frac{1}{2} \int (1+x^2)^{-1/2} 2x \, dx$$

$$\frac{y^{-3+1}}{-3+1} = \frac{1}{2} \frac{(1+x^2)^{-1/2+1}}{-1/2+1} + C$$

$$-\frac{1}{2y^2} = \frac{1}{\cancel{2}} \frac{(1+x^2)^{1/2}}{1/\cancel{2}} + C$$

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$-\frac{1}{2y^2} = (1+x^2) + C$ —— (*)

$y(0) = -1$ put $x=0$, $y=-1$ put in eq (*) we get

$\Rightarrow \frac{-1}{2} = 1 + C \Rightarrow = \frac{-1}{2} - 1$

put $\frac{-3}{2}$ in eq (*)

$C = \frac{-3}{2}$

$1+x^2 = \frac{-3}{2} = \frac{-1}{2y^2}$

$2(1+x^2) - 3 = \frac{-1}{y^2}$

$\left(\frac{1}{2(1+x^2)-3}\right) = y^2$

$\frac{1}{3-2(1+x^2)} = y$

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$$y = \frac{1}{(3-2(1+x^2)}$$

$$y = \frac{1}{(3-2)} \Rightarrow y = \frac{1}{1}$$

So: $y = 1$ is the separable equation.

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"Part B"

$y' = e^{-y}(2x-4) \quad y(5)=0$

Solution:-

$$y' = e^{-y}(2x-4) \quad y(5)=0$$

$$\frac{dy}{dx} = e^{-y}(2x-4)$$

By separating variables and Integration.

$$\int e^{y}dy = \int (2x-4)\,dx$$

$$e^{y} = \frac{\cancel{2}x^2}{\cancel{2}} - 4x + C \quad \text{—} \; (*)$$

put $x=5$ and $y=0$ in $(*)$

$$e^{0} = 5^2 - 4(5) + C$$
$$1 = 25 - 20 + C$$
$$1 = 5 + C$$
$$C = -4 \text{ put in } (*)$$
$$\boxed{e^{y} = x^2 - 4x - 4}$$

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Question no 2

Solve the following NP using linear differential method.

1) Explain the steps for solving Linear differential equation.

Answer:-

Steps:-

1) Substitute $y = uv$ and $dy/dx = u\,dv/dx + v\,du/dx$

into

$dy/dx + P(x)y = Q(x)$

2) Factor the parts involving v

3) put the v term equal to zero (this gives a differential equation in u and x which canbe solved in the next step.

4) Solve using separation of variables to find u.

5) Substitute u back into the equation we

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got at step 2

6) Solve that to find v.

7) Finally, substitude u and v into
$y = uv$ to get our solution.

"Question 2"

"Part B"

$$\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1$$

$$y[\pi/4] = 3\sqrt{2} \quad 0 \le x \le \pi/2$$

Solution:-

$$\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1$$

$$y[\pi/4] = 3\sqrt{2} \quad 0 \le x \le \pi/2$$

Dividing both side by $(\cos x)$

$$\frac{dy}{dx} + \cot x\, y = 2\cos^2 x \sin x - \sec x$$

which is linear differential equation

Compare it with

$$\frac{dy}{dx} + P(x)\, y = Q(x)$$

$P(x) = \cot x$, $Q(x) = 2\cos^2 x \sin x - \sec x$

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I.F = Integrating factor.

$$= e^{\int P(x)dx} = e^{\int \cot x\, dx} = e^{\ln \sin x} \quad I.F = \sin x$$

$$y(I.F) = \int IF\, Q(x)\, dx + C$$

$$y\sin x = \int \sin x \cdot (2\cos^2 x \sin x - \sec x)\,dx + C$$

$$= \int 2\cos^2 x \sin^2 x - \int \frac{\sin x}{\cos x} + C$$

$$= \frac{1}{2}\int \sin^2 2x + \ln\cos x + C$$

$$= \frac{1}{2}\int \frac{1+\cos 4x}{2} + \ln\cos x + C$$

$$y\sin x = \frac{1}{4}x - \frac{1}{4}\frac{\sin 4x}{4} + \ln\cos x + C$$

put $x = \frac{\pi}{4}$ and $y = 3\sqrt{2}$

$$y\sin\left(\frac{\pi}{4}\right) = \frac{1}{4}\left(\frac{\pi}{4}\right) - \frac{1}{4}\frac{\sin 4(\pi/4)}{4} + \ln\cos\frac{\pi}{4} + C$$

$$3\sqrt{2}\sin\left(\frac{\pi}{4}\right) = \frac{1}{4}\left(\frac{\pi}{4} - \frac{1}{4}\frac{\sin 4(\pi/4)}{4} + \ln\cos\frac{\pi}{4} + C\right.$$

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"Part C"

$x' + 2x = \sin t$

Solution:-

$$x' + 2x = \sin t$$
$$\frac{dx}{dt} + 2x = \sin t$$
$$\frac{dx}{dt} + P(t)\cdot x = Q(t)$$
$$P(t) = 2 \;,\; Q(t) = \sin t$$

Integrate the following

$$e^{\int 2dt} = e^{2t}$$
$$e^{2t} = \int e^{2t}\cdot \sin t + C$$

$$\int e^{2t}\cdot \sin t + C \; dt$$

$$\frac{e^{2t}}{2}\cdot \sin t - \int \frac{e^{2t}}{2}\cdot \cos t \; dt$$

$$\frac{e^{2t}}{2}\cdot \sin t \cdot \frac{1}{2} \int e^{2t}\cdot \cos t \, dt$$

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$$\frac{e^{2t}\sin t}{2} - \frac{1}{2}\left(e^{2t}.\cos t + \frac{1}{2}\int e^{2t}\right.$$

$$\frac{e^{2t}\sin t}{2} - \frac{e^{2t}\cos t}{4} - \frac{1}{4}\int e^{2t}\sin t$$

$$\frac{4}{3}\left(\frac{e^{2t}\sin t}{2} - \frac{e^{2t}\cos t}{4}\right)$$

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"Question 3"

Solve the following IVP for the exact equation and find the interval of validity for the solution.

i) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0 \quad y(0) = -3$

Solution:-

$$2xy - 9x^2 + (2y + x^2 + 1) \longrightarrow (*)$$

As the above equation is exact differential equation there exist a function.

$\phi(x,y)$ Such that

$$\frac{\partial \phi}{\partial x} = M , \frac{\partial \phi}{\partial y} = N$$

Here:

$$M = 2xy - 9x^2 , N = 2y + x^2 + 1$$

Now

$$\frac{\partial \phi}{\partial x} = 2xy - 9x^2 \longrightarrow (1) , \frac{\partial \phi}{\partial y} = 2y + x^2 + 1 \longrightarrow (2)$$

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Integrate w.r.t to "x" b/s

$\phi = \int (2xy - 9x^2)\, dx$

$\phi(x,y) = x^2y - 3x^3 + f(y)$ — 3

Differentiate equation 3 w.r.t "y"

$\frac{\partial \phi}{\partial y} = x^2 - 0 + f'(y) = x^2 + f'(y)$ → 4

Comparing 2 and 4

$\cancel{x^2} + f'(y) = 2y + \cancel{x^2} + 1$

$\frac{\partial f}{\partial y} = 2y + 1$

$f(y) = y^2 + y$ → put this in eq 3

$\phi(x,y) = x^2y - 3x^3 + f(y)$

Hence the given DE Can be written a

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$d(x^2y - 3x^3 + y^2 + y) = 0$

Integrate both sides.

$x^2y - 3x^3 + y^2 + y = C \longrightarrow (A)$

Now put

$x = 0 \; , \; y = -3$ in eq (A)

$0 = 0 + 9 - 3 = C$

$C = 0$

Hence the solution is

$x^2y - 3x^3 + y^2 + y = 6$

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ii) $\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))y' = 0$

$y(5) = 0$

Solution:-

$$\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))y' = 0$$

The given equation Canbe written as:

$$\left(\frac{2ty}{t^2+1} - 2t\right)dt + (\ln(t^2+1) - 2)\,dy = 0$$

Here $M = \frac{2ty}{t^2+1} - 2t$, $N = \ln(t^2+1) - 2$

$\frac{\partial M}{\partial y} = \frac{2t}{t^2+1}$, $\frac{\partial N}{\partial t} = \frac{1}{t^2+1} \cdot 2t$

AS $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$.

So the given equation is exact its Solution is given by

$$\int M dt + \int (\text{Terms of } N \text{ free of } t)\, dy = C$$

$y =$ Constant

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$$\int\left(\frac{2ty}{t^2+1}-2t\right)dt+\int(-2)dy=C$$

$$\boxed{y\ln(t^2+1)-t^2-2y=C} \rightarrow *$$

put $t=5$ and $y=0$

$$0\ln(5^2+1)-5^2-2(0)=C$$

$$\boxed{C=-25} \rightarrow \text{put in } *$$

$$y\ln(t^2+1)-t^2-2y=-25$$

Answer