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Subject:- calculus and analytical Geometry.

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Question # 1

Ans $\dfrac{3x^4-2x^3+5}{x^3+1}$

with respect to x :-

Sol :

$$\frac{3x^4-2x^3+5}{x^3+1}$$

Now apply Quatiant Rule

$$\frac{x^3+1 \; \frac{d}{dx}(3x^4-2x^3+5) - 3x^4-2x^3+5 \; \frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$\Rightarrow \frac{(x^3+1)(12x^4-6x^2+0) - 3x^4-2x^3+5 \cdot (3x^2+0)}{(x^3+1)^2}$$

$$= \frac{(x^3+1)(12x^4-6x^2) - (3x^4-2x^3+5)}{(x^3+1)^2}$$

$$= \frac{12x^7 - 6x^5+12x^4-6x^2-9x^6+6x^5-5}{(x^3+1)^2}$$

$$\Rightarrow \frac{12x^7 - 6x^5+12x^4-6x^2-9x^6+6x^5-15x^2}{(x^3+1)^2}$$

$$\Rightarrow \frac{12x^7-9x^6+12x^4-21x^2}{(x^3+1)^2}$$

Ans

Question # 1

B> $\frac{(x^3+1)^2}{x3-1}$

with respect to x.

Sol

by Quotient Rule

$$= \frac{x3-1 \frac{d}{dx}(x^3+1)^2 - (x^3+1)^2 \frac{d}{dx}(x^3-1)}{(x^3+1)^2}$$

$$= x3-1 \cdot \frac{2(x^3+1)\frac{d}{dx}(x^3+1) - (x^3+1)^2 \cdot 3}{(x^3-1)^2}$$

$$= (2x^3-1) \frac{2(x^3+1)\cdot 2x^2 - (x3+1)^2 \cdot 3x^2}{(x^3-1)^2}$$

$$= \frac{(2x^3-2)(2x^5+2x^2) - (x^3)^2 + (1)^2 + 2x^3)}{(x3-1)^2}$$

$$= \frac{4x^8+4x^5-4x^5-4x^2-3x^8-3x^2-6}{(x3-1)^2}$$

$$= \frac{x^8-6x^5-7x^2}{(x3-1)^2}$$

Ans.

Question #4:

A

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

If $X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$

Then

$$X = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

So $\begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

2) $\begin{bmatrix} 2+3 & 2+1 \\ -5+2 & -3+2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

$$\begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

L.H.S = R.H.S

Question # 4

(B):-

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

Sol

if $X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$

Then L.H.S = R.H.S

Put x value

$$\Rightarrow \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

So L.H.S = R.H.S

(C):-

$$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Sol:

If $X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

Then L.H.S = R.H.S

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Put value to make
L.H.S = R.H.S

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Thus L.H.S = R.H.S

Question #3

If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

find $A^2 + BC$

Sol

$A^2 = A \cdot A$ $A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1\times1 + 4\times2 & 1\times4 + 4\times1 \\ 2\times1 + 1\times2 & 2\times4 + 1\times1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} \rightarrow A^2$$

$$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3\times1 + 2\times0 & -3\times0 + 2\times1 \\ 4\times1 + 0\times0 & 4\times0 + 2\times1 \end{bmatrix}$$

$$A^2+BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2+BC = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$

Question 2:

$$B = \int \frac{1}{(8x+7)^8} dx$$

Let

$$8x+7 = t \quad \cdots (1)$$

$$\frac{d}{dx}(8x+7) = \frac{dt}{dx}$$

$$\Rightarrow 8 = \frac{dt}{dx}$$

$$= 7(8)\,dx = dt \quad \cdots (ii)$$

put (i) and (ii) in the given integral

$$\int \frac{1}{(8x+7)^8} dx = \int \frac{dt}{t^8}$$

$$= \int t^{-8} dt$$

$$= \frac{t^{-7}}{-7} = \frac{1}{7t^7}$$

$$= \int \frac{1}{(8x+7)^8} dx = \frac{1}{7(8x+7)^7} + C$$

Question # 2

A:

$$\int \frac{1}{\sqrt{x^5}}\,dx =$$

$$\int \frac{1}{x^{5-1/2}}\,dx$$

$$\int \frac{1}{x^{11/2}}\,dx$$

$$\int x^{-11/2}\,dx$$

$$= \frac{x^{-11/2+1}}{-11/2+1} + C$$

$$= \frac{x^{-9}}{-9/2}$$

$$= \frac{2}{-9}x^{-9}$$ Ans

Question # 3

B:

$$\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)}\,dx$$

by partial fractions

Sol

$$\frac{4x^2+8x}{(x^2)(x^2+2x+3)} = x_3$$

$$\frac{4x^2+8x}{(x^3)(x+3)(x+1)} = \frac{A}{x} + \frac{B}{(x+3)} + \frac{C(x+1)}{x^3}$$

$$4x^2+8x = A(x^3)(x-1)+B(x^3)(x+3)+ (Cx+D)(x+3)(x-1)$$

Put $x=0$

$$4(0)^2+8(0) = A(0)^3(0-1)+B(0)^3(0+3) + (C(0)+D)(0+3)(0-1)$$

$$0 = -3D$$

$$D = 0$$

$$\boxed{D = -1/3}$$

Q3 part (A)

Q3 (a) find $\int \frac{-x+9}{2x^2-8x+6}\,dx$

$$\frac{-x+9}{2x^2-8x+6} = \frac{-x+9}{2x^2-8x-6x+6}$$

$$= (2x^2-8x)-(6x-6)$$

$$2x(x^2-1)-6(x-1)$$

$$(x-1)(2x-6)$$

$$\frac{-x+9}{(x-1)(2x-6)}$$

Let $\frac{-x+9}{(x-1)(2x-6)} = \frac{A}{(x-1)} + \frac{B}{(2x-6)}$ ---(i)

Eq(i) is multiplied by $(x-1)(2x+6)$

$$-x+9 = A(x-1)+B(2x+6) \text{ ---(ii)}$$

put $x=1$ in (ii)

$$-1+9 = A(1-1)+B(2(1)+6) \Rightarrow 8 = B8$$

$$\frac{8}{8} = \frac{B8}{8} \Rightarrow B = 1$$

Q3 part (A)

put -3 in eq (ii)

$$-3+9 = A(-3-1) + B(2(-3)+6)$$

$$6 = A(-4) + 0$$

$$A = -\frac{6}{4} = -\frac{3}{2} \qquad A = \frac{3}{2}$$

$$\left(A = -\frac{3}{2}\right)$$

put values of A and B in (i)

$$\frac{-x+9}{2x^2-8x+6} = \frac{-3/2}{(x-1)} + \frac{1}{(2x+6)}$$

$$\Rightarrow \int \frac{-x+9}{2x^2-8x+6}\,dx = \int \frac{-3/2}{(x-1)}\,dx + \int \frac{dx}{2x+6}$$

$$= -\frac{3}{2}\ln|x-1| + \ln|2x+6|$$

$$= \ln|(x-1)^{-3/2}(2x+6)| + c \text{ Ans}$$