Digital signal processing
Assignment

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13788

Electrical

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Q(a) Determine the response $y(n)$, $n \geq 0$, of the system described by the second-order different equation.

$$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$$

To the input $x(n)=4^n u(n)$

Solution

Consider the difference equation

$$y(n)-3y(n-1)-4y(n-2)=x(n)-1\ 2x(n-1) \quad (1)$$

The homogenous equation of the system is

$$y(n)-3y(n-1)-4y(n-2)=0$$

The characteristic equation of the system is

$$\lambda-3\lambda^{-1}-4\lambda^{-2}=0$$

$$\lambda^2-3\lambda-4=0$$

Determine the root of the characteristic equation.

$\lambda^2 - 4\lambda + \lambda - 4 = 0$

$\lambda(\lambda-4) + 1(\lambda-4) = 0$

$(\lambda-4)(\lambda+1) = 0$

$\lambda = -1, 4$

The homogenous solution is,

$y(n) = c_1(-1)^n u(n) + c_2(4)^n u(n)$

Since 4 is a characteristic root and the

Excitation is

$x(n) = 4^n u(n)$

we assume a particular solution of the

Form $y_p(n) = Kn\,4^n u(n)$

Then $Kn\,4^n u(n) - 3K(n-1)4^{n-1} u(n-1) - 4K(n-2)4^{n-2} u(n-2)$

$= 4^n u(n) + 2(4)^{n-1} u(n-1)$

For $n=2$

$K(32-12) = 4^2 + 8 = 24$

$K = \frac{6}{5}$

The Total solution is.

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$y(n) = y_p(n) + y_h(n)$

$= \left[\frac{6}{5} n 4^n + C_1 4^n + C_2 (-1)^n\right] u(n)$

The solve for C_1 and C_2 we Assume that

$y(-1) = y(-2) = 0$ Then

$y(0) = 1$ and

$y(1) = 3y(0) - 1\ 4 + 2 = 9$

Hence $C_1 + C_2 = 1$ and

$\frac{24}{5} + 4C_1 - C_2 = 9$

$4C_1 - C_2 = \frac{21}{5}$

There Fore

$C_1 = \frac{26}{25}$ and $C_2 = \frac{-1}{25}$

The Total is

$y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n\right] u(n)$

(b) Determine the impulse response and unit response of the system describe by the difference equation.

$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$.

Solution.

Consider the difference Equation.

$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n)$

To obtain the homogenous equation set input

$x(n) = 0$

$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$

Determine the solution to the homogenous equation.

$y_h(n) = \lambda^n$

substitute the solution obtained in the homogenous equation.

$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$

$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$(\lambda - 0.2)(\lambda - 0.4) = 0$

There Fore the roots are

$\lambda_1 = 0.2, \quad \lambda_2 = 0.4$

Thus, the general From of the solution to the homogenous equation is,

$y_h(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$

$y(n) = C_1(0.2)^n + C_2(0.4)^n$ ......(1)

$\lambda = 0.2, \quad \lambda = 0.4$ hence

$y_h(n) = C_1 \frac{1^n}{5} + C_2 \frac{2^n}{5}$

with $x(n) = \delta(n)$, the initial condition are

$y(0) = 1,$

$y(1) - 0.6y(0) = 0$

$y(1) = 0.6$

Hence $C_1 + C_2 = 1$ and

$\frac{1}{8}C_1 + \frac{2}{5} = 0.6$

$C_1 = -1, \quad C_2 = 3,$

therefore $h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$

The step response is

$$n = \sum_{k=0}^{n} h(n-k), \quad n > 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= \frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[\left(\frac{1}{5}^{n+1} - 1\right)\right] u(n)$$

x ——— ∝ ——— ∝ ——— x ——— x ——— x ———

Q2(a) Determine the causal signal $x(n)$ having the z-Transform

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Take inverse z-Transform using partial Fraction Method.

<u>solution</u>.

The z-Transform is,

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

The Expression is written as

$$X(z) = \frac{1}{\left(1-\frac{2}{z}\right)\left(1-\frac{1}{z}\right)^2}$$

$$= \frac{1}{\left(\frac{z-2}{z}\right)\left(\frac{z-1}{z}\right)^2}$$

$$= \frac{1}{\frac{(z-2)(z-1)}{z^3}}$$

$$= \frac{z^3}{(z-2)(z-1)^2} \quad \text{——— (1)}$$

$X(z)$ has a simple pole at $P_1 = 2$ and a doble $P_2 = P_3 = 1$ partial Fraction in such a Case the Apropiate partial Fraction enpression is.

$$X(z) = \frac{z^3}{(z-2)(z-1)^2} = \frac{A_1}{z-2} + \frac{A_2}{z-1} + \frac{A_3}{(z-1)^2}$$

The Problem is to determine the Coefficient A_1, A_2, and A_3.

we proved as in the case of defined pole To determine A_1 we multiply both sides to by $(z-2)$ and evaluate the result $z = -2$

$$(z-2)X(z) = A_1 + \frac{z-2}{z-1} A_2 + \frac{z-2}{(z-1)^2} A_3$$

which we evaluated at $z=2$.

$$A_1 = \frac{(z-2)X(z)}{z} \Big|_{z=2}$$

$$A = 4$$

$$A_2 = A_1 + \frac{z-2}{z-1}$$

$$A_2 = -3$$

$$A_3 = A_1 + \frac{z-2}{z-1} A_2$$

$$= -1$$

Hence $x(n) = [4(2)^n - 3 - n]u(n)$

x——x——x—x——x——x——x——x——x——x

(b) Determine the Partial Friction exponansion of the following proper Function.

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

Solution. First we eliminate the negative power by multiplying both numenator and de-nominator by z^2 then.

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

The poles $X(z)$ are $P_1 = 1$ and $P_2 = 0.5$

Consequently the exponansion of the Form

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

A very simple method to determine A_1 and A_2 is to multiply the Equation by the ~~demon~~ denominator Term

$(z-1)(z-0.5)$ thus we obtain

$$z = (z-0.5)A_1 + (z-1)A_2 \quad \text{—— (1)}$$

Now if we set $z = P_1 = 1$ in eq(1) we eliminate the term involving A_2

Hence $1 = (1-0.5)A_1$

Thus we obtain the result $A_1 = 2$ next we return eq(1) and $P_2 = 0.8$ thus eliminating the Term involving A_1 So we have

$$0.5 = (0.8-1)A_2$$

and have $A_2 = -1$ Therefore the result of the partial Fraction expansion is

$$\frac{X(z)}{z} = \frac{z}{z-1} = \frac{1}{z-0.8} \quad \underline{\text{Ans}}$$

Q3 (a)

At $w=0$ we have

$$H(0) = \frac{b_0}{(1-P)^2} = 1$$

Hence $b_0 = (1-P)^2$

At $w = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4) - 1jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2}+jP\sqrt{2})^2}$$

Hence $$\frac{(1-P)^4}{((1-P/\sqrt{2})^2+P^2/2)^2}$$

$= 1/2$ or equivalently

$$\sqrt{2}(1-P)^2 = 1+P^2\sqrt{2}P$$

The value of $P = 0.32$ satisfies this Equation Consequently the system Function For This desired Filter is

$$H(z) = \frac{0.46}{(1 - 0.32z^{-1})^{20}}$$

Q. 3(b)

solution

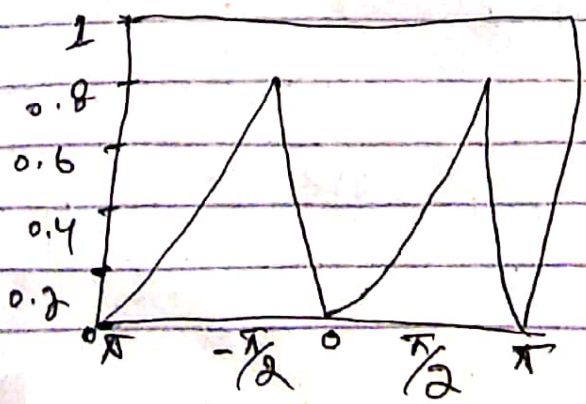
clearly the filter must have poles at

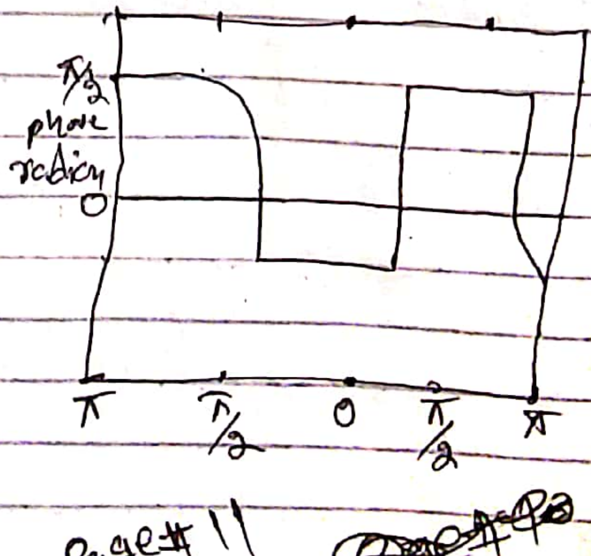
$$P_{1,2} = re^{\pm j\pi/2}$$

and zeros at $z=1$ and $z=-1$, Consequently, the is system is

$$H(z) = \frac{h(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= h\frac{z^2-1}{z^2+r^2}$$





The gain Factor is determined by evaluating the Frequency response $H(\omega)$ of the filter at $\omega = \pi/2$. Thus we have

$$H\left(\frac{\pi}{2}\right) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{2}$$

$$\frac{2-2\cos(r\pi/9)}{1+r^4+2r^2\cos(r\pi/9)} = 1/2$$

or equivalently

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation Therefore,

The system Function For the Desired Filter is

$$H(z) = 0.15 \frac{1 - z^{-2}}{1 + 0.7 z^{-2}}$$

Q4(a)

The Fourier Transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1/2)}$$

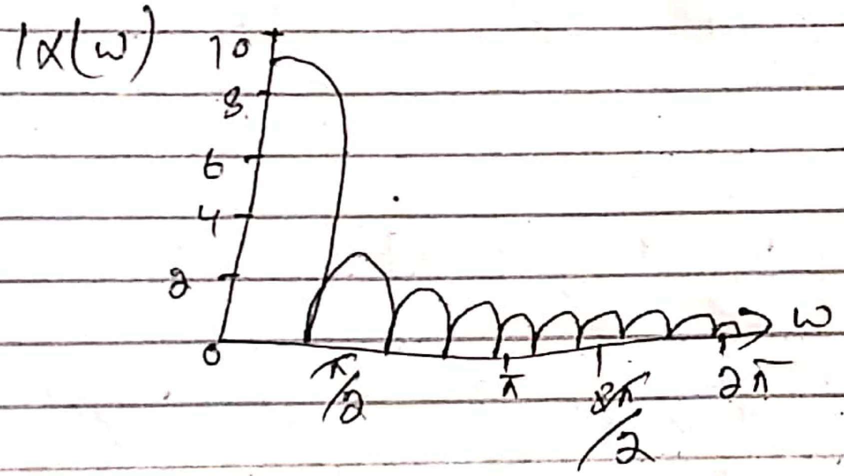
The magnitude and phase of $X(\omega)$ are

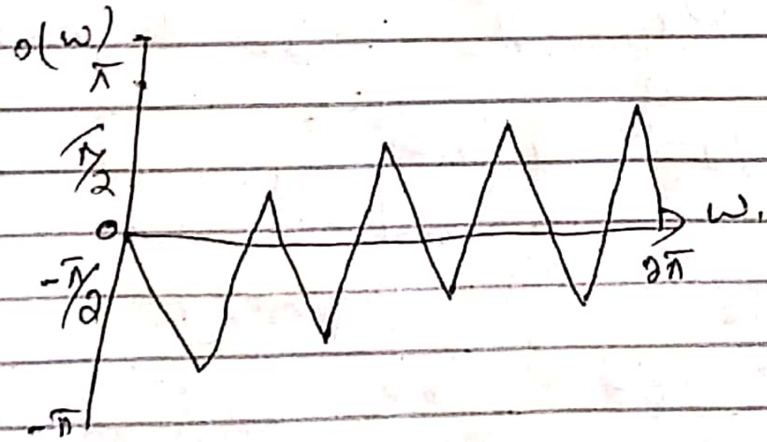
$L=10$ The N-Point DFT of $x(n)$ is simply $X(\omega)$ envalted at the set of N equally speed.

Frequencies $\omega_k = 2\pi k/N$, $k = 0, 1 \ldots N-1$. Hence

$$X(k) = \frac{1 - e^{-j2\pi k L/N}}{1 - e^{-j2\pi k/N}}, \quad k = 0, 1 \ldots N-1$$

$$= \frac{\sin(\pi k L/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$





if N is selected such that $N = L$, then the DFT become.

$$X(K) = \begin{cases} 4 & K=0 \\ 0, & K=1,2,\ldots 2-1 \end{cases}$$

This There is only one non zero value in DFT This is apparent From observation of,

$X(w)$ since $X(w)=0$

at the Frequencies $w_K = 2\pi K/2$

$K \neq 0$. The reador should verify

that $x(n)$
Can be recovered From $X(K)$

by performing and

1-Point DFT.

Q4 (b)

The first step is to determine the Matrix W_4 By exploiting the periodaicity property of W_4 and the Symmetry property.

$$W_N^{K+N/2} = -W_N^{K}$$

the Matrix W_4 may be expressed as.

$$W_4 = \begin{bmatrix} W_4 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^2 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^4 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -j \\ 1 & j & -1 & j \end{bmatrix}$$

Then $y_4 = W_4 Y_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$

The DFT of Y_4 may be determine by Conjugating the Element in W_4 to obtain W_4 and then Applying the ~~df~~ Formula.