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13789

BSCS

Differential equation

Q.1) a) i) Define differential equation along with 2 example

Ans) A differential equation is an equation that relate one or more function and their derivatives in application, the functions genrally represent physical quantities the derivatives represent their rate of change and differential equation define relation between the two.

$$dy/dx = f(x)$$

here 'x' is an independent variable and 'y' is dependent variable

e.g $dy/dx = 5x$

$dy/dx = 7x$

B) Define a seperable differential equation?

Ans) Seperable differential equation is used when the differential equation can be written in form ~~$dy/dx + p(x)y$~~ ($dy/dx = f(y)g(x)$ where 'f' is the function of 'y' only 'g' is the function of x only. Taking an initial condition

e.g

$$\int \frac{1}{12000-S} \frac{dS}{dt} dt = \int \frac{3}{200} dt$$

$$\Rightarrow \int \frac{1}{12000-S} dS = \int \frac{3}{200} dt$$

i) Solution of IVP

$$y' = \frac{xy^3}{\sqrt{1+x^2}}, \quad y(0) = -1.$$

Sol:

$$x = 0, \quad y = -1$$

$$\Rightarrow \frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\Rightarrow \frac{dy}{y^3} = \frac{x}{\sqrt{1+x^2}}\,dx$$

Taking Integration on b/s.

$$\Rightarrow \int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}}\,dx$$ → solving R.H.S by subst method.

Let $u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\Rightarrow \frac{y^{-3+1}}{-3+1} =$$

$$\Rightarrow \frac{y^{-2}}{-2} = \int \frac{1}{\sqrt{u}} \cdot \frac{du}{2} = \int \frac{1}{2\sqrt{u}}\,du = \sqrt{u} = \sqrt{1+x^2} + C$$

$$= -\frac{1}{2y^2} = \sqrt{1+x^2} + C$$

Now put $y = -1, x = 0$ we get

$$y' = \frac{xy^3}{\sqrt{1+x^2}}, \quad y(0) = -1.$$

Sol:

$x = 0, \quad y = -1$

$$\Rightarrow \frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\Rightarrow \frac{dy}{y^3} = \frac{x}{\sqrt{1+x^2}}\,dx$$

Taking Integration on b/s.

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Let $u = 1+x^2$

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$$= -\frac{1}{2y^2} = \sqrt{1+x^2} + C$$

Now put $y = -1, x = 0$ we get

$$= -\frac{1}{2(-1)^2} = \sqrt{1+(0)^2} + C \Rightarrow C = -1/2$$

$$\boxed{\Rightarrow \frac{1}{2y^2} = \sqrt{1+x^2} - \frac{1}{2}}$$ Solution of IVP.

(ii) SD equation.

$$\frac{dx}{dt} = \frac{t}{x}$$

Sol:

$$\frac{dx}{dt} = \frac{t}{x}$$

$$\Rightarrow x\,dx = t\,dt$$

Taking integral on b/s.

$$\int x\,dx = \int t\,dt$$

$$\frac{x^2}{2} = \frac{t^2}{2} + c$$

$$\Rightarrow x^2 = t^2 + 2c$$

$$\boxed{\Rightarrow x^2 = t^2 + c \quad \text{or} \quad x = \sqrt{t^2+c}}$$ Ans.

Q_2 (a) 1^{st} step is to convert D.E into its standard form. i.e $y' + P(x)y = Q(x)$.

(b) then we will find the integrating factor by $e^{\int P(x)dx}$.

(c) multiply b/s with the integrating factor.

(d) simplify the relation.

(e) take integral on both sides.

(f) put the value of x and y.

(g) find c.

(h) put value of 'c' in seperable equation.

(i) $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin x - 1$

Sol:

divide by $\cos(x)$ on b/s we get

$$\Rightarrow y' + \frac{\sin x}{\cos x}y = 2\frac{\cos^3 x}{\cos x}\sin x - \frac{1}{\cos x}$$

$$\Rightarrow y' + \tan(x)y = 2\cos^2 x\sin x - \sec x.$$

$$\Rightarrow \frac{dy}{dx} + \tan(x)y = 2\cos^2 x\sin x - \sec x. \rightarrow (a)$$

Now finding integrating factor.

$\cancel{e^{\int \ldots dx}}$

$e^{\int \tan x\, dx}$.

$\Rightarrow e^{\ln \sec x + c}$

Now multiply I.F on b/s of eq (a) (4)

$\Rightarrow \sec x \frac{dy}{dx} + \sec(x)\tan(x)\, y = 2\cos^2 x \sin x \sec x - \sec^2 x$

Above eq can be written as

$\Rightarrow \frac{d}{dx}\left(\sec(x)\, y\right) = 2\cos x \sin x - \sec^2 x$

Now taking integral

$\Rightarrow \int \frac{d}{dx}\left(\sec(x)\, y\right) = \int 2\cos x \sin x\, dx - \int \sec^2 x\, dx$

$\Rightarrow \sec(x)\, y = \cancel{2}\left(\frac{1}{\cancel{2}} \sin^2 x\right) - \tan x + C_1$

$\Rightarrow \sec(x)\, y = \sin^2 x - \tan x + C$

$\Rightarrow y = \frac{\sin^2 x}{\sec x} - \frac{\tan x}{\sec x} + C$

$\Rightarrow y = \sin^2 x \cos x - \sin x + C$ — (b)

Now $y = 3\sqrt{2}$ and $x = \frac{\pi}{4}$ so

$\Rightarrow 3/\sqrt{2} = \sin^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) - \sin \pi/4 + C$

$\Rightarrow 3/\sqrt{2} = \left(\frac{1}{\sqrt{2}}\right)^2 \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= 3/\cancel{\sqrt{2}} = \frac{1}{2}\frac{1}{\cancel{\sqrt{2}}} - \frac{1}{\cancel{\sqrt{2}}} + C$

$\Rightarrow \quad 3 = -\frac{1}{2} + c$

$= \quad 3 + \frac{1}{2} = c$

$\Rightarrow \quad \frac{7}{2} = C$

Now eq (b) becomes.

$$\boxed{\Rightarrow \quad y = \sin^2 x \cos x - \sin x + 7/2}$$ Ans

(ii) $x' + 2x = \sin t$

Sol:

$\frac{dx}{dt} + 2x = \sin t$ — (a)

Find I.F

$e^{\int 2 dx} = e^{2x} + c$

multiply eq(a) by I.F on b/s.

$e^{2x} \cdot \frac{dx}{dt} + e^{2x} \cdot 2x = e^{2x} \cdot \sin t$

$\frac{d}{dt}(e^{2x} \cdot x) = e^{2x} \cdot \sin t$.

Taking integral on b/s

$\int \frac{d}{dt}(e^{2x} \cdot x) = e^{2x} \cdot \sin t$

$\Rightarrow e^{\cdot} \cdot x = e^{2x} \sin t$ (6)

$$\boxed{\Rightarrow x = \sin t}$$

Answer

Q(3)(i) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0, y(0) = -3$

Sol:

~~[illegible]~~

In this equation we ~~have~~ can take $M = (2xy - 9x^2)$ & $N = 2y + x^2 + 1$.

Lets take a function ψ and integrate N w.r.t. to y.

$$\psi = \int N(x,y)\, dy.$$

$$= \int (2y + x^2 + 1)\, dy.$$

$$= \cancel{\int (y^2 + x^2y + y)}\ h(x)$$

$$= (2y + x^2y + y) + c$$

$$\psi - c = 2y + x^2y + y$$

$$\Rightarrow C = 2y + x^2y + y \rightarrow (a) \quad \boxed{-9 = 2y + x^2y + y}$$ Ans

Now put $x = 0$ and $y = -3$ we get

$$\Rightarrow c = 2(-3) + (-3) \Rightarrow c = -9$$

Now put this in eq (a)

(ii) $\frac{2+y}{t^2+1} - 2t - \left(\left(2 - \ln(t^2+1)\right) \right) y' = 0 \quad , y(5)=0$ ①

<u>Sol:</u> M= N=

Taking derivation of N w.r.t to y

$\int (2 - \ln(t^2+1) dy$

$\Rightarrow \int 2 \, dy - \int \ln(t^2+1) \, dy$.

$= 2y -$ solving by substitution we get

$- \quad 2y = \ln(1+t^2) - 2(t - \cot an(t)) + c$ — (a)

Now put $y=0$ and $t=5$

$\Rightarrow \quad 0 = \ln(1+(5)^2) - 2(5 - \cot an(5)) + c$

$c = \ln(1+(5)^2) - 2(5 - \cot an(5))$

put value of c in eq (a).

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