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Q1 $dy/dt = e^{y-1} \sec(y)(1+t^2)\, y(0) = 0$

Solution:-

$y(0)$ So $x=0$ $y=0$

$dy = e^y . e^{-1} \sec(y)(1+t^2)\, dt$

$\frac{1}{e^y \cdot \sec(y)} dy = (1+t^2)\, e^{-t}\, dt$

As $\cos(y) = \frac{1}{\sec(y)}$

$\int e^{-y} \cos y\, dy = \int (1+t^2) e^{-1}\, dt$

using integrating by parts

$e^{-y} \int \cos y\, dt - \int(\int \cos y \frac{d}{dy} e^{-y}) = (1+t^2)$

$\int e^{-t} \int(\int e^{-t} \frac{d}{dt}(1+t^2)$ eq 1

L.H.S.

$e^{-y} \int \cos y\, dx - \int(\int \cos y \cdot \frac{d}{dy} e^{-y})$

$e^{-y} \sin y - \int(\sin e^{-y}(-1))$

$e^{-1} \sin y + \int(\sin y \cdot e^{-y})$

$e^{-1} \sin y + \int e^{-y} \sin y)$

again using integration by parts.

$e^{-y} \sin y + e^{-y}(-\cos y) - \int(\sin y \frac{d}{dy} e^{-y})$

$e^{-y} \sin y + e^{-y} \cos y - \int(\cos y\, e^{-y})$

Since $\int \cos y e^{-y} \cdot$ L.H.S

Since is again Same to the first one

So L.H.S will become L.H.S $= e^{-y}(\sin y$

$\cos y) -$ L.H.S.

2 L.H.S $= e^{-y}(\sin y - \cos y)$

L.H.S $= \dfrac{e^{-y}(\sin y - \cos y)}{2}$

Now taking R.H.S

$\int (1+t^2)\, e^{-t}\, dt$

$= -(1+t^2)\int e^{-t} - \int\left(\int e^{-t}\, d/dt\,(1+t^2)\right)$

$= -(1+t^2)\, e^{-t} - \int(-e^{-t}\,(2t)$

$= -(1+t^2)\, e^{-t} + \int (2t)\, e^{t}$

again using integration by parts.

$= -(1+t^2)\, e^{-1} + (2t\int e^{-t} - \int e^{-t}\, d/dt\, 2t))$

$= -(1+t^2)\, e^{-t} + (-2t e^{-t} - \int(-e^{-1}\, 2)$

$= (1+t^2)\, e^{-t} + (-2t\, e^{-t} + \int 2e^{-t}))$

$= -(1+t^2)\, e^{-t} + (-2t\, e^{-t} - 2e^{-t}) + C$

$= -(1+t^2)\, e^{-t} + 2t\, e^{-t} - 2e^{-t} + C$

$= e^{-t} - e^{-t}\, t^2 - 2te^{-t} + C$

$-(t^2+2t+3)e^{-t}+C = R.H.S$

Now take $L.H.S = R.H.S$

$$e^{-y}\left(\frac{\sin y - \cos y}{2}\right) = (t^2+2t+3)e^{-t}+C$$

we know that

$t=0 \qquad y=0$

Put in above

$(0-1) = -3+C$

$C = 5/2$

Put value of C

$$\frac{e}{2}^{-y}(\sin y - \cos y) = -(x^2+2t+3)e^{-t} + 5/2 \cdot An$$

Q2) $\left(\sqrt{x+y}+\sqrt{x-y}\right)dx - \left(\sqrt{x+y}-\sqrt{x-y}\right)dy = 0$

Solution: $\frac{dy}{dx} \; \frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$

This is homogenous Differential eq in x & y to solve this put.

$$y = vx$$

$$\Rightarrow dy/dx = v + \frac{dv}{dx}$$

They eq ① becomes

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}} \times \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}+\sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}\left(1+\sqrt{1-v2}\right)}{\cancel{2}v}$$

$$v + \frac{x\,dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

(Q2)

$$x\frac{dV}{dx} = \frac{1+\sqrt{1-V_2}}{V} - V$$

$$x\frac{dV}{dx} = \frac{1+\sqrt{1-V^2} - V^2}{V}$$

$$x\frac{dV}{dx} = \frac{\sqrt{1-V^2}\left(1+\sqrt{1-V^2}\right)}{V}$$

$$\frac{V\,dV}{\sqrt{1-V_2}\left(+\sqrt{1-V^2}\right)} = \frac{dx}{x}$$

Taking integrals on b/s

$$\int \frac{V\,dV}{\sqrt{1-V^2}(1+\sqrt{1-V^2}} = \int \frac{dx}{x}$$

- put $1+\sqrt{1-V^2} = t$

$\Rightarrow \frac{1}{2}\ 1-V^2)^{-1/2}\ (-2V)\,dV = dt$

$$\frac{V\,dV}{\sqrt{1-V}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

$-\ln t = \ln x + \ln c$

$-\ln\left(1+\sqrt{1-V_2} = \ln cx\right.$

$\ln\left(1+\sqrt{1-V^2} = -\ln cx\right.$

$\cancel{\ln}\left(1+\sqrt{1-V^2}\right) = \cancel{\ln}\,(x)^{-1}$

Q2

$$= 1 + \sqrt{1-V^2} = 1/cx$$

$$: 1 + \frac{\sqrt{x^2 - y^2}}{x^2} = 1/cx$$

$$= x + \sqrt{x^2-y^2} = 1/c$$

$$x + \sqrt{x^2-y^2} = C_1 \quad :- \quad 1/c = C_1$$

which is a Requirred Solution

Question 3)

$$(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$$

Solution:-

$$\Rightarrow F(D)y = f(x)$$

As it is non homogenous linear equ

So Solution will be.

$$y = y_c + hp \quad -① $$

Complementary Solution y^c

$D^4+D^2=0 \Rightarrow D^2(D^2+1)=0$

Either $D^2=0 \Rightarrow \boxed{D=0}$

$D^2+1=0 \Rightarrow D^2=-1$

$D=\sqrt{-1} \Rightarrow D=i$ or $D=\boxed{0+i}$

Roots are real & complex

$$y_c = C_1 e^{0x} + e^{0x}(C_2\ C_3 x + C_3 \sin x)$$

$$y_c = C_1 + C_2 \cos x + C_3 \sin x$$

$$y_p = \frac{1}{f(D)} F(x)$$

$$y_p = \frac{1}{D^4+D^2}(3x^2+4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$f(D) = D^4 + D^2$

at $D = 0 \Rightarrow f(0) = 0$

So $f'(D) \; 4D^3 + 2D$

Now also for $D = 0 \Rightarrow f(D) = 0$

again differentiating

$f''(D) : 12D + 2$

So for $D = 0$

$f''(0) = 12(0) + 2 = 2$

So replacing $1/f(D)$ with $\dfrac{x^2}{f''(D)}$

$\Rightarrow y_D \; \dfrac{x^2 \, 3x^2}{12D+2} + \dfrac{x^2}{12D+2} \, 4\sin x - \dfrac{x^2}{12D+2} \, 2\cos x$

Putting $D = 0$ in all

$$y_P = \frac{x^2 \, 3x^2}{12(0)+2} + \frac{x^2 . 4\sin x}{12(0)+2} - \frac{2x^2 \cos x}{12(0)+2}$$

$$y_P = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3x^4}{2} + 2x^2 \sin x - x^2 \cos x$$

So; putting in equation (i)

$y = C_1 + C_2 \cos x + C_3 \cos + 3/2 \, x^4 + 2x^2 \sin x$
$- x^2 \cos x .$

$y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + 3/2 \, x^4$

Ans