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Section :- B

Subject :- Differential equations

Q1:- Solve the following objective type questions

① The order of matrix A is $m \times p$ and the order of B is $p \times n$ Then the order of matrix AB is ?

Solve:- Order of AB $m \times n$

② The number of non zero rows in echelon form ?

$\Rightarrow$ No of non-zero rows in echelon form is called rank of the matrix

③ If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Solve :- $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix} \Rightarrow |B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$

$\Rightarrow a - 8 = 0$

$\boxed{\Rightarrow a = 8}$

④ If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ Then $|A| = ?$

Solve :-

$|A| = -2i^2 - i^2 = 2+1 \boxed{= 3}$

⑤ The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

$\Rightarrow$ Scalar matrix.

⑥ The solution of $\frac{dy}{dx} + 2xy = y$?

Solve $\frac{dy}{dx} + 2xy = y$

$\frac{dy}{dx} = y - 2xy$

$\frac{dy}{dx} = y(1-2x)$

$\int \frac{1}{y} dy = \int (1-2x)\, dx$

$\ln y = \int 1\, dx - 2 \int x\, dx$

$\ln y = x - \cancel{2}\frac{x^2}{\cancel{2}} + c$

$$\boxed{y = e^{x - x^2 + C}}$$

⑦ The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)}$$ is

$\Rightarrow$ squaring b/s

$$\left(\frac{dy}{dx}\right)^6 = 1+\left(\frac{dy}{dx}\right)^2$$

$\boxed{\text{order} = 1}$ $\boxed{\text{degree} = 6}$

⑧ $$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$$

$\Rightarrow$ order = 2

Degree = undefine because the derivative of dependent variable is appear in the domain of the tansidental function.

⑨ If differential equation $2\frac{dy}{dx} + x^2y = 2x+3$ $y(0) = 5$ is?

$\Rightarrow$ Homogeneous equation.

$P=4$

10. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is?

$$= 1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$\Rightarrow (bc^2 - cb^2) - a(c^2 - b^2) + a^2(c-b)$$

$$\Rightarrow cb(c-b) - a(c+b)(c-b) + a^2(c-b)$$

$$\boxed{\Rightarrow (a-b)(b-c)(c-a)}$$

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p=5

Q# 2 A: Part:-

Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Solve:-

as the product of factors which are linear in a, b, c

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$\Rightarrow a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$\Rightarrow ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

taking common abc

P=6

$$\Rightarrow abc\,(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$\Rightarrow abc\left[bc(c-b) - ac(c+a) + ab(b-a)\right] \text{ ans}$$

Q2:- **B part:-**

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

solve

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic equ $\Rightarrow |A - \lambda I| = 0 \longrightarrow (1)$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinate

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} = 0$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow (B)$$

again $\begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix}$ expand by R_1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda) \Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1 \big((-1)(2-\lambda) - (-1)(-1)\big)$$

$$- 1 \big((-1)(-1) - (-1)(3-\lambda)\big) \Big]$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-(1+3-\lambda)$$

$$\Rightarrow (3-\lambda)(\lambda^2-5\lambda+5)+(3+\lambda)-(4-\lambda)$$

$$\Rightarrow 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$\Rightarrow \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \longrightarrow (a)$$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \longrightarrow (b)$$

P=9

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

expand by C_1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\boxed{-\lambda^2+6\lambda-8} - (c)$$

Put (a), (b) and (c) in (B)

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2$$

$$+6\lambda-8$$

$\Rightarrow -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda - \lambda^2 + 6\lambda$

$-8 - \lambda^2 + 16\lambda - 8$

$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda$

$+ 6\lambda + 6\lambda + 16 - 16$

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

No By sythatic division

we get

$\lambda(\lambda - 2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$\lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16 = 0$

By factorization method

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$(\lambda - 4)(\lambda - 4)$

$\lambda = 4 \quad , \lambda = 4$

so

$\boxed{\lambda_1 = 0 \quad , \lambda_2 = 2 \quad , \lambda_3 = 4 \quad , \lambda_4 = 4}$ ans

P= 11

Q # 3:- $(x^2 - 3y^2)dx - 2xy\,dy = 0$

$x = 2 \quad y = 6$

solve :- $(x^2 + 3y^2)dx - 2xy\,dy = 0$

$\Rightarrow (x^2 + 3y^2)dx = 2xy\,dy$

Dividing both sides by $2xy\,dx$

we get

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow (*)$$

let $y = vx$

Diff:- $dy = v\,dx + x\,dv$

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Dividing by dx

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \quad \text{— (a)}$$

Put (a) in (*)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\,\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + \frac{x\,dv}{dx} + \frac{1}{2}\left[\frac{1}{v} + \frac{3}{v}\right]$$

Mutiplying B.s By "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2\cancel{x}\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying b.s By $\frac{dx}{dv}$

So we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiplying b/s by $\frac{v}{x(1+v^2)}$

we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

take "$\int$" on both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

take 'e' on both sides

$$e^{\ln|1+v^2|} = e^{\ln(xc)}$$

$$1+v^2 = xc$$

$$I + v^2 = xc$$

put $v = y/x$

$$1 + (y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2 + y^2 = x^3 c \longrightarrow (**)$$

put $x = 2$, $y = 6$ in eq $(**)$

$$(4) + (36) = 8c$$

$$c = \frac{40}{8} \qquad \boxed{c = 5}$$

so $x^2 + y^2 = 5x^3$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

taking "$\sqrt{\ }$" on both sides

$\boxed{y = +x\sqrt{5x-1}}$ $\boxed{y = -x\sqrt{5x+1}}$ or

$\boxed{y = \pm x\sqrt{5x-1}}$ Ans