IQRA NATIONAL UNIVERSITY

INU

DEFINING POSSIBILITIES, CREATING OPPORTUNITIES

DEPARTMENT: BE (CIVIL)

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SECTION : "A"

SUBJECT : Calculus

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Question No 1 : THE FUNCTION $g(t)$ is defined by $g(t) = 0 \quad t < 0$

$$t^2 \quad 0 \le t \le 3$$

$$2t + 3 \quad 3 < t \le 4$$

$$12 \quad t > 4$$

a) State any point of discontinuity

b) Find, if they exist

i) $\lim_{t \to 3} g$

<u>Solution:</u> To check possibility of discontinuity of the function is at $t = 0$ & 4

→ First at $t = 0$

$$g(t) = t^2$$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

2 For R.H.L

$$\lim g(1+h) = \lim_{h\to 0} (1+h)^2$$

$$\lim_{h\to 0} 1+h^2+2h$$

Apply limits :

$$1+0^2+2(0)$$

$$=1$$

For L.H.L

$$\lim_{h\to 0} g(1-h) = 2t+3$$

$$= \lim_{h\to 0} 2(1-h)+3$$

$$= \lim_{h\to 0} 2-2h+3$$

Apply limit

$$= 2-2(0)+3$$

$$= 5$$

$$R.H.L \neq L.H.L = g(t) = 5$$

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Now! at $t = 4$

$$g(4) = 2(4) + 3$$

$$= 8 + 3$$

$$= 11$$

For R.H.L

$$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$$

$$= \lim_{h \to 0} 2 + 2h + 3$$

Apply limits

$$= 2 + 2(0) + 3 = 5$$

For L.H.L

$$\lim_{h \to 0} g(1-h) = 12$$

$$g(4) = R.H.L \neq L.H.L$$

Point of discontinuity is

at $t = 4$

b) FIND, IF THEY EXIST

i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R·H·L

$$\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$$

$$= \lim_{h \to 3} 1 + h^2 + 2h$$

$\Rightarrow$ Apply limits

$$= 1 + 3^2 + 2(3) = 16$$

L·H·L

$$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t+3$$

$$= \lim_{h \to 3} 2(1-h)+3$$

$$= \lim_{h \to 3} 2 - 2h + 3$$

Apply limits

$$= 2 - 2(3) + 3$$

$$= 2 - 6 + 3$$

$$= -1$$

R·H·L $\neq$ L·H·L

(DOES NOT EXIST
SINCE L·H·L is
NEGATIVE)

-(-ive)-

Question No 2nd:

FIND THE MACLAURIN'S SERIES FOR $Y(x) = x^2 + \sin x$

Sol

$$Y(x) = x^2 + \sin x$$

As we know maclurim Series is:

$$Y(x) = Y(x_0) + Y'(x_0)(x - x_0) + Y''(x_0)\frac{(x-x_0)^2}{2!} + \cdots$$

Put $x_0 = 0$

$$Y(x) = Y(0) + (x-0)Y'(0) + \frac{(x-0)^2 Y''(0)}{2!} + \cdots$$

$$Y(x) = Y(0) + xY'(0) + \frac{x^2 Y''(0)}{2!} + \cdots \quad \text{(i)}$$

NOW! Find $Y(0) = ?$

$$Y(1) = x^2 + \sin x$$

$$Y(0) = 0 + \sin 0$$

$$Y(0) = 0 + 0$$

$$= 0 \Rightarrow \boxed{Y(0) = 0}$$

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$$Y(x) = x^2 + \sin x$$

$$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$$

$$y'(x) = 2x + \cos x$$

$$y'(0) = 2(0) + \cos 0$$

$$= 0 + 1$$

$$\boxed{y'(0) = 1}$$

Since $y'(x) = 2x + \cos x$

$$\frac{d}{dx} y(x) = 2 \frac{d}{dx} x + \frac{d}{dx} \cos x$$

$$= 2 + \sin x$$

$$y''(x) = 2 - \sin x$$

$$y''(0) = 2 - \sin 0$$

$$= 2 - 0 = 2$$

$$\boxed{y''(0) = 2}$$

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Now!

$$y''(x) = 2 - \sin x$$

$$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$$

$$= 0 - \cos x$$

$$y'''(x) = 0 - \cos x$$

$$y'''(0) = -\cos x$$

$$\boxed{y'''(0) = -1}$$

Putting all values in equation (i)

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots$$

$$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \cdots$$

$$= x + x^2 - \frac{x^3}{3!} + \cdots$$

So!

$$\boxed{y(x) = x + x^2 - x^3/3! + \cdots}$$

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Question No 3rd
Part "A" $1 + xy = x^2 + y^2$ find $y'' = ?$

Given
$$1 + xy = x^2 + y^2$$
$$y'' = ?$$

Sol

Taking $\frac{d}{dx}$ on both sides

$$1 + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$1 + \left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) = 2x + 2y\,\frac{dy}{dx}$$

$$1 + x\,\frac{dy}{dx} + y = 2x + 2y\,\frac{dy}{dx}$$

$$1 + y + x\frac{dy}{dx} = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y - 1$$

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a $(x-2y)\dfrac{dy}{dx} = 2x-y-1$

$$\frac{dy}{dx} = \frac{2x-y-1}{x-2y} \quad — \quad (*)$$

$$\Rightarrow y' = \frac{2x-y-1}{x-2y} \longrightarrow (1)$$

$\Rightarrow$ Diff again:

$$\frac{d}{dx}y' = \frac{d}{dx}\left(\frac{2x-y-1}{x-2y}\right)$$

USE QUOTENT Rule

$$y'' = \frac{(x-2y)\frac{d}{dx}(2x-y-1) - (2x-y-1)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

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$$= \frac{(x-2y)\left(2-\frac{dy}{dx}\right)-(2x-y-1)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

We know value of y' from Eq (1)

$$y'' = \frac{(x-2y)\left(2\left(\frac{2x-y-1}{x-2y}\right)\right)-(2x-y-1)\left(1-2\left(\frac{2x-y-1}{x-2y}\right)\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)(2-2x-y-1)}{(x-2y)^3} - \frac{(2x-y-1)(1-2(2x-y-1))}{(x-2y)^3}$$

$$\Rightarrow y'' = \frac{-2x-y-3}{(x-2y)^2} - \frac{(2x-y-1)(1-2(2x-y-1)}{(x-2y)^3}$$

$$y'' = \frac{-2x-y-3}{(x-2y)^2} - \frac{(2x-y-1)(1-2(2x-y-1)}{(x-2y)^3}$$

Question NO 3rd :

Part B :

$$\ln(y) = \ln\left(x^3(1+x)^9 e^{6x}\right)$$

$\frac{d\ln(y)}{dx} =$

Sol : $\ln\left(x^3(1+x)^9\right) + \ln e^{(6x)}$

$$= \ln x^3 + \ln(1+x)^9 + 6x$$

$$= 3\ln x + 9\ln(1+x) + 6x$$

Now!!

$$\frac{d\ln(y)}{dx} = \frac{dx}{dx}\, 3\ln x + 9\ln(1+x) + 6x)$$

$$= 3\frac{d}{dx}\ln x + 9\frac{d}{dx}\ln(1+x) + 6\frac{dx}{dx}$$

$$= 3 \cdot \frac{1}{x} + 9 \cdot \frac{1}{1+x} + 6$$

$$\frac{d\ln(y)}{dx} = \frac{3}{x} + \frac{9}{x+1} + 6$$

$$\boxed{\frac{d}{dx}\ln(y) = \frac{3}{x} + \frac{9}{x+1} + 6}$$