



DEPARTMENT: BE (CIVIL)

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SECTION : "A"

SUBJECT : Calculus

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Question NO 1: THE FUNCTION $g(t)$ is defined by $g(t) = 0 \quad t < 0$

$$t^2 \quad 0 \leq t \leq 3$$

$$2t+3 \quad 3 < t \leq 4$$

$$12 \quad t > 4$$

a) State any point of discontinuity

b) Find, if they exist

i) $\lim_{t \rightarrow 3} g$

Solution:

To check possibility of discontinuity of the function is at $t = 0$ & 4

→ First at $t = 0$

$$g(t) = t^2$$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

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For R.H.L

$$\lim_{h \rightarrow 0} g(1+h) = \lim_{h \rightarrow 0} (1+h)^2$$

$$\lim_{h \rightarrow 0} 1+h^2+2h$$

Apply limits:

$$1+0^2+2(0)$$

$$= 1$$

For L.H.L

$$\lim_{h \rightarrow 0} g(1-h) = 2t+3$$

$$= \lim_{h \rightarrow 0} 2(1-h)+3$$

$$= \lim_{h \rightarrow 0} 2-2h+3$$

Apply limit

$$= 2-2(0)+3$$

$$= 5$$

$$R.H.L \neq L.H.L = g(t) = 5$$

Now! at $t = 4$

$$g(4) = 2(4) + 3$$

$$= 8 + 3$$

$$= 11$$

For R.H.L

$$\lim_{h \rightarrow 0} g(1+h) = \lim_{h \rightarrow 0} 2(1+h) + 3$$

$$= \lim_{h \rightarrow 0} 2 + 2h + 3$$

Apply limits

$$= 2 + 2(0) + 3 = 5$$

For L.H.L

$$\lim_{h \rightarrow 0} g(1-h) = 12$$

$$g(4) = \text{R.H.L} \neq \text{L.H.L}$$

Point of discontinuity is

at $t = 4$

u
b) FIND, IF THEY EXIST

i) $\lim_{t \rightarrow 3} g$

For $g(t) = t^2$

R.H.L

$$\lim_{h \rightarrow 3} g(1+h) = \lim_{h \rightarrow 3} (1+h)^2$$

$$= \lim_{h \rightarrow 3} 1 + h^2 + 2h$$

\Rightarrow Apply limits

$$= 1 + 3^2 + 2(3) = 16$$

L.H.L

$$\lim_{h \rightarrow 3} g(1-h) = \lim_{h \rightarrow 3} 2t + 3$$

$$= \lim_{h \rightarrow 3} 2(1-h) + 3$$

$$= \lim_{h \rightarrow 3} 2 - 2h + 3$$

Apply limits

$$= 2 - 2(3) + 3$$

$$= 2 - 6 + 3$$

$$= -1$$

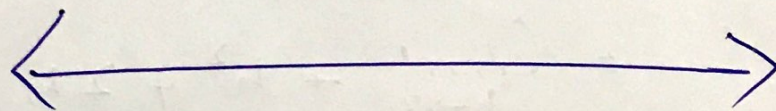
$$R \cdot H \cdot L \neq L \cdot H \cdot L$$

(DOES NOT EXIST)

SINCE L.H.L is

NEGATIVE.

— (-IVE) —



Question No 2nd:

FIND THE MACLAURIN'S SERIES

FOR $Y(x) = x^2 + \sin x$

Sol

$$Y(x) = x^2 + \sin x$$

As we know maclurin Series
is:

$$Y(x) = Y(x_0) + Y'(x_0)(x-x_0) + \frac{Y''(x_0)(x-x_0)^2}{2!} + \dots$$

Put $x_0 = 0$

$$Y(x) = Y(0) + (x-0)Y'(0) + \frac{(x-0)^2 Y''(0)}{2!} + \dots$$

$$Y(x) = Y(0) + xY'(0) + \frac{x^2 Y''(0)}{2!} + \dots \quad \text{--- (1)}$$

NOW! Find $Y(0) = ?$

$$Y(x) = x^2 + \sin x$$

$$Y(0) = 0 + \sin 0$$

$$Y(0) = 0 + 0$$

$$Y(0) = 0$$

$$\Rightarrow \boxed{Y(0) = 0}$$

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$$y(x) = x^2 + \sin x$$

$$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$$

$$y'(x) = 2x + \cos x$$

$$y'(0) = 2(0) + \cos 0$$

$$= 0 + 1$$

$$y'(0) = 1$$

Since $y'(x) = 2x + \cos x$

$$\frac{d}{dx} y(x) = 2 \frac{d}{dx} x + \frac{d}{dx} \cos x$$

$$= 2 + \sin x$$

$$y''(x) = 2 - \sin x$$

$$y''(0) = 2 - \sin 0$$

$$= 2 - 0 = 2$$

$$y''(0) = 2$$

x

Now!

$$y''(x) = 2 - \sin x$$

$$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$$

$$= 0 - \cos x$$

$$y'''(x) = 0 - \cos x$$

$$y'''(0) = -\cos x$$

$$y'''(0) = -1$$

Putting all values in equation (i)

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \dots$$

$$= x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

$$= x + x^2 - \frac{x^3}{3!} + \dots$$

So!

$$y(x) = x + x^2 - \frac{x^3}{3!} + \dots$$

Question NO 3rd

Part "A" $1 + xy = x^2 + y^2$ find $y'' = ?$

Given

$$1 + xy = x^2 + y^2$$

$$y'' = ?$$

Sol

taking $\frac{d}{dx}$ on both sides

$$1 + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$1 + \left(x \frac{dy}{dx} + y \frac{dx}{dx} \right) = 2x + 2y \frac{dy}{dx}$$

$$1 + x \frac{dy}{dx} + y = 2x + 2y \frac{dy}{dx}$$

$$1 + y + x \frac{dy}{dx} = 2x + 2y \frac{dy}{dx}$$

$$x \frac{dy}{dx} = 2y \frac{dy}{dx} + 2x - y - 1$$

a

$$(x = 2y) \frac{dy}{dx} = 2x - y - 1$$

$$\frac{dy}{dx} = \frac{2x - y - 1}{x - 2y} \quad (*)$$

$$\Rightarrow y' = \frac{2x - y - 1}{x - 2y} \rightarrow (1)$$

\Rightarrow Diff again :

$$\frac{d}{dx} y' = \frac{d}{dx} \left(\frac{2x - y - 1}{x - 2y} \right)$$

USE
QUOTIENT
Rule

$$y'' = \frac{(x - 2y) \frac{d}{dx} (2x - y - 1) - (2x - y - 1) \frac{d}{dx} (x - 2y)}{(x - 2y)^2}$$

~~Q~~

$$= \frac{(x-2y)\left(2 - \frac{dy}{dx}\right) - (2x-y-1)\left(1 - 2\frac{dy}{dx}\right)}{(x-2y)^2}$$

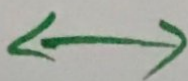
We know value of y' from
Eq (1)

$$y'' = \frac{(x-2y)\left(2\left(\frac{2x-y-1}{x-2y}\right)\right) - (2x-y-1)\left(1 - 2\left(\frac{2x-y-1}{x-2y}\right)\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)(2 - 2x - y - 1)}{(x-2y)^3} - \frac{(2x-y-1)(1 - 2(2x-y-1))}{(x-2y)^3}$$

$$\Rightarrow y'' = \frac{-2x-y-3}{(x-2y)^3} - \frac{(2x-y-1)(1-2(2x-y-1))}{(x-2y)^3}$$

$$y'' = \frac{-2x-y-3}{(x-2y)^3} - \frac{(2x-y-1)(1-2(2x-y-1))}{(x-2y)^3}$$



Question NO 3rd:

Part B:

$$\ln(y) = \ln(x^3(1+x)^9 e^{6x})$$

Sol: $\frac{d \ln(y)}{dx} =$

$$\ln(x^3(1+x)^9) + \ln e^{(6x)}$$

$$= \ln x^3 + \ln(1+x)^9 + 6x$$

$$= 3 \ln x + 9 \ln(1+x) + 6x$$

Now!

$$\frac{d \ln(y)}{dx} = \frac{dx}{dx} (3 \ln x + 9 \ln(1+x) + 6x)$$

$$= 3 \frac{d}{dx} \ln x + 9 \frac{d}{dx} \ln(1+x) + 6 \frac{dx}{dx}$$

$$= 3 \cdot \frac{1}{x} + 9 \cdot \frac{1}{1+x} + 6$$

$$\frac{d \ln(y)}{dx} = \frac{3}{x} + \frac{9}{x+1} + 6$$

$$\frac{d}{dx} \ln(y) = \frac{3}{x} + \frac{9}{x+1} + 6$$