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Mid term Assignment (PAPER)

LINEAR Algebra

MARKS = 30

SUBMITTED To :-

SIR. M. SHAKEEL

SUBMITTED BY :->

M. ISLAM
ID = 6844
BS(SE) Section B
8th Semester

IQRA NATIONAL UNIVERSITY.

ID = 6844 M.ISLAM. 2

Question No1:→

My ID = 6844

$$A = \begin{bmatrix} 1 & ID_3 & 3 & 0 & 5 \\ 0 & 1 & -ID_2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID_3 \end{bmatrix}$$

SOLUTION:→

Put $ID_3 = 4$ & LAST $= 4$ in (A)

$$A = \begin{bmatrix} 1 & 4 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Let The argumented matrix be

$$A = \left[\begin{array}{cccc|c} 1 & 4 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

Let suppose

$$x_1 + 4x_2 + 3x_3 = 5 \longrightarrow 1 \text{ equation}$$

$$x_2 + (4)x_3 = 7 \longrightarrow 2$$

$$x_3 = -6 \;—\; 3$$

$$x_4 = 4 \;—\; 4$$

Now from equation 4 is

$$x_4 = 4 \longrightarrow (4)$$

so

$$\boxed{x_4 = 4}$$

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Now from equ (3) is

$$x_3 = -6 \longrightarrow (3)$$

So

$$\boxed{x_3 = -6}$$

Now from equ 2 is

$$x_2 + (-4)x_3 = 7 \longrightarrow (2)$$

Put The Value in equ (2)

$$x_2 + (-4)(-6) = 7$$

$$x_2 + 24 = 7$$

$$\frac{\cancel{24}\, x_2}{\cancel{24}} = 7/24$$

$$\boxed{x_2 = 7/24}$$

Now from equ 1 is

$$x_1 + 4x_2 + 3x_3 = 5 \longrightarrow (1)$$

Put The Value of x_2, x_3 in equ (1)

$$x_1 + 4(7/24) + 3(-6) = 5$$

$$x_1 + 1.16 + (-18) = 5$$

$$x_1(-16.8) = 5$$

$$x_1 = 5 + 16.8$$

$$\boxed{x_1 = 21.8}$$

So The Solution is

$$ss\ \{x_1 x_2, x_3, x_4 = 4, -6, 7/24, 21.8\}.$$

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Question No 2 :→

(A)

SOLUTION :→

Given Information

Suppose, we are given with following matrix :

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

We have to find the elementry row-operation to transform matrix A into matrix B.
We have to find the reverse row-operation to get A from B.

Step 1 : **From A to B**

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$R_3 = R_3 - 2R_2$$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} = \underline{\underline{B}}$$

So the row operation is $R_3 = R_3 - 2R_2$

Step 2 :- **From B to A**

Reverse Row operation :-

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

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$R_3 = 2R_2 + R_3$

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} = A$$

So The row operation is
$R_3 = 2R_2 + R_3$

Hence showed

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Q 2.→
PART (B)

a) $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is The echolen form.

SOLUTION:→

Let $A = \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$

→ Yes, matrix A is in echelon form because of its definition, as echelon form of a matrix states That "If a Column Conditions a leading entry then all zero".
- In matrix A, it satisfies The difinition of echelon form of a matrix.
So, it is in echolen form.

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b) $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form.

SOLUTION :⇒

Let $B = \begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

⇒ Yes, matrix B is in echelon form because of its definition which states that "If a Column Contains a leading entry then all entries, below that leading entry are Zero"

- According to definition, matrix B is Columns Contains leading entries as 1 and below that all entries are Zero, so matrix B is in echelon form.

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c) $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is The reduced row echelon form.

SOLUTION :⇒

Let $C = \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 5/5 & 0/5 & 0/5 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ $R_1/5$

$C = \begin{bmatrix} 1 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

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— Yes, matrix C is in reduced row echelon form because reduced row echelon form states that " In reduced row echelon form the leading co-efficient must be 1 in each row is to the right of the leading co-efficient in the row above it.

= According to definition matrix C satisfies the definition properties, so it is an reduced row echelon form.

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d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Solution:⇒

$$\text{Let } D = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

⇒ No, matrix D is not in reduced row echelon form, because if a matrix is in reduced row echelon form then its row (non-zero) contains, its first entries as a number "1" which is known as leading 1 i.e, the first non-zero entry is 1.
Also if there are any rows containing only zero entries than they allowed.
So, it is not in reduced row echelon form.

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Question No 3:→

(a) PART :-

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Answer :→

D/B row Echelon & reduced row Echelon form :-

Row Echelon form

1- Row echelon form from a matrix form of a matrix is defined as " The leading entry is row echelon form in echelon form in ~~each~~ each row (column) is the only ~~zero~~ non-zero entry in its row (column).

2- Echelon form of a matrix isn't Unique which means there are infinite answers possible, when we perform row reduction or elementry operation

3- Each row Containing a non-zero number has the number 1 appearing in the row's first non-zero Column. Such entry will be known as "leading entry/one"

4- The entries only below the first leading non-zero entry that must be zero, not necessory for above one's.

Reduced Row Echelon form.

1- Reduced row echelon form is defined as " In reduced row echelon form The left most non-zero entry of a row is equal to 1. The leftmost non-zero entry of a row is the only non-zero entry in its column.

2- Reduced Row echelon form is Unique which means when we apply elementry row operation on an a matrix it will Produce the same answers, no matter now we perform The same row operation

3- In reduced row echelon form, The left most non-zero entry of a row is equal to 1. The left most non-zero entry of a row is The non-zero entry an its Column.

4- The entries above and below The first 1 in each row must all be 0.

Examples

$$\begin{bmatrix} 1 & 6 & 2 & -8 \\ 0 & 1 & 14 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 6 \\ 0 & 1 & 7 & 0 & 1 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

Practical use of Reduced Row Echelon Form :-

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1- This type of matrix is used to solve system of Linear equation.

2- Reduced Row Echelon form used in balancing chemical equations.

3- Such matrix is used to solve Computer operations.

* Example of Reduced Row Echelon Form :->

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question No 3 :->

(B)

$$\begin{bmatrix} 1 & \text{ID2} & 8 \\ 2 & 8 & -1 \\ -\text{ID3} & 0 & 0 \\ 1 & -4 & \text{IDF.L} \end{bmatrix} \longrightarrow A$$

SOLUTION :->

My ID = ~~6478~~ 6844

Put The Value In A

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ID2 = 8 ID3 = 4

ID-First-Last = 64

Putting Value -

$$\begin{bmatrix} 1 & 8 & 8 \\ 2 & 8 & 1 \\ -4 & 0 & 0 \\ 1 & -4 & 64 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 8 & 8 \\ 2 & 8 & -1 \\ -4 & 0 & 0 \\ 1 & -4 & 64 \end{bmatrix} \quad R_3 - R_2$$

$$= \begin{bmatrix} 1 & 8 & 8 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -4 & 64 \end{bmatrix} \quad R_4/64$$

$$= \begin{bmatrix} 1 & 8 & 8 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -4 & 0 \end{bmatrix} \quad R_4/4$$

$$= \begin{bmatrix} 1 & 8 & 8 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{So}$$

$$S.S = \left\{ \begin{bmatrix} 1 & 8 & 8 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right.$$

Ans $\Rightarrow$

End Paper -