IQRA NATIONAL UNIVERSITY

Digital Signal Processing

Final Assignment Spring 2020

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Question - 1

(A) <u>Part:</u> $y(n), \quad n \geq 0$

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

The characteristic equation is;

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2$ Hence,

$$y_h(n) = c_1 2^n + c_2 n 2^n$$

The particular Solution is

$$y_p(n) = K(-1)^n u(n).$$

Subsituting this solution into the difference equation, we obtain

$$\Rightarrow K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$, $K(1+4+4) = 2$

$\Rightarrow K = 2/9$ The total Solution is,

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n \right] u(n)$$

From the initial conditions;

we obtain, $y(0) = 1$, $y(1) = 2$

Then,

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c_1 = 7/9$$

$$\Rightarrow 2c_1 + 2c_2 - 2/9 = 2$$

$$\Rightarrow c_2 = 1/3$$

Question - 1

(B) Part :

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Solution :-

The characteristic equation is ;

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = 1/2 \,,\, 1/5$$ Hence,

$$y_h(n) = C_1 \left(1/2\right)^n + C_2 \left(1/5\right)^n$$

With $x(n) = \delta(n)$ we have

$$y(0) = 2,$$

$$y(1) - 0.7y(0) = 0$$

$$\Rightarrow \quad y(1) = 1.4$$

Hence, $c_1 + c_2 = 2$

And,

$$\frac{1}{2} c_1 + \frac{1}{5} = 1.4$$

$$1.4 = \frac{7}{5}$$

$$\Rightarrow c_1 + \frac{2}{5} c_2 = \frac{14}{5}$$

These equations yield,

$$c_1 = \frac{10}{3}, \quad c_2 = -\frac{4}{3}$$

$$h(n) = \left[\frac{10}{3} \left(\frac{1}{2}\right)^n - \frac{4}{3} \left(\frac{1}{5}\right)^n \right] u(n)$$

The step response is,

$$S(n) = \sum_{k=0}^{n} h(n-k)$$

$$S(n) = \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$S(n) = \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$S(n) = \frac{10}{3}\left(\frac{1}{2}\right)^n \left(2^{n+1} - 1\right)u(n) - \frac{1}{3}$$

$$\left(\frac{1}{5}\right)^n \left(5^{n+1} - 1\right)u(n).$$

Question - 2

(A) Part:

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Solution:-

Taking inverse z-transform.

$$\frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence,

$$x(n) = [4(2)^n - 3 - n]\,u(n)$$

Question - 2

(B) Part:

$$X(z) = \frac{1}{1 - az^{-1}}$$

$$|z| > |a|$$

Using the complex inversion integral.

Solution:- We have,

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1 - az^{-1}} dz = \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z - a}$$

where c is circle at radius greater then $|a|$. we shall evaluate this integral Using (3,4,2) with $f(z) = z^n$.

we distinguish two cases.

1- If $n \geq 0$, $f(z)$ has at radius only zeros and hence no poles inside c. The only pole inside c is $z = a$.

Hence,

$$x(n) = f(z_0) = a^n \qquad n \geq 0$$

2- If $n<0$ $f(z) = z^n$ has an nth-order pole at $z=0$, which is also inside C. Thus there are contributions from both poles. For $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j} \oint_C \frac{1}{z^2(z-a)} dz = \frac{1}{z-a}\bigg|_{z=0} + \frac{1}{z}\bigg|_{z=a} = 0$$

If $n = -2$, we have

$$x(-2) = \frac{1}{2\pi j} \oint_C \frac{1}{z^2(z-a)} dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\bigg|_{z=0} + \frac{1}{z^2}\bigg|_{z=a} = 0$$

By continuing in the same way we can show that $x(n) = 0$.

for $n<0$, Thus

$$x(n) = a^n u(n)$$

Question - 3

(A) part : $H(z) = \dfrac{b_0}{(1-pz^{-1})^2}$

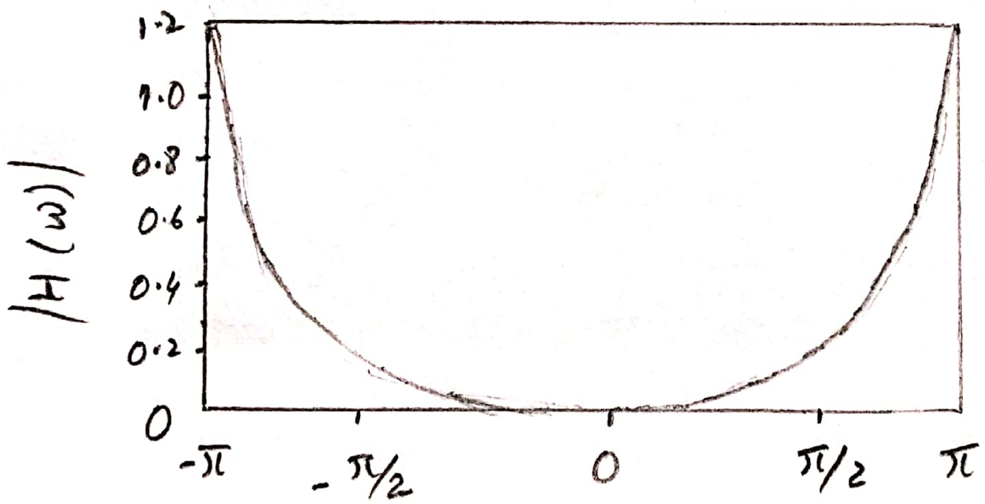
$H(0) = 1$ & $|H(\pi/4)|^2 = 1/2$

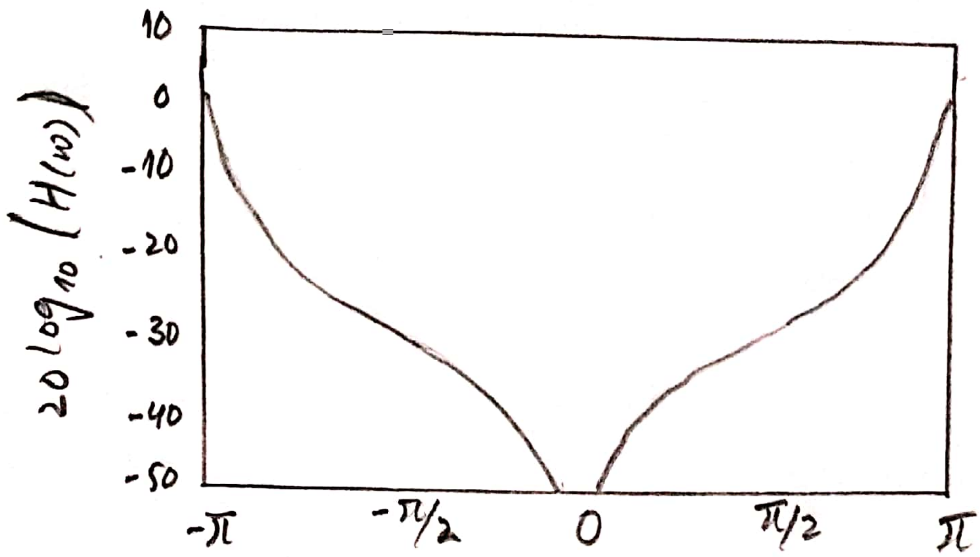
Solution :-

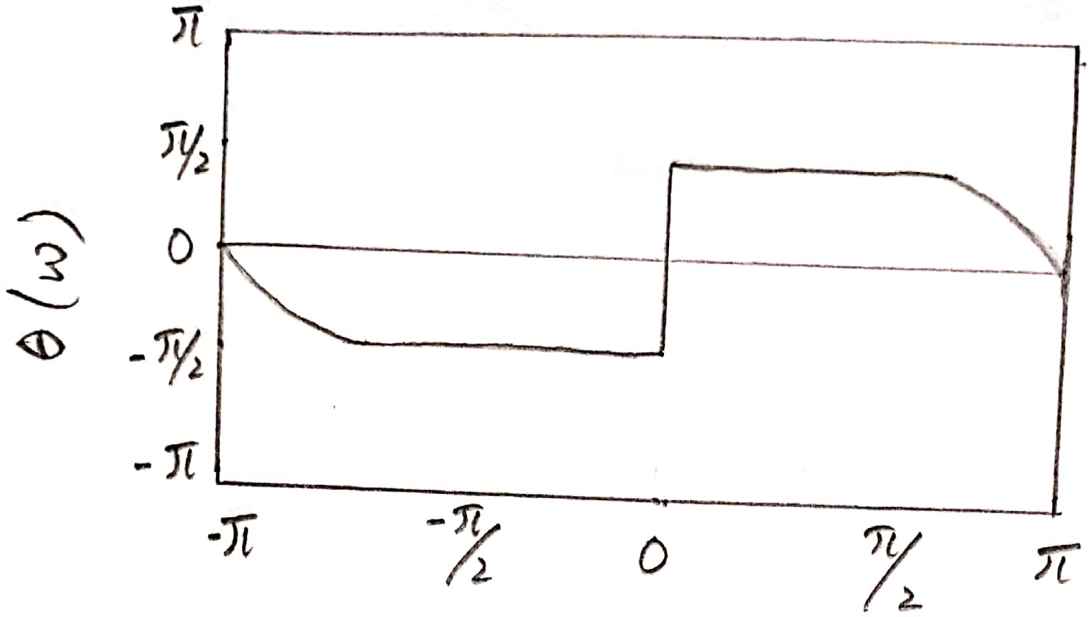
At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence, $b_0 = (1-p)^2$







At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{\left(1-Pe^{-j\pi/4}\right)^2}$$

$$= \frac{(1-P)^2}{\left[1-P\cos(\pi/4)+ jP\sin(\pi/4)^2\right]}$$

$$= \frac{(1-P)^2}{\left(1-P/\sqrt{2}+jP/\sqrt{2}\right)^2}$$

Hence,

$$= \frac{(1-P)^4}{\left[(1-P/\sqrt{2})^2+P^2/2\right]^2}$$

$$= 1/2$$

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Equivalently,

$$\sqrt{2}\,(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

⇒ The system function for desired filter,

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Question - 3

(B) part :-

Solution :- The filter must have poles at,

$$P_{12} = re$$

And zero at $z = 1$ And $z = -1$.

consequently. the same system funtion is,

$$H(z) = G\,\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$H(z) = G \frac{z^2 - 1}{z^2 + r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$.

$$H(\pi/2) = G \frac{2}{1 - r^2} = 1$$

$$G = \frac{1 - r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$.

we have;

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \cdot \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

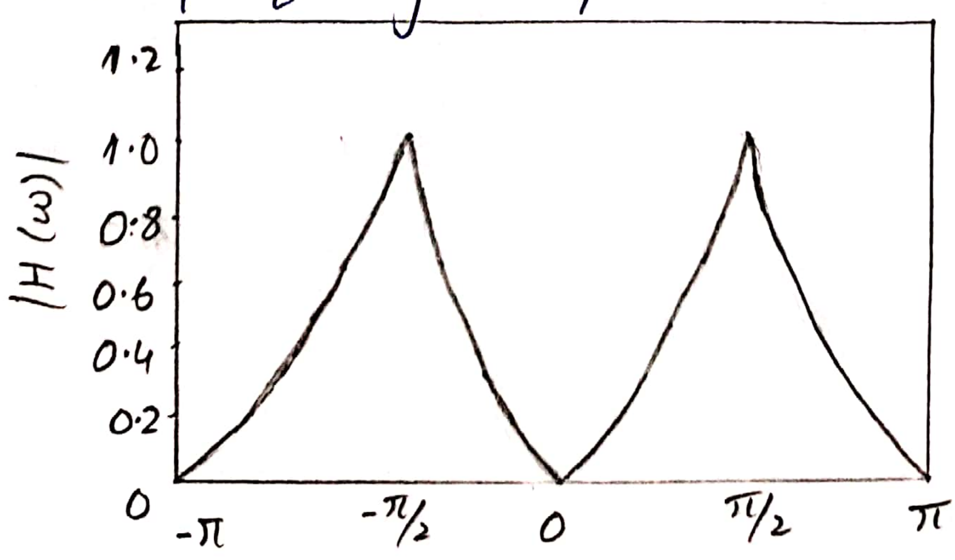
Equivalently;

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation. Therefore, the system function for the desired filter is;

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

Its frequency response is illustrated



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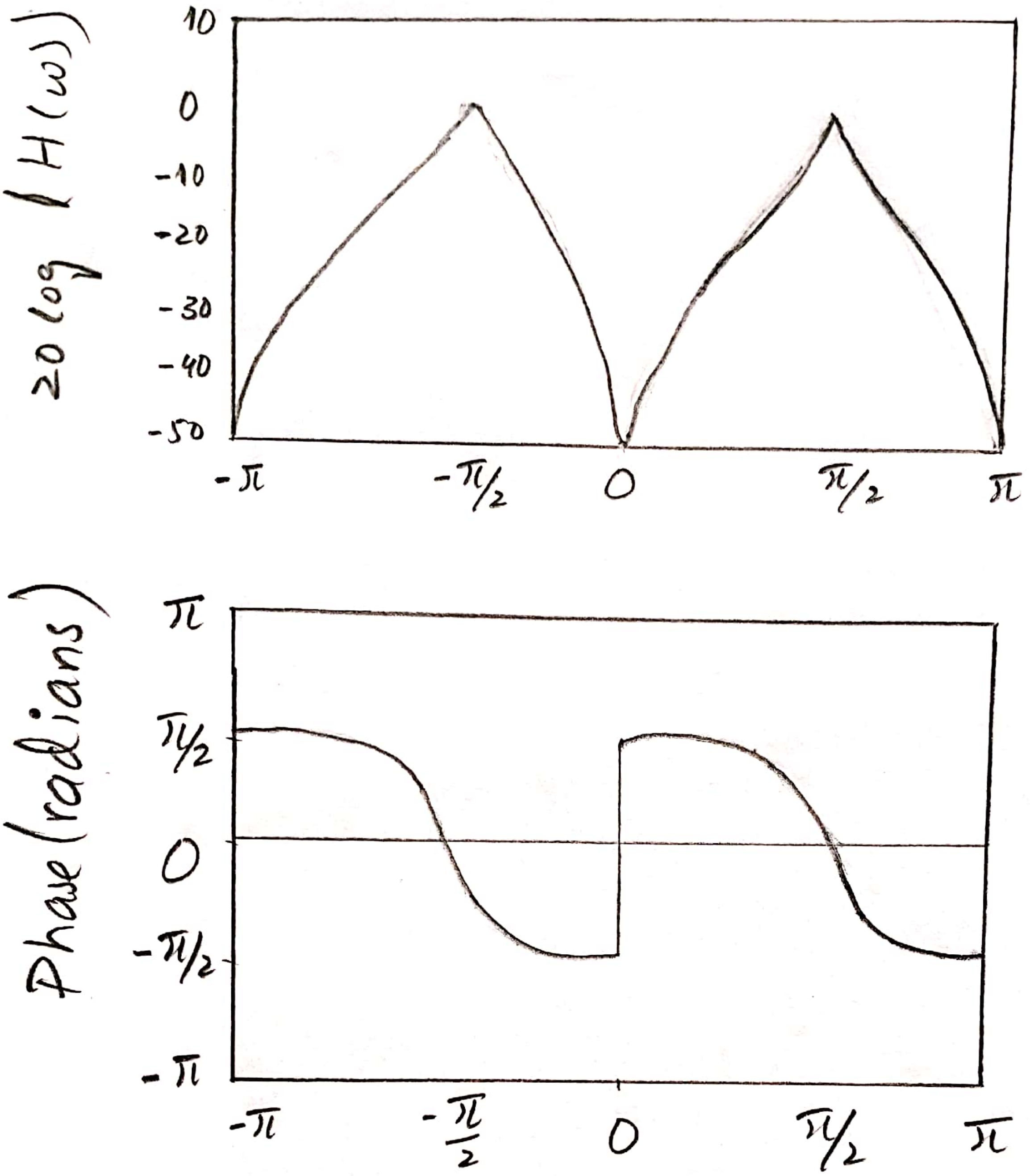

Magnitude and phase response of a simple bandpass filter is,

$$H(z) = 0.15\left[(1 - z^{-2}) / (1 + 0.7\, z^{-2})\right].$$

Question - 4

(A) Part :

$$x(n) = \begin{cases} 1 , & 0 \leq n \leq L-1 \\ 0 , & \text{otherwise} \end{cases}$$

Solution :- The Fourier transform of this sequance is ;

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$X(\omega) = \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated for $L=10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequencies

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$$\omega_k = 2\pi k/N,$$

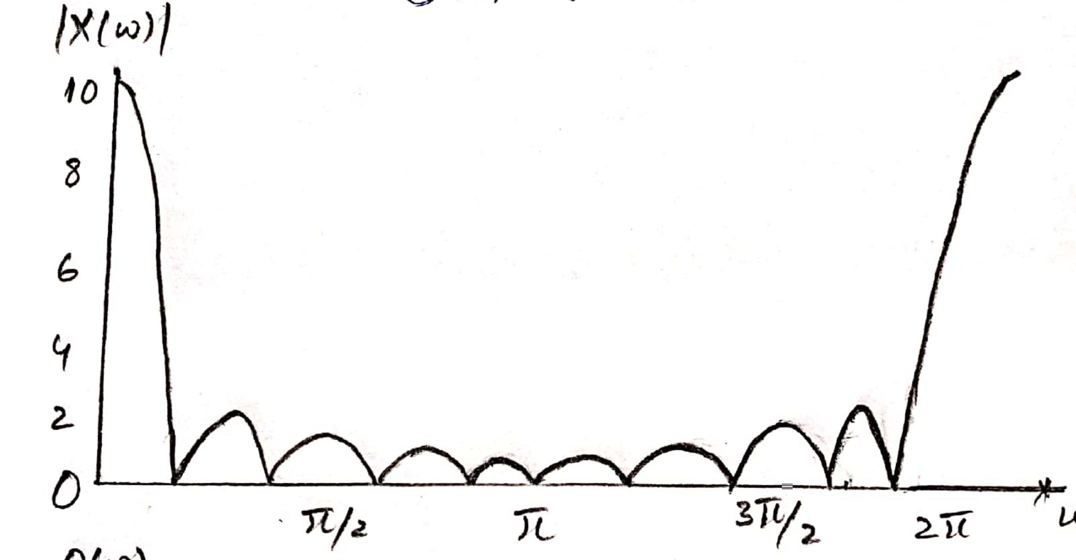
$$k = 0, 1, 2, \cdots N-1$$

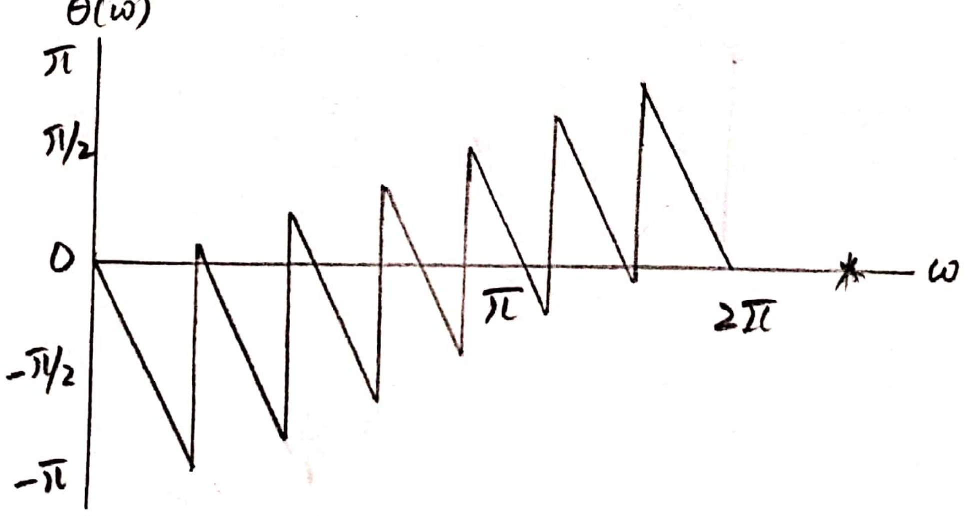
Hence,

$$X(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}}$$

$$k = 0, 1, \cdots, N-1$$

$$X(k) = \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)N}$$





If N is selected such that $N = L$ then the Discrete Fourier Transform becoms,

$$X(k) = \begin{cases} L, & k = 0 \\ 0, & k = 1, 2, \ldots L-1 \end{cases}$$

Thus, there is only one nonzero value in the DFT, This is apparent from observation of $X(\omega)$, since $X(\omega) = 0$ at the frequencies $\omega_k = 2\pi k/L$

$k \neq 0$;

The reader should verify that $x(n)$ can be recoverd from $X(k)$ by performing an L-point IDFT.

Provides a plot of the N-point DFT, in magnitude and phase, for $L = 10$, $N = 50$ and $N = 100$. Now the spectral charactristics of the sequence are more clearly evident.

Question - 4

(B) Part:

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Solution :-

Each Sequence consists of four nonzero points. For the purposes of illustrating the operations involved in circular convolution, it is desired to graph each sequence as points on a circle. Thus the sequences $x_1(n)$ and $x_2(n)$ are graphed as illustrated. we note that the Sequences are graphed in a counterclock-wise direction on a circle.

Now, $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified.

Begining with $m=0$ we have

$$x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2[(-n)]_N$$

$x_2(-n)_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle.

The product sequence is obtained by multiplying $x_1(n)$ with $x_2(-n)_4$, point by point. Finally we sum the values in the product sequence to obtain.

$$x_3(0) = 14$$

For $m = 1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)_4$$

It is easily verified that $x_2(1-n)_4$ is simply the sequence $x_2(-n)_4$ rotated counter-clockwise by one unit in time.

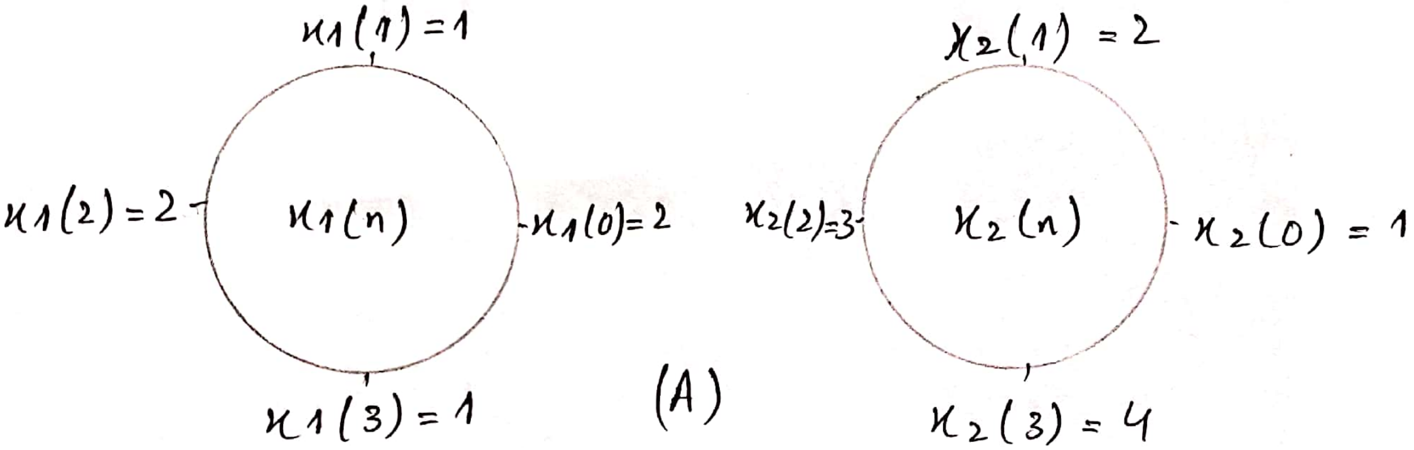
This rotated sequence multiplies $x_1(n)$ to yield the product sequence, also finally, we sum the values in the product sequence to obtain $x_3(1)$, Thus

$$x_3(1) = 16$$

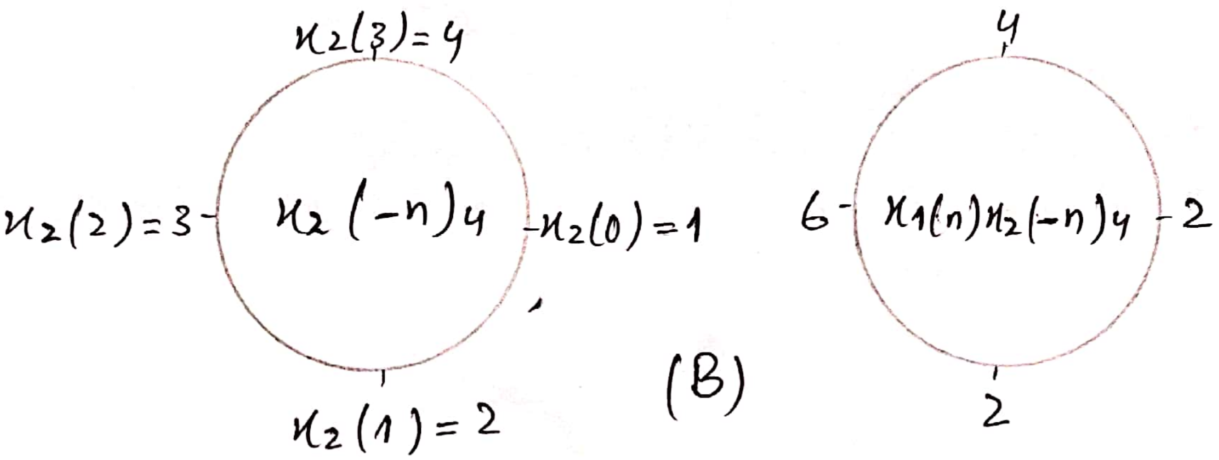
For $m = 2$ we have

$$x_3(2) \sum_{n=0}^{3} x_1(n)\, x_2(2-n)_4$$

Now $x_2(2-n)_4$ is the folded sequence.



(A)



(B)

Folded sequence

Product sequence

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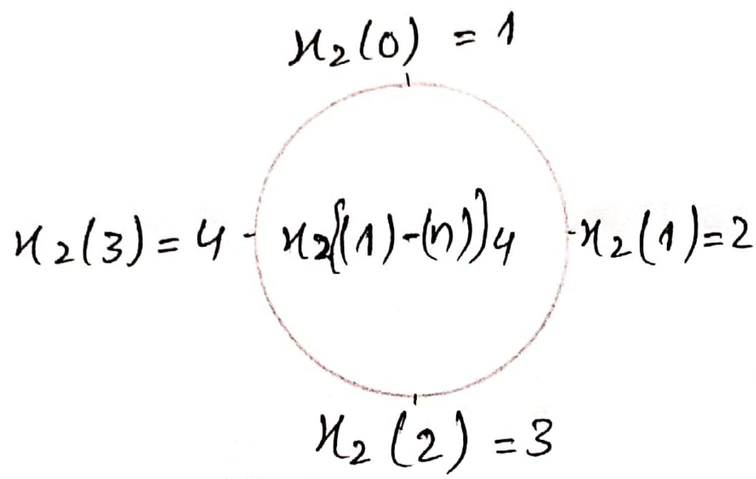
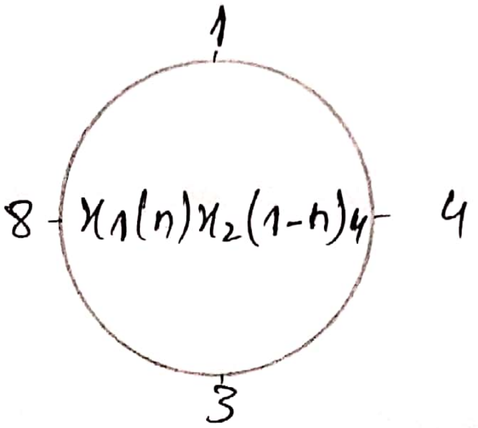
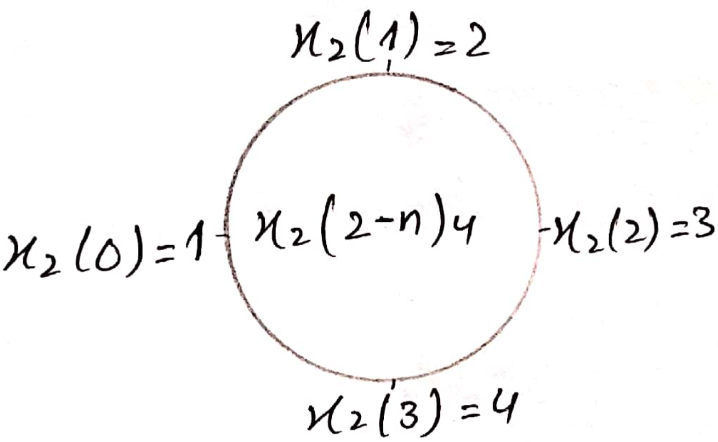


(C)

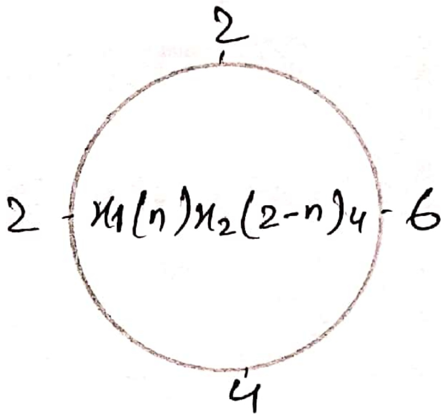


Folded sequence rotated by one unit in time

Product Sequence

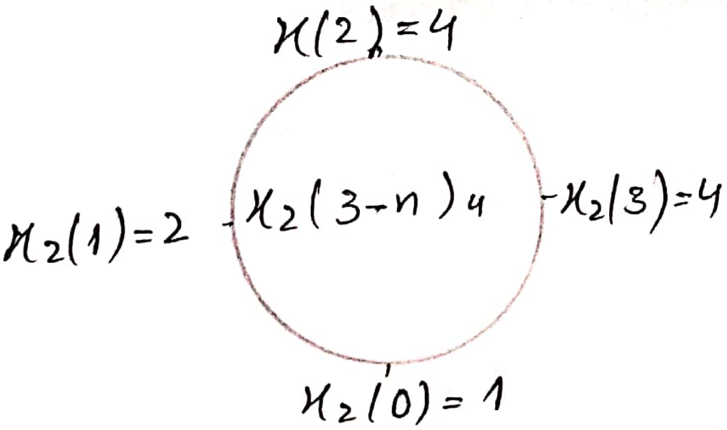


(D)

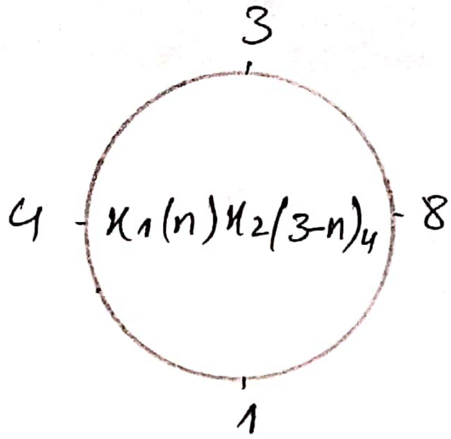


Folded Sequence rotated by two unit in time.

Product Sequence



(E)



Folded Sequence rotated by three unit in time.

Product Sequence