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Subject # Linear Algebra

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Department # BEE

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QUESTION NO 1

PART (A)

Express the equation of Plane passing through the Points $A(2,-2,1)$, $B(-1,0,3)$, $C(5,-3,4)$

Sol:-

$A(2,-2,1)$, $B(-1,0,3)$
$C(5,-3,4)$.

We can get two vectors in the plane by subtracting pass of Points in the plane.

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ -3 \\ 4 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$$

The cross Product of these two vectors in the unique direction orthogonal

So both hence in the direction of the normal vector the plane.

(2)

$$\begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix} \times \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ -15 \\ 3 \end{bmatrix} \quad \begin{matrix} 3 & & -2 \; -2 \\ ;6 & & \times \\ & & -3 \;\; 1 \end{matrix}$$

$$-2-6 \;,\; -12-3 \;,\; -9+12$$

$$(-8, 15, 3)$$

The equation of a plane is $ax + by + cz = d$ where

$\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is the normal vector to the plane we can plug in $a = -8$, $b = -15$, $c = 3$ to get

$$\boxed{-8x - 15y + 3z = d.}$$

Plug in the point $(-1, 0, 3)$ to get

$$-8(-1) - 15(0) + 3(3) = d$$

or

$$d = 8 - 0 + 9$$

$$\boxed{d = 17}$$

The equation of the plane is hence

$$\boxed{-8x - 15y + 3z = 17}$$

Ans

QUESTION NO 1

PART (B)

Express a pair of planes whose intersection is the given line,

$x = 2 - 3t, \; y = 3 + t, \; z = 2 - 4t.$

Soln:-

$x = 2 - 3t.$
$y = 3 + t$
$z = 2 - 4t.$

$x - 2 = -3t \implies t = \frac{x-2}{-3}$

$y - 3 = t \implies t = \frac{y-3}{1}$

$z - 2 = -4t \implies t = \frac{z-2}{-4}$

$$\frac{x-2}{3} = \frac{y-3}{1} = \frac{z-2}{-4}$$

for 1st plane takes 1st and 2nd.

$$\frac{x-2}{-3} = \frac{y-3}{1}$$

$$x-2 = -3y+9$$

$$x+3y-11=0$$

for 2nd plane take 1st and 3rd.

$$\frac{x-2}{-3} = \frac{z-2}{-4}$$

$$-4x+8 = -3z+6$$

$$-4x+3z+2=0$$

or

$$4x-3z-2=0$$ Ans

QUESTION NO 2

$L(x,y) = (x+1, y, x+y)$ illustrate that L is Linear transformation?

Sol:-

$$L(x,y) = (x1, y, x+y)$$

$$L(x,y) = (x+1, y, x+y)$$

let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$$u+v = (x_1, y_1) + (x_2, y_2)$$

$$u+v = (x_1+x_2, y_1+y_2)$$

Put "L" on both side

$$L(u+v) = L(x_1+x_2, y_1+y_2)$$

$$L(u+v) = (x_1+x_2+1, y_1+y_2, x_1+x_2+y_1+y_2)$$

given that $u = (x+y)$

$$L(u) = L(x_1, y_1) = (x+1, y+, x_1+y-1)$$

$$L(v) = L(x_2, y_2) = (x_2+1, y_2, x_2+y_2)$$

$t(u)+t(v) = (x_1+x_2+2, y_1+y_2, x_1+x_2+y_2+y_2)$

$\boxed{\text{Since } 1 \neq 2.}$

Ans

QUESTION NO 3

Using the matix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ the interpret to decode the massage 77 54 38 71 49 29 68 51 33 76 48 40 86 53 52.

Soultion:-

find $A^{-1} = ?$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 16 \\ 18 \\ 15 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix} \quad x_4 = \begin{bmatrix} 8 \\ 16 \\ 12 \end{bmatrix}$$

16	8	15	20	15	7	18	1	16	1	14	19
P	H	O	T	O	G	R	A	P	A	N	S

Photograph Plans.

Ans

QUESTION NO 4

find an equation of the Plane Passing through the Point $(-1,3,2)$ and perpendicular to the vector $n = (0,1,-3)$.

Solution:

Equation of Plane

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

Given that

$$P=(x_0,y_0,z_0)=(-1,3,2)$$

$$n=(a,b,c)=(0,1,3)$$

So

$$0(x-(-1))+1(y-3)-3(z-2)$$

$$0(x+1)+1(y-3)-3(z+2)$$

$$0+y-3-3z+6$$

$$\Rightarrow y-3z-3+6$$

$$\Rightarrow y-3z+3$$ Ans

QUESTION NO 5

Find an Eigen values and Eigen vectors of matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

Solution:-

Eigen values.

We known that $Ax = \lambda x$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

then

$$x_1 + x_2 = \lambda x_1 \quad \text{—— (i)}$$
$$-2x_1 + 4x_2 = \lambda x_2 \quad \text{—— (ii)}$$

So

$$x_1 - \lambda x_1 + x_2 = 0$$

$$= (1-\lambda) x_1 + x_2 = 0$$

Cy $-2x_1 + 4x_2 - \lambda x_2 = 0$

$= -2x_1 + (4-\lambda)x_2 = 0$

$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow 2x_i = x_2 = 0$

$\Rightarrow -2x_1 + 4x_2 = 3x_2 \quad -(ii)$

$\Rightarrow -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$x_1 = \frac{1}{2} x^2$

let $x^2 = r$

where $r \neq 0$

so,

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & r \\ r & \end{bmatrix}$

eigen vector for $\lambda_2 = 2$ put in eq (i) and eq (ii)

$x_1 + x_2 = 2x_1 \quad -(i)$

$-2x_1 + 4x_2 = 2x_2 \quad (ii)$

$= -x_1 + x_2 = 0$

$= x_1 - x_2 = 0$

$= x_1 = x_2$

$$\Rightarrow -2x_1 + 4x_2 = 2x \quad \text{— ii}$$

$$= -2x_1 + 2x_2 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$x_1 = \gamma$ then $x_2 = \gamma$

So,

$$\boxed{x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \gamma \\ \gamma \end{bmatrix}}$$

Ans

THE END