

Date = 14/4/2020

For

ID = 16372

Name = Muhammad Amir

Linear Algebra

BS-CS 2nd semester

mid term.

Teacher:

Date: \_\_\_/\_\_\_/20\_\_\_

page 1

Q1

$$\begin{bmatrix} 1 & 3 & 3 & 0 & 5 \\ 0 & 1 & -2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

For solving this system, we will write the system of equations corresponding to this matrix. We will write each non-zero equation so that its one basic variable is expressed in terms of any free variable appearing in equation

System of equations corresponding to the matrix is

$$x_1 + 3x_2 + 3x_3 = 5$$

$$x_2 - 2x_3 = 7$$

$$x_3 = -6$$

$$x_4 = 3$$

Now solving

$$x_2 - 2x_3 = 7$$

$$x_2 = 7 + 2x_3$$

$$= 7 + 2(-6)$$

$$x_2 = -5$$

$$x_1 + 3x_2 + 3x_3 = 5$$

$$\begin{aligned}x_1 &= 5 - 3x_2 - 3x_3 \\&= 5 - 3(-5) - 3(-6) \\&= 5 + 15 + 18\end{aligned}$$

$$x_1 = 38$$

Thus

$$x_1 = 38$$

$$x_2 = -5$$

$$x_3 = -6$$

$$x_4 = 3$$

Ans



Q2: The two matrices are

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

(A) converting 1st into 2nd

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} R_3 - 2R_2$$

Converting 2nd into 1st

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad R_3 + 2R_2$$



(B)

a)

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & \pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

is in echelon form because

i) Each leading of a row is in a column to the right of the leading entry of the row above it.

ii) All entries in a column below a leading entry are zero

b)

$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

is in echelon form because

- i) All non-zero rows are above the rows of all zeros.
- ii) Each leading of a row is in a column to the right of the leading entry of the row above it.
- 3) All entries in a column below a leading entry are zero.

c) 
$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

is in echelon form because

1) Each leading entry of a row is in a column to the right of the leading entry of the row above it.

2) All entries in a column below a leading entry are zero.



d) 
$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

is not in echelon form. It can be written in echelon form as

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

In this matrix the non zero rows are above the rows of zero and all entries in a column below a leading entry are zero.

(3) a)

**Echelon Form:** A matrix is in echelon form if

i) All non zero rows are above any rows of all zeros.

ii) Each leading of a row is in a column to the right of the leading entry of the row above it.

iii) All entries in a column below a leading entry are zero.

**Example:**

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix}$$

**Reduced Echelon Form:** A matrix is in reduced echelon form if.

- i) It is already in echelon form.
- ii) The leading entry in each nonzero row is 1
- iii) Each leading 1 is the the only non-zero entry in the column.

**Example**

$$\begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$



## Practical use of Reduced Echelon Form

Reduced echelon form can be used for solving system of linear equations.

From the system of linear equations, form the Augmented matrix and bring it to reduced echelon form.

e.g. the matrix is

$$\left[ \begin{array}{cccccc} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{array} \right]$$

Reduced echelon form is

$$\begin{bmatrix} 1 & 6 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

Associated system is

$$x_1 + 6x_2 + 3x_4 = 0$$

$$x_3 - 4x_4 = 5$$

$$x_5 = 7$$

which can then be solved for finding values of  $x_1, x_2, x_3, x_4$  and  $x_5$ .

B) The matrix is

$$\begin{bmatrix} 1 & 2 & 8 \\ 2 & 8 & -1 \\ -3 & 0 & 0 \\ 1 & -4 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 8 \\ 0 & 4 & -17 \\ -3 & 0 & 0 \\ 1 & -4 & 12 \end{bmatrix} \quad R_2 - 2R_1,$$



$$\begin{bmatrix} 1 & 2 & 8 \\ 0 & 4 & -17 \\ 0 & 6 & 24 \\ 0 & -6 & 4 \end{bmatrix} \quad \begin{array}{l} R_3 + 3R_1 \\ R_4 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 8 \\ 0 & 4 & -17 \\ 0 & 0 & 28 \\ 0 & 0 & 28 \end{bmatrix} \quad \begin{array}{l} R_3 + R_4 \\ R_4 + R_3 \end{array}$$

