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Assignment. Final Term.

DSP.

Q1) a#

sol:

$y_h(n) = C_1(-1)^n + C_2(4)^n$

we assume a solution in form

$y_p(n) = k(4)^n u(n)$

$y_p(n) = kn(4)^n u(n)$

Put in the given eq.

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + x(n)$

$\rightarrow$ $kn(4)^n u(n) - 3k(n-1)(4)^{n-1}u(n-1) - 4k(n-2)4(n-2)$

$= (4)^n u(n) + 2(4)^{n-1} u(n-1)$

For $n=2$, $u = 6/3$

$y_p(n) = \frac{6}{3} n (4)^n u(n)$

So the total solution on the equation

$y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{3} n(4)^n \; n \geq 0$

C_1 & C_2 are determine.

$y(0) = C_1 + C_2$

$y(1) = -C_1 + 4C_2 + \frac{24}{5}$

Computting above by setting.

$y(-1) - y(-2) = 0.$

$c_1 + c_2 = 1$

$-c_1 + 4c_2 + \frac{24}{4} = 9.$

$y_{25}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6n}{5}(4)^n \; n \geq 0$

Q1 b// Ans #

Sol:

$y(z) = \frac{x(z)}{1 - 0.6z^{-1} + 0.8z^{2}}$

$x(z) = 1$

$\therefore H(z) = \frac{Y(z)}{X(z)}$

$$\frac{1}{(-\frac{1}{5}z^{-1})\ (1-\frac{2}{5}z^{-1})}$$

$$H(z) = \frac{1}{1-\frac{5}{5}z^{-1}} + \frac{2}{1-\frac{2}{5}z^{-1}}$$

$$h(n) = \left\{-1\left(\tfrac{1}{5}\right)^n + 2\left(\tfrac{2}{5}\right)^n\right\}u(n)$$

Q2 A#

Sol:

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$\frac{A}{(1-2z)^{-1}} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}.$$

$$A = 4,\ B = -3,\ C = -1$$

Hence $x(n) = [4(2)^n - 3 - n]u(n)$

Q2 b

sol: 1st we have to eleminate the negative power so we will divide & multiply z^2

$$x(z) = \frac{z^2}{z^2 - 1.5z^{-1} + 0.5z^{-2 \cdot 2}}$$

$$= \frac{z^2}{z^2 - 1.5z^{-1} + 0.5}.$$

$$= \frac{z^2}{z^2 + 1z^{-1} - 0.5z^{-1} + 0.5}$$

$$= \frac{z^2}{z(z-1)^{-0.5}(z-1)}$$

$$x(z) = \frac{z^2}{(z-0.5)(z-1)}$$

$$P_1 = 1,\ P_2 = 0.5.$$

$$\frac{x(z)}{z} = \frac{z}{(z-1)(z-0.5)}.$$

$$\frac{z}{(z-1)(z-0.5} = \frac{A}{(z-1)} + \frac{B}{(z-0.5)}$$

$$z = A(z-0.5) + B(z-1)$$

Now for $z = 1$

$$1 = A(1-0.5) + B(0)$$
$$1 = A(0.5)$$
$$A = 2$$

for $z = 0.5$

$$0.5 = A(z-0.5) + B(0.5-0.1)$$
$$0.5 = B(-0.5)$$
$$B = -1$$

Put the value of A & B in equation

$$\frac{xz}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}.$$

Q3(a)

Sol

At $w=0$ we have

$$H(0) - \frac{b_0}{(1-P)^2} = 1$$

Hence $b_0 = (1-P)^2$

At $\omega = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}.$$

$$= \frac{(1-P)^2}{(1-P\cos(\frac{\pi}{4}) + jP\sin(\frac{\pi}{4}))^2}.$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2} + jP/\sqrt{2})^2}.$$

$P = To.$

Hence $\frac{(1-p)^4}{[(1-p/\sqrt{2})^2+p^2/2]^2}=\frac{1}{2}.$

$$\sqrt{2}\,(1-p)^2 = 1+p^2-\sqrt{8}p.$$

$$H(z)=\frac{0.46}{(1-0.32z^{-1})^2}\ \underline{\text{Ans}}$$

Q3 (b)

Sol.

$$p_{1,2} = re^{\pm j\pi/2}$$

& zero at $z=1$ & $z=-1$

$$H(z)=\frac{G_1(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G_1\frac{z^2-1}{z^2+r^2}$$

The gain factor is determine by evaluating the frequency $H(\omega)$ of the filter at $\omega=\pi/2$. Thus we have.

$$H\left(\frac{\pi}{2}\right)=G\left(\frac{2}{1-r^2}\right)=1$$

$$G_1=\frac{1-r^2}{2}.$$

$$\left[H=\left(\frac{4\pi}{9}\right)\right]^2=\frac{(1-r^2)^2}{4}\ \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}=\frac{1}{2}$$

$$1.94(1-r^2)^2=1-1.88r^2+r^4$$

$$H(z)=0.15\ \frac{1-z^{-2}}{1+0.7z^{-2}}.$$

Q4 A

$$x(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)}$$

$$= \frac{\sin(\omega/2)}{\sin(\omega/2} e^{-j\omega(L-1)/2}$$

Hence.

$$x(K) = \frac{1-e^{-j\pi\, KL/N}}{1-e^{-2j2\pi/K/N}} \quad K = 0, 1 \cdots N, 1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}.$$

Q4b

Sol

$$W_N^{K+N/2} = W_N^{k}.$$

The matrix may be extend be

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^1 & W_4^0 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then $x_4 = W_4\, x_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$