Department of Electrical Engineering
Assignment
Date: 25/06/2020

Course Details

Course Title:	Signals & Systems	**Module:**	04
Instructor:	Engr Mujtaba Ihsan	**Total Marks:**	50

Student Details

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Q1.	(a)	**Show** with a help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain.	Marks 06+08
	(b)	If $\mathbf{x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]}$ $\mathbf{h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]}$ **Produce** Y(z) and y[n]	CLO 3
Q2.		$\mathbf{f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}}$ **Retrieve** the Fourier series for the given function.	Marks 10 CLO 3
Q3.		If $\mathbf{X(z) = 2z^2 + 2z / (z^2 + 2z - 3)}$ **Retrieve** x[n] using inverse Z-transform method.	Marks 10 CLO 3
Q4.		**Express** the transfer function using the given data. $\mathbf{A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = [1 \quad 2] \quad D = [0]}$	Marks 09 CLO 3
Q5.		**Apply** Fourier transform on the signal, **x (t) = e** $^{-a\,\lvert t \rvert}$ **u (t**) where u (t) is a unit step function.	Marks 07 CLO 3

Q1 (a) show with a help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain

Ans: Let $x(t)$ be a continuous-time signal with a fourier transform of $X(jw)$.

So,

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)\, e^{jwt}\, dw.$$

Differentiating both sides w.r.t "t"

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) \frac{d}{dt}\{e^{jwt}\} dw$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) \{e^{jwt} \cdot jw\} dw$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{jw\, X(jw)\}\, e^{jwt}\, dw$$

So, in F-domain.

$$F\left\{\frac{d}{dt} x(t)\right\} = jw\, X(jw).$$

If a function is differential in time domain it is multiplied by jw in frequency domain, and also differentiated in time ~~ϕ~~ domain it is multiplied with 'jw" in frequency domain.

we know that differentiation in time domain corresponds to multiplication by jw in frequency domain. we might suspect that multiplication by jt in the time domain corresponds roughly to differentiation in frequency domain.

As we know that.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Differentiate both sides w.r.t "ω"

$$\frac{dX}{d\omega}(j\omega) = \int_{-\infty}^{\infty} -j\cancel{t}\; x(t)\, e^{-j\omega t}\, dt$$

$$\frac{dX}{d\omega}(j\omega) = -jt \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

$$\frac{dX}{d\omega}(j\omega) = -jt\, f\{x(t)\}.$$

so, $\boxed{-jt\, x(t) \xleftrightarrow{f} d/dt\; X(j\omega).}$

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Q1 (b)

If $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

produce $Y(z)$ and $y(n)$.

soln:-

$Y(z) = H(z)X(z)$

Find $y[n]$.

$X(z) = 2 - 4z^{-2} + 2z^{-3}$

$H(z) = 3 + 1z^{-1} + 2z^{-2}$

Now,

$Y(z) = H(z) * X(z)$

$Y(z) = (2 - 4z^{-2} + 2z^{-3})(3 + z^{-1} + 2z^{-2})$

$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4}$
$+ 6z^{-3} + 2z^{-4} + 4z^{-5}$

$\Rightarrow 6 + 2z^{-1} - 8z^{-2} - 2z^{-3} + 6z^{-4} + 4z^{-5}$

To Find $y[n]$ use the delay property.

$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] - 2\delta[n-3]$
$+ 6\delta[n-4] + 4\delta[n-5].$

[redacted]

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~~Qt~~

~~Ans~~

Q2:- $f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$

Retrieve the Fourier series of the following.

a_0 as:-

$$= \frac{1}{\pi} \int_{\pi}^{0} \frac{-\pi}{2}\, dx + \int_{0}^{\pi} \frac{\pi}{2}\, dx$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2} \int_{\pi}^{0} 1\, dx + \frac{\pi}{2} \int_{0}^{\pi} 1\, dx \right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{2}(x)\Big|_{-x}^{0} + \frac{\pi}{2}(x)\Big|_{0}^{x} \right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{2}(0)-(-\pi) + \frac{\pi}{2}(\pi)-(0) \right]$$

$$= \frac{1}{\pi}\left[-\frac{\cancel{\pi}}{2} + \frac{\cancel{\pi}}{2} \right]$$

$$= \frac{1}{\pi}\left[0 \right]$$

$$= 0$$

A_n as:- $\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$

$$= \frac{1}{\pi} \int_{-\pi}^{0} -\frac{\pi}{2} \cos nx\, dx + \int_{0}^{\pi} \frac{\pi}{2} \cos nx\, dx$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2} \int_{-\pi}^{0} \cos nx\, dx + \frac{\pi}{2} \int_{0}^{\pi} \cos nx\, dx \right]$$

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$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}(\sin nx)\Big|_{-\pi}^{0} + \frac{\pi}{2}(\sin nx)\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}(\sin n(0) - \sin n(-\pi)) + \frac{\pi}{2}(\sin n(\pi) - \sin n(0))\right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}(0) + \frac{\pi}{2}(0)\right]$$

$$= 0$$

Now for b_n.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\sin nx\, dx + \int_{0}^{\pi}\frac{\pi}{2}\sin nx\, dx\right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{2}(-\cos nx)\Big|_{-\pi}^{0} + \frac{\pi}{2}(-\cos nx)\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}(-\cos n(0) - (-\cos n(-\pi)) + \frac{\pi}{2}(\cos n(\pi) - (-\cos n(0))\right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}(-2) + \frac{2\pi}{2}\right]$$

$$= \frac{1}{n\pi}\left[\frac{2\pi}{2} + \frac{2\pi}{2}\right]$$

$$= \frac{1}{n\pi}\left(\frac{4\pi}{2}\right) = \frac{4\pi}{n2\pi} \quad \frac{1}{2n}$$

$$= \frac{1}{2n}$$

$$\begin{cases} 0 & \text{if } n \text{ is even} \\ \dfrac{1}{2n} & \text{if } n \text{ is odd} \end{cases}$$

$$= 0 + 0 + 0 + \cdots + \frac{1}{2}\sin x + \frac{1}{4}\sin 2x + \frac{1}{6}\sin 3x + \frac{1}{8}\sin 4x \cdots\cdots$$

will be the fourier series.

Q3.

$$x(z) = \frac{2z^2 + 2z}{z^2 + 2z - 3}$$

Retrieve the fourier series for the given function.

soln:

$$x(z) = \frac{2z^2 + 2z}{(z-1)(z+3)}$$

$$\frac{x(z)}{(z)} = \frac{2z^2 + 2z}{z^2+2z-3} = \frac{A}{(z-1)} + \frac{B}{(z+3)} \text{ —— } (A)$$

$$2z^2 + 2z = A(z+3) + B(z-1) \longrightarrow (B)$$

put $z = -3$ in eq (B)

$$2(-3)^2 + 2(-3) = A(\cancel{-3}+\cancel{3}) + B(-3-1)$$

$$\Rightarrow 2(9) - 6 = B(-4)$$

$$18 - 6 = B(-4)$$

$$B = \frac{12}{-4}$$

$$\boxed{B = -3}$$

Now

put $z = 1$ in eq (B)

$$2(1)^2 + 2(1) = A(1+3) + B\cancel{(1-1)}^{0}$$

$$2 + 2 = A(4)$$

$$A = \frac{4}{4}$$

$$\boxed{A = 1}$$

Now putting the values of A and B in eq (A)

$$\frac{2z^2 + 2z}{(z-1)(z+3)} = \frac{1}{(z-1)} + \frac{-3}{(z+3)}$$

$$X(z) = 1\,\frac{z}{z-1} - 3\,\frac{z}{z+3}$$

Taking Inverse Z-Transform.

$$x[n] = 1\,u[n] - 3\,(-3)^k.$$

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Q. Express the transfer function using the given data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = [0]$$

soln.

$$\frac{Y(s)}{X(s)} = H(s)$$

$$H(s) = C(sI - A)^{-1} B + D$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right.$$

$$\left. -1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right] + 0$$

$$H(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & +1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Adj = (s+2)\, s + 1 = s^2 + 2s + 1$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} + \frac{1}{s^2 + 2s + 1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \times \frac{1}{s^2 + 2s + 1} \begin{bmatrix} s + 0 \\ 1 \end{bmatrix}$$

$$H(s) = \frac{\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}}{s^2 + 2s + 1} \Rightarrow \boxed{H(s) = \frac{s+2}{s^2 + 2s + 1}}$$

Q5:- Apply fourier transform on the signal $x(t) = e^{-a|t|}$ $u(t)$ is a unit step function.

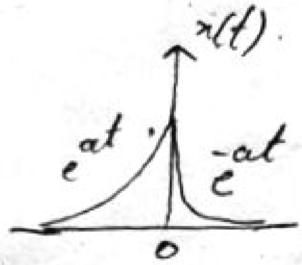
Sol/n: The fourier transform of the given function $x(t)$ is given by.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

so,

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0. \end{cases}$$



$$X(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{(a+j\omega)}\left[e^{-\infty} - e^{0}\right]$$

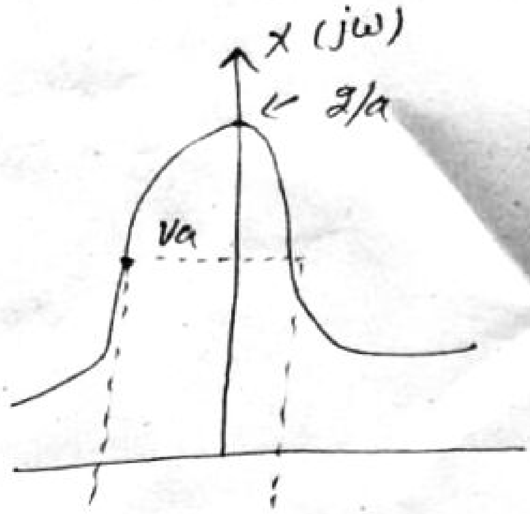
$\because e^{-\infty} = 0$

$\because e^{0} = 1$

$$= \frac{1}{(a-j\omega)}\left[1-0\right] - \frac{1}{(a+j\omega)}\left[0-1\right]$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a + \cancel{j\omega} + a - \cancel{j\omega}}{a^2 - (j\omega)^2}$$



$$\boxed{X(j\omega) = \frac{2a}{a^2 + \omega^2}}$$