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Subject E-m-F

Assignment 02

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Q12 The value of E at $P(p=2, \phi=40, z=3)$
 for move a $20 \mu\text{C}$
 charged a distance of $6 \mu\text{m}$.

Ans The direction of a_p : incremental work
 is given by $dw = -qE \cdot dL$, where in
 this case, $dL = dp a_p = 6 \times 10^{-6} a_p$ Thus
 $dw = -(20 \times 10^{-6} \text{C})(100 \text{V/M})(6 \times 10^{-6} \text{M}) = -12 \times 10^{-9} \text{J}$
 $\Rightarrow -12 \text{ nJ}$

• The direction of a_ϕ : In this case dL
 $= 2d\phi a_\phi = 6 \times 10^{-6} a_\phi$ & so

$$dw = -(20 \times 10^{-6})(300)(6 \times 10^{-6}) = -24 \times 10^{-9} \text{J} = -24 \text{ nJ}$$

• The direction of a_z : Here $dL = dz a_z = 6 \times 10^{-6} a_z$
 $dw = -(20 \times 10^{-6})(300)(6 \times 10^{-6}) = -3.6 \times 10^{-8} \text{J} = -36 \text{ nJ}$

• The direction of E : Here, $dL = 6 \times 10^{-6} a_E$,

$$a_E = \frac{100 a_p - 200 a_\phi + 300 a_z}{(100^2 + 200^2 + 300^2)^{1/2}}$$

$$\Rightarrow 0.267 a_p - 0.535 a_\phi + 0.808 a_z$$

Thus

$$dw = -(20 \times 10^{-6}) [100 a_p - 200 a_\phi + 300 a_z] \cdot [0.267 a_p - 0.535 a_\phi + 0.808 a_z] \times (6 \times 10^{-6})$$

$$\Rightarrow -44.9 \text{ nJ}$$

The direction of $G = 2ax - 3ay + 4az$: in this case, $dL = 6 \times 10^6 a \cdot G$

$$aG = \frac{2ax - 3ay + 4az}{[2^2 + 3^2 + 4^2]^{1/2}} = 0.371ax - 0.557ay + 0.743az$$

Now

$$dW = -(20 \times 10^6) [100a_p - 200a_\phi + 300az] \cdot [0.371ax - 0.557ay + 0.743az] (6 \times 10^6)$$
$$\Rightarrow -(20 \times 10^6) [37.1(a_p \cdot ax) - 55.7(a_p \cdot ay) - 74.2(a_\phi \cdot ax) + 111.4(a_\phi \cdot ay) + 222.9] (6 \times 10^6)$$

Where at P, $(a_p \cdot ax) = (a_\phi \cdot ay) = \cos(40) = 0.766$

$(a_p \cdot ay) = \sin(40) = 0.643$, $\& (a_\phi \cdot ax)$

$$\Rightarrow -\sin(40) = -0.643$$

Substituting these resultion

$$dW = -(20 \times 10^6) [28.4 - 35.8 + 47.7 + 85.3 + 222.9] (6 \times 10^6) = -41.8 \text{ J}$$

Q2 Let $E = 400a_x - 300a_y + 500a_z$ -----
 ----- distance 1mm in the direction
 specified by.

(a) $a_x + a_y + a_z$ i.e. 1/rate

$$\begin{aligned} dW &= -qE \cdot dL \\ &= -4(400a_x - 300a_y + 500a_z) \cdot \frac{a_x + a_y + a_z}{\sqrt{3}} (10^{-3}) \\ &= \frac{-(4 \times 10^{-3})}{\sqrt{3}} (400 - 300 + 500) = \boxed{-1.39 \text{ J}} \end{aligned}$$

(b) $-2a_x + 3a_y - a_z$. The computation
 is similar that of part a, but
 we change the direction.

$$\begin{aligned} dW &= -qE \cdot dL \\ &\Rightarrow -4(400a_x - 300a_y + 500a_z) \cdot \frac{(-2a_x + 3a_y - a_z)}{\sqrt{14}} (10^{-3}) \\ &\Rightarrow \frac{-(4 \times 10^{-3})}{\sqrt{14}} (-800 - 900 - 500) = \boxed{2.35 \text{ J}} \end{aligned}$$

Q₃

IF $E = 120 \hat{a}_\rho$ V/m find the incremental work done by the electric field on a charge of 2 mC moving from a distance of 2 mm from the z-axis to a distance of 2 mm from the z-axis.

(A)

$P(1,2,3)$ toward $Q(2,1,4)$ The vector along this direction will be $Q - P = (1, -1, 1)$ form which $a_p Q = \frac{a_x - a_y + a_z}{\sqrt{3}}$

also now write

$$\begin{aligned} dW &= -qE \cdot dL \\ &= -(50 \times 10^{-6}) \left[120 \hat{a}_\rho \cdot \frac{a_x - a_y + a_z}{\sqrt{3}} \right] (2 \times 10^{-3}) \\ &= -(50 \times 10^{-6}) (120) \left[(a_p \cdot a_x) - (a_p \cdot a_y) \right] \frac{1}{\sqrt{3}} (2 \times 10^{-3}) \end{aligned}$$

At P , $\phi = \tan^{-1}(2/1) = 63.4^\circ$. Thus $(a_p \cdot a_x) = \cos(63.4^\circ) = 0.447$ & $(a_p \cdot a_y) = \sin(63.4^\circ) = 0.894$ substituting these we obtain $dW = 3.14 \mu\text{J}$.

(B) $Q(2,1,4)$ toward $P(1,2,3)$. A little thought in order here. Note that the field has only a radial component & does not depend ϕ or z . Note also that P & Q are the same radius ($\sqrt{5}$) from z -axis. But different ϕ & z coordinates. Two points at the same

z location and problem would not change. then moving along a straight line between P & Q would involve moving along a chord of a circle whose radius is $\sqrt{5}$. Halfway along this line point of symmetry in field (make a sketch to see this).

This means that when starting from either point the initial force will be same. Thus the answer $dW = 31$ HJ

as part a. This is also found by going through the same procedure as part a, but with the direction (role of P & Q) reversed.

Q₄

Compute the value of G ---
 --- using the Path.

(A)

Straight line of segments $A(1, -1, 2)$
 to $B(1, 1, 2)$ to $P(2, 1, 2)$ In general

We have.

$$\int_A^P G \cdot dL = \int_A^P 2y dx$$

The change of x occurs when
 moving between B & P during which
 $y = 1$ Thus

$$\int_A^P G \cdot dL = \int_B^P 2y dx = \int_1^2 2(1) dx = \boxed{2}$$

(B)

Straight line segment $A(1, -1, 2)$ $C(2, -1, 2)$
 to $P(2, 1, 2)$. In case the change in x occur
 when moving from A to C , during

which $y = -1$. Thus

$$\int_A^P G \cdot dL = \int_A^C 2y dx = \int_1^2 2(-1) dx = \boxed{-2}$$

Q5

For $G = 3xy^3ax + 2zay$. Now things ---
 --- in that path does matter.

(A)

Straight line $y = x-1, z = 1$ we obtain

$$\int G \cdot dl = \int_2^4 3xy^2 dx + \int_1^3 2z dy = \int_2^4 3x(x-1)^2 dx + \int_1^3 2(1) dy = \boxed{90}$$

(B)

Parabola $6y = x^2 + 2, z = 1$ we obtain

$$\int G \cdot dl = \int_2^4 3xy^2 dx + \int_1^3 2z dy$$

$$\Rightarrow \int_2^4 \frac{1}{12} x (x^2 + 2)^2 dx + \int_1^3 2(1) dy = \boxed{82}$$