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PAPER : Digital Signal

SUBJECT: Processing

DEPARTMENT: BE(E)

SEMESTER : 6TH Semester

DATE : 27/6/2020

SUBMITTED TO

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Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	50

Student Details

Name:		**Student ID:**	

Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks** 7 **CLO** 2
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks** 7 **CLO** 2
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks** 6 **CLO** 2
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	**Marks** 6 **CLO** 2
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition H(0) = 1 and $\left\lvert H(\frac{\pi}{4})\right\rvert^2 = \frac{1}{2}$.	**Marks** 6 **CLO** 3

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	**Marks** 6 **CLO** 3
Q 4	(a)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \geq L$	**Marks** 6 **CLO** 2
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$ $$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$	**Marks** 6 **CLO** 2

Q1/ANSWER

Part (a)

Solution :

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

This characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2 \quad \text{Hence}$$

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is

$$y_p(n) = K(-1)^n u(n)$$

Substituting this solution into the different equation, we obtain

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$$
$$(-1)^n u(n) - (-1)^{n-1} u(n-1) \quad \text{For}$$

$n = 2$, $K(1+4+4) = 2 \Rightarrow K = 2/9$. The total solution is

$$y(n) = [C_1 2^n + C_2 n 2^n + 2/9(-1)^n] u(n)$$

From the initial conditions, we obtain $y(0) =$

$y_0 = 1$, $y(1) = 2$ Then

$$C_1 + 2/9 = 1$$

$$\Rightarrow C_1 = 7/9,$$

$$2C_1 + 2C_2 - 2/9 = 2$$

$$\boxed{C_2 = 1/3}$$

Q1 / ANSWER /

Part (b)

Solution:

$$y(n) - 0.7y(n-1) + 0.1y(n-1) = 2x(n) - x(n-2)$$

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = 1/2 \,,\, 1/5$ Hence

$$y_h(n) = c_1 \tfrac{1}{2}^n + c_2 \tfrac{1}{5}^n$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2,$$

$$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence

$$c_1 + c_2 = 2 \;\&\; \tfrac{1}{2}c_1 + \tfrac{1}{5} = 1.4 = 7/5$$

$$\Rightarrow c_1 + \tfrac{2}{5}c_2 = \tfrac{14}{5}$$

These equation yield

$$c_1 = 10/3 \,,\, c_2 = -4/3$$

$$h(n) = \left[\frac{10}{3}\left(\tfrac{1}{2}\right)^n - \tfrac{4}{3}\left(\tfrac{1}{5}\right)^n\right]u(n)$$

$$x'(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\tfrac{1}{2}\right)^{n-k} - \tfrac{4}{3}\sum_{k=0}^{n}\left(\tfrac{1}{5}\right)^{n-k}$$

NEXT

Q 1 / ANSWER / CONTINUE /

Part (b)

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \left(2^{n+1} - 1\right) u(n) - \frac{1}{3} \left(\frac{1}{5}\right)^{n} \left(5^{n+1} - 1\right) u(n)$$

Q2

Part (a)

Solution

$$X(z) = 1/(1-2z^{-1})(1-z^{-1})^2$$

$$\frac{X(z)}{z} = \frac{z^2}{(2z-1)(z-1)^2}$$

$$\frac{x(z)}{z} = \frac{A_1}{2z-1} + \frac{B_1}{z-1} + \frac{C_1}{(z-1)^2}$$

Find A, B, & C

$A = 4$

$B = -3$

$C = -1$

Hence

$$x(n) = [4(2)^n - 3 - n]\,u(n)$$

Q 2 / ANSWER /

Part (b)

~~Solution~~ :

Evaluate the inverse z-transform of

$X(z) = 1/1-az^{-1} \quad |z| > |a|$

Using the complex inversion integral.

Solution:

We have

$x(n) = 1/2\pi j \oint_C z^{n-1}/1-az^{-1}\, dz = \frac{1}{2\pi j} \oint z^n dz/z-a$

where C is a circle at radius great than $|a|$. We shall evaluate this integral using (3.4.2) with $f(z) = z^n$, we distinguish two cases.

1. If $n \geq 0$, $f(z)$ has only zeros & hence no poles inside C. The only pole inside C is $z = a$. Hence

$x(n) = f(z_0) = a^n \quad n \geq 0$

2. If $n<0$, $f(z) = z^n$ has an n^{th}-order pole at $z=0$, which is also inside C. Thus there are contributions from both poles. for $n=-1$ we have

$x(-1) = \frac{1}{2\pi j} \oint_C \frac{1}{z^2(z-a)} dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$

By continuing in the same way we can show that $x(n)=0$ for $n<0$. Thus $x(n) = a^n u(n)$

Q3 ANSWER/CONTINUE/

Determine the values of b_0 & ρ such that the frequency response $H(w)$ satisfies the conditions

$$H(0) = 1$$

and

$$|H(\pi/4)|^2 = 1/2$$

Part a

Solution :

At $w = 0$ we have

$$H(0) = b_0/(1-\rho)^2 = 1$$

$$b_0 = (1-\rho)^2$$

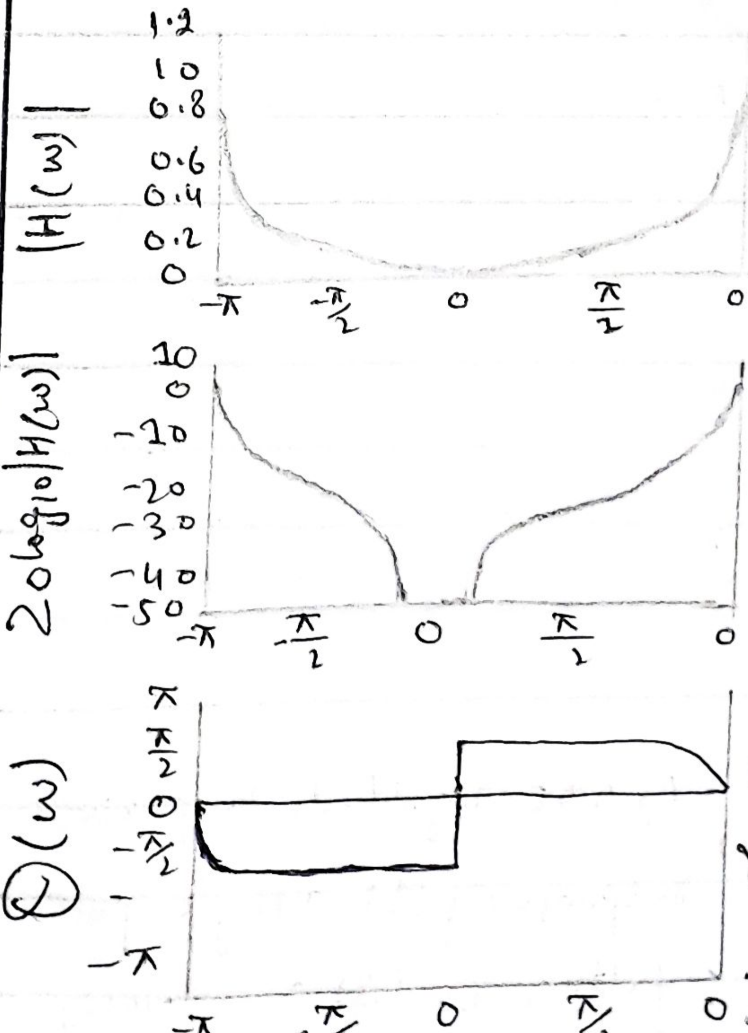


fig 4.46 Magnitude & phase response of a simple highpass filter:
$H(z) = [(1-a)/2][(1-z^{-1})/(1+az^{-1})]$
with $a = 0.9$

NEXT

Q3/ANSWER/

Part (a)

Solution:

A two pole lowpass filter has the system function

$$H(z) = \frac{b_0}{(1 - p z^{-1})^2}$$

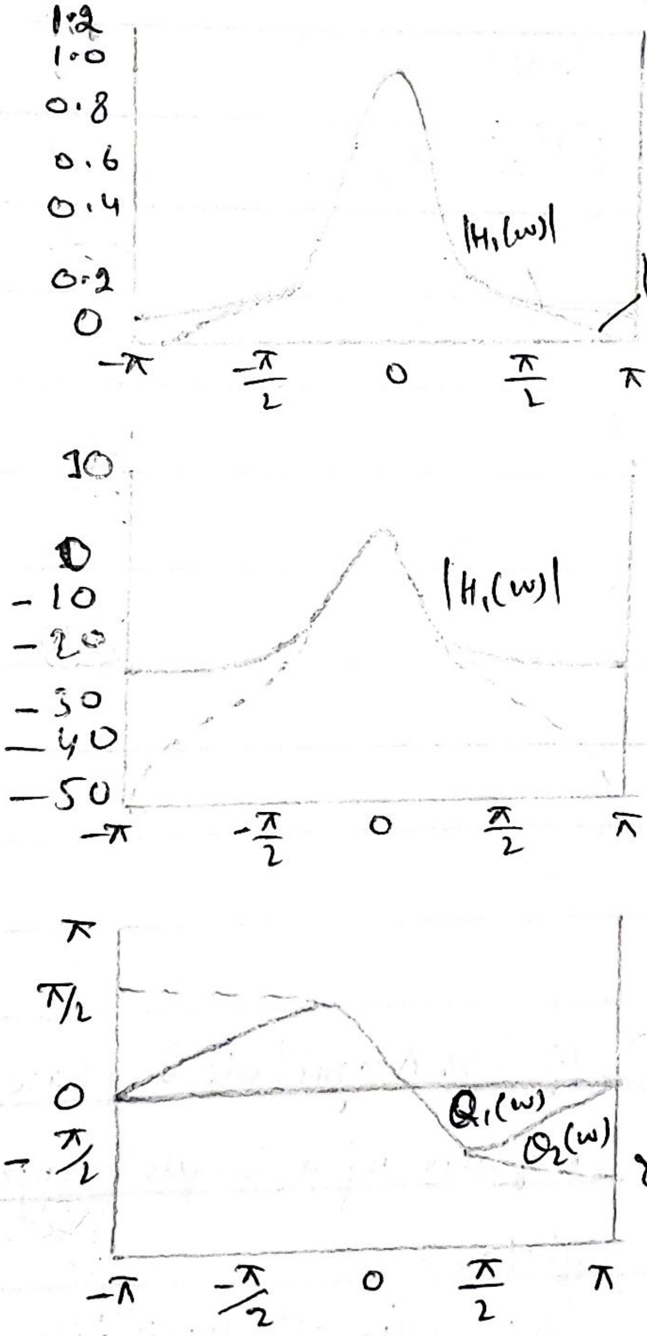


Fig. 4.45 Magnitude & phase response of (1) a single-pole filter & (2) a one-pole, one-zero filter

$H_2(z) = [(1-a)/2][(1+z^{-1})/(1-az^{-1})]$

& $a = 0.9$

NEXT

At $\omega = \pi/4$

$$H(\pi/4) = (1-p)^2 / (1 - pe^{-j\pi/4})^2$$

$$= (1-p)^2 / (1 - p\cos(\pi/4) + jp\sin(\pi/4))^2$$

$$= (1-p)^2 / (1 - p/\sqrt{2} + jp/\sqrt{2})^2$$

Hence

$$= (1-p)^4 / [(1-p/\sqrt{2})^2 + p^2/2]^2 = 1/2$$

Or equivalently

$$\sqrt{2}(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

The value of $p = 0.32$ satisfies this equation. Consequently the system function for the derived filter is

$$H(z) = 0.46 / (1 - 0.32z^{-1})^2$$

Q3 / ANSWER / 8 / WT I NOE / 3

Q3 PART (b)

Solution :

clearly, the filter must have poles at

$$p_{1,2} = r e^{\pm j\pi/2}$$

and zeros at $z = 1$ & $z = -1$, consequently the system function is

$$H(z) = G \cdot (z-1)(z+1) / (z-jr)(z+jr)$$

$$= G \; z^2-1 / z^2+r^2$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$. Thus we have

$$H(\pi/2) = G \; 2/1-r^2 = 1$$

$$G = 1-r^2/2$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = 1/2$$

or equivalently

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

NEXT

Q3/PART b/CONTINUE:

~~The value of $r^2 = 0.7$ satisfies this equation.~~
Therefore, the system function for the desired filter is

$$H(z) = 0.15 \; 1 - z^{-2} / 1 + 0.7 z^{-2}$$

Its frequency response is illustrated in fig 4.47.

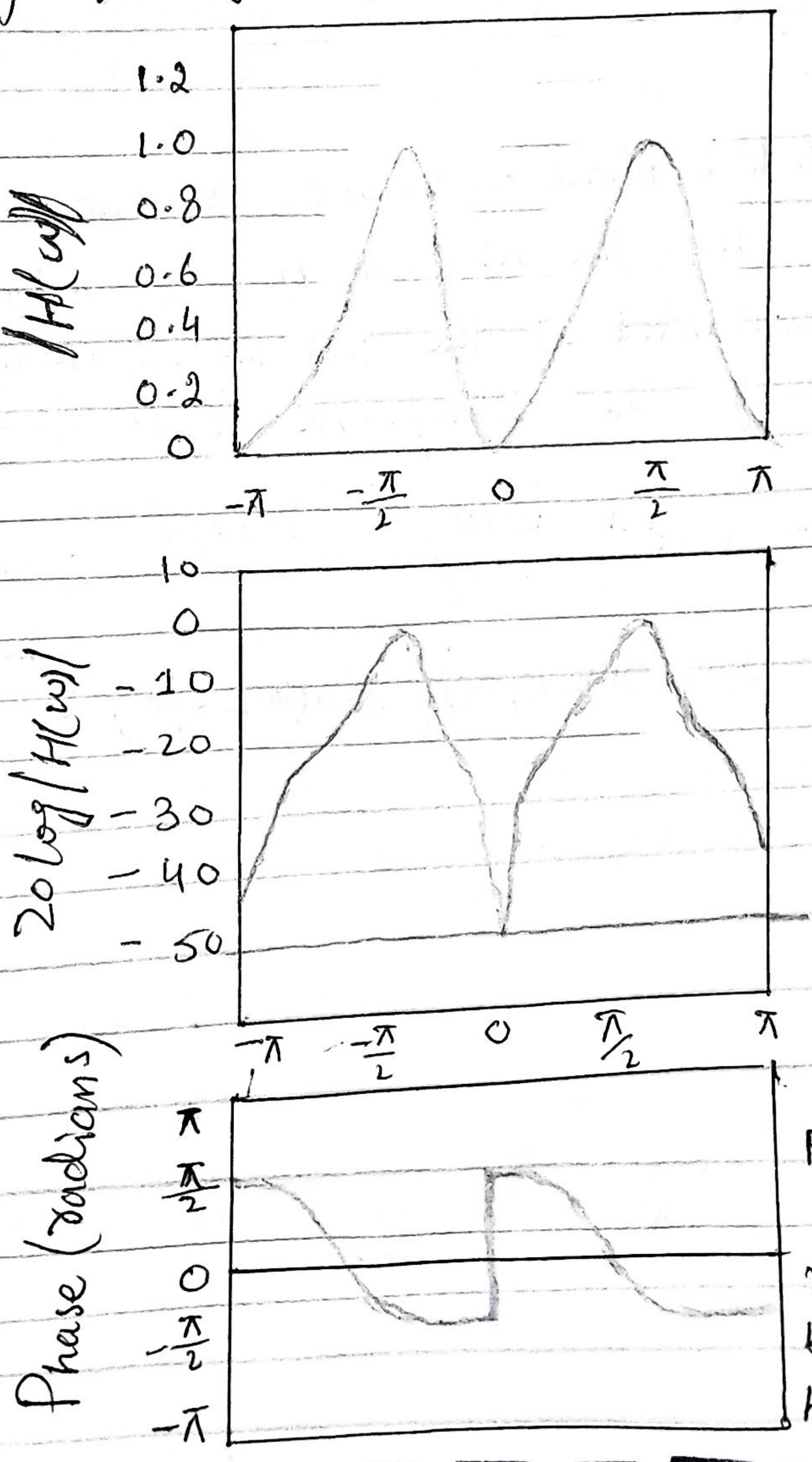


Fig. 4.47
Magnitude and Phase response of a Simple bandpass filter ~~as~~ $H(z) = 0.15\left[(1-z^{-2})/(1+0.7z^{-2})\right]$

Q4

PART (a)

Solution:

The fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

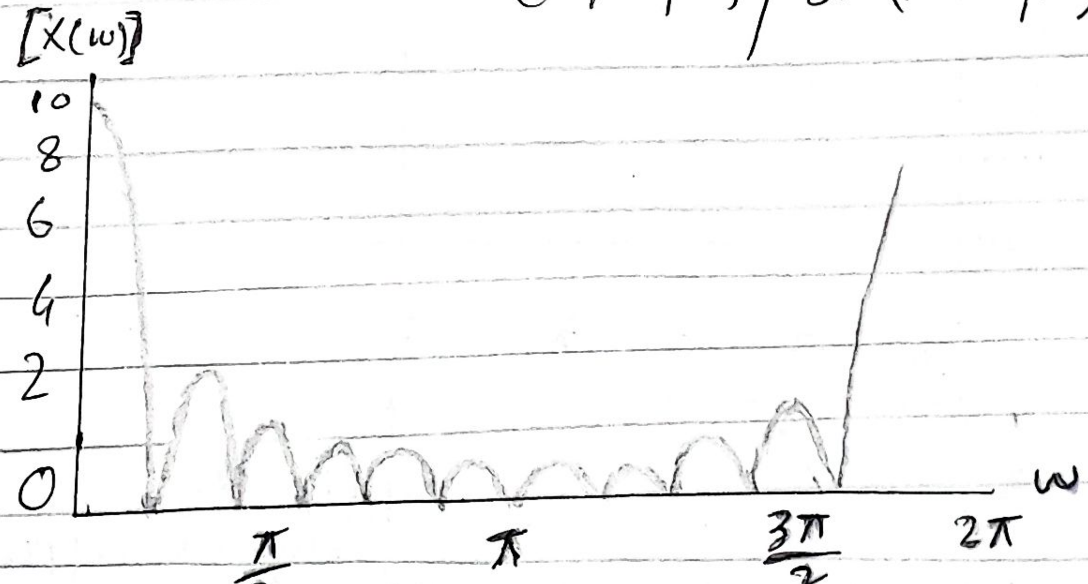
$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude & phase of $X(\omega)$ are illustrated in fig. 5.5 for $L = 10$. The N-Point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequency $\omega_K = 2\pi K/N, K = 0, 1 \cdots N-1$.

Hence

$$X(K) = 1-e^{-j2\pi KL/N} \Big/ 1-e^{-j2\pi K/N} \quad K = 0 \cdot 1 \cdots N-1$$

$$= \sin(\pi KL/N) \Big/ \sin(\pi KL/N) \, e^{-j\omega K(L-1)/N}$$



Q4/ANS/CONTINUE:

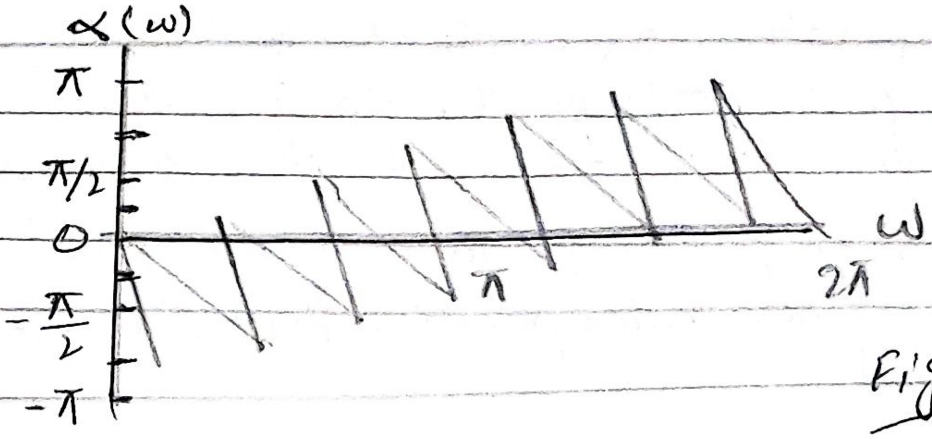


Fig. 5.5

If N is selected such that $N = L$, then the DFT becomes

$$X(K) = \begin{cases} L & K = 0 \\ 0 & K = 1, 2, \ldots, L-1 \end{cases}$$

Thus there is only one nonzero value in the DFT. This is apparent from observation of $X(\omega)$. Since $X(\omega) = 0$ at the frequencies $\omega_L = 2\pi K/L$, $K \neq 0$. The reader should verify that $x(n)$ can be recovered from $X(K)$ by performing an L-point IDFT.

Although the L-point DFT is different to uniquely represent the sequences $x(n)$ in the frequency domain. It is apparent that it does not provide sufficient detail to yield a good picture of the spectral characteristics of $x(n)$. If we wish to have better picture we must evaluate (interpolate) $X(\omega)$ at more closely spaced frequencies, say $\omega_K = 2\pi K/N$, where $N > L$. In effect we can

NEXT

View this computation as explanding the size of the sequence from ~~L~~ L points to N point by appending N-L zeros to the sequence X(n). That is zero padding then the N-point DFT ~~provides~~ provides finer interpolation than the L-point DFT.

Q4/ANSWER/

Part (b)

Solution:

Each sequence consists of four nonzero points. for the purposes of illustrating the operations involved in circular convolution, it is desirables to graph each sequence as point on a circle. Thus the sequences $x_1(n)$ & $x_2(n)$ are graphed as illustrated in f 5.8(a). We note that the sequences are graphed in a counterclock'wise direction on a circle. This establishes the reference direction in rotating one of the sequence relative to the other.

Now, $x_3(m)$ is obtained by circularly convoluting $x_1(n)$ with $x_2(n)$ as specified by (5.2.39). Beginning with $m=0$ we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

$X_2((-n))_4$ is simply the sequence $x_2(n)$ folded & graphed on a circle as illustrated in fig (5.8(b)). In other words, the folded sequence is simply $x_2(n)$ graphed in a clock-wise direction.

NEXT

Q4/ ANSWER/ (CONTINUE)

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$ point by point. This sequence is also illustrated in fig 5.8(b). Finally, we sum the values in the product sequence to obtain.

$$x_3(0) = 14$$

for $m = 1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

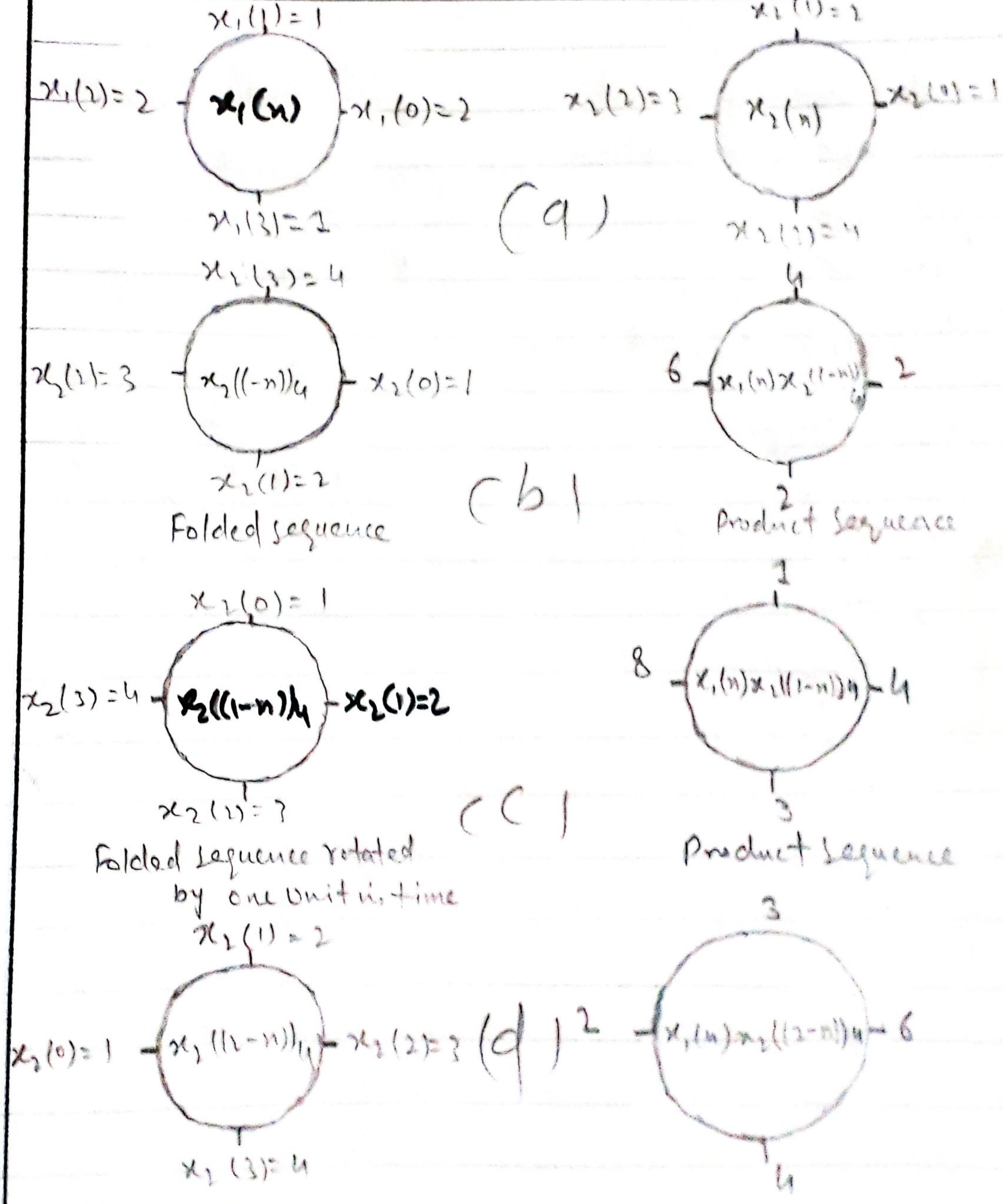
It is easily verified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ rotated counter wise by one unit in time as illustrated in fig 5.8(c). This rotated sequence multiplies $x_1(n)$ to yield the product sequence, also illustrated in fig 5.8(c). Finally, we sum the values in the product sequence to obtain $x_3(1)$.

Thus $x_3(1) = 16$

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence in fig 5.8(b) rotated two units of time in the counterclockwise direction. The resultant sequence is illustrated in fig 5.8(d)

NEXT

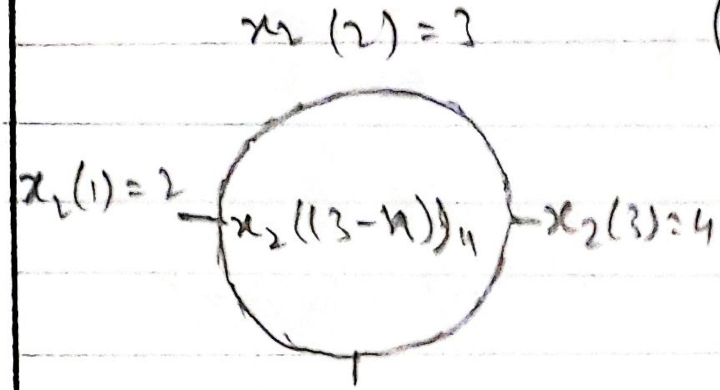

$x_1(1)=1$
$x_1(2)=2$
$x_1(n)$
$x_1(0)=2$
$x_1(3)=1$
$x_2(1)=2$
$x_2(2)=3$
$x_2(n)$
$x_2(0)=1$
$x_2(3)=4$
(a)
$x_2(3)=4$
$x_2(1)=3$
$x_2((-n))_4$
$x_2(0)=1$
$x_2(1)=2$
Folded sequence
(b)
4
6
$x_1(n)x_2((-n))_4$
2
2
Product Sequence
$x_2(0)=1$
$x_2(3)=4$
$x_2((1-n))_4$
$x_2(1)=2$
$x_2(2)=3$
Folded sequence rotated by one unit in time
(c)
1
8
$x_1(n)x_2((1-n))_4$
4
3
Product sequence
$x_2(1)=2$
$x_2(0)=1$
$x_2((2-n))_4$
$x_2(2)=3$
$x_2(3)=4$
(d)
3
2
$x_1(n)x_2((2-n))_4$
6
4

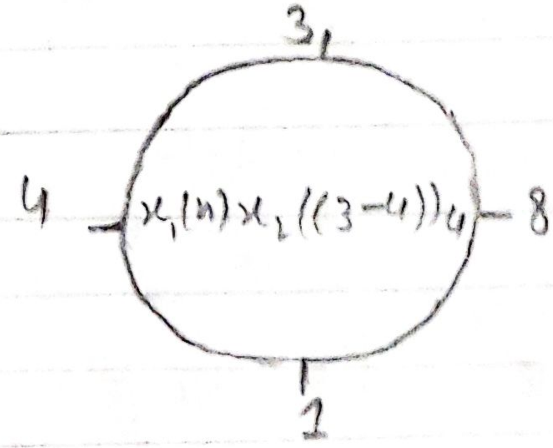
Next

Folded sequence rotated
by two units in time

(e)

Product sequence





Folded sequence rotated
by three units in
time

Product Sequence

along with the product sequence $x_1(n)\,x_2((2-n))_4$. By summing the four terms in the product sequence we obtain $x_3(2) = 14$

For $m = 3$ we have

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\,x_2((3-n))_4$$

The folded sequence $x_2((-n))$ is now rotated by three units in time to yield $x_2((3-n))_4$ & the resultant sequence is multiplied by $x_1(n)$ to yield the Product sequence as illustrated in fig 5.8(e). The sum of the values in the product sequence is $x_3(3) = 16$

We observe that if the computation above is continued above. Therefore, the Circular Convolution of the two sequences $x_1(n)$ & $x_2(n)$ yields the sequence

$$x_3(n) = \{14, 16, 14, 16\}$$