Linear Algebra

Final Paper

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Q1 Consider the following vectors R^3.

$$v_1 = \begin{bmatrix} ID1 \\ ID2 \\ ID3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} ID2 \\ ID3 \\ ID4 \end{bmatrix}, \quad v_3 = \begin{bmatrix} ID3 \\ ID4 \\ ID5 \end{bmatrix}$$

Solution:-

My ID is $\boxed{13063}$ so this becomes.

$$v_1 = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \quad v_2 \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}, \quad v_3 \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$

Now:

$$a_1v_1 + a_2v_2 + a_3v_3 = 0$$

$$a_1(1,3,0) + a_2(3,0,6) + a_3(0,6,3) = 0$$

$$(1a_1 + 3a_2 + 0a_3, \; 3a_1 + 0a_2 + 6a_3, \; 0a_1 + 6a_2 + 3a_3) = 0$$

$$1a_1 + 3a_2 + 0a_3 = 0$$
$$3a_1 + 0a_2 + 6a_3 = 0$$
$$0a_1 + 6a_2 + 3a_3 = 0$$

Let $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 0 & 6 \\ 0 & 6 & 3 \end{bmatrix}$

$$|A| = 1\begin{vmatrix} 0 & 6 \\ 6 & 3 \end{vmatrix} - 3\begin{vmatrix} 3 & 6 \\ 0 & 3 \end{vmatrix} + 0\begin{vmatrix} 3 & 0 \\ 0 & 6 \end{vmatrix}$$

$$= 1(0-36) - 3(9-0) + 0(18-0)$$

$$= -36 - 27 + 0$$

$$= -63$$

$$\boxed{|A| \neq -63}$$

So the system is Linearly Independant.

Q2(a)

Given data:-

Cost per unit	Product X	Product Y
Materials	Rs 450	Rs 400
Labor	Rs 250	Rs 350
Overhead	Rs 150	Rs 150

Find the total cost.

Solution:-

$$\begin{bmatrix} 450 & 400 \\ 250 & 350 \\ 150 & 150 \end{bmatrix} \begin{bmatrix} 1000 \\ 500 \end{bmatrix}$$

so

$$\begin{bmatrix} 450\times1000 + 400\times500 \\ 250\times1000 + 350\times500 \\ 150\times1000 + 150\times500 \end{bmatrix}$$

This becomes

$$= \begin{bmatrix} 450000 + 200000 \\ 250000 + 175000 \\ 150000 + 75000 \end{bmatrix}$$

$$= \begin{bmatrix} 650000 \\ 425000 \\ 225000 \end{bmatrix}$$

so The total material = 650000

The total cost of labours = 425000

And the total cost of overhead = 225000

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Q2 b) Explain the Linear tranformation Properties with the help of above ~~example~~ problem as an example.

- $T(u+v) = T(u) + T(v)$
- $T(cu) = cT(u)$

Solution:-

Linear Tranformation:-

Let $U(F)$ and $V(F)$ are vector spaces. A Mapping F such that $f: U \to V$ is a linear tranformation of U in to V.

if (i) $f(x+y) = f(x) + f(y)$
(ii) $f(ax) = a f(x)$

$x, y \in V$, $a \in F$, $f(x), f(y) \in V$

Properties of LT $T: U \to F$

(i) $f(0) = 0'$, when 0 is identify.
(ii) $f(-\alpha) = -f(\alpha) \ \forall \alpha \in U$
(iii) $f(\alpha - \beta) = f(\alpha) - f(\beta) \ \forall \alpha, \beta \in U$

examples:- a $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \to \begin{bmatrix} x-y \\ x+y \\ 2x \end{bmatrix}$

$T\left(\begin{bmatrix} 6 \\ 8 \end{bmatrix}\right) \to \begin{bmatrix} 6-8 \\ 6+8 \\ 2(6) \end{bmatrix} = \begin{bmatrix} -12 \\ 14 \\ 12 \end{bmatrix}$

$T\left(\begin{bmatrix} 6 \\ 8 \end{bmatrix}\right) \to \begin{bmatrix} -2 \\ 14 \\ 12 \end{bmatrix}$ $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \to \begin{bmatrix} x-y \\ x-y \\ y \end{bmatrix}$

e.g. $T(u+v) = T(u) + T(v)$

$u = \begin{bmatrix} a1 \\ a2 \end{bmatrix}$, $v = \begin{bmatrix} b1 \\ b2 \end{bmatrix}$

$T\left(\begin{bmatrix} a1 \\ a2 \end{bmatrix}\right) + \begin{bmatrix} b1 \\ b2 \end{bmatrix}\Big)$ $T\left(\begin{bmatrix} a1+b1 \\ a2+b2 \end{bmatrix}\right)$

$= \begin{bmatrix} a1+b1 - (a2+b2) \\ a1+b1 + (a2+b2) \\ 2(a1+b1) \end{bmatrix}$ $= \begin{bmatrix} a1+b1 - a2-b2 \\ a1+b1 + a2+b2 \\ 2a1+2b1 \end{bmatrix}$

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<u>example</u>:- $T(u+v) = T(u) + T(v)$

$u = \begin{bmatrix} 6 \\ 8 \end{bmatrix}, v = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

$T = \left(\begin{bmatrix} 6 \\ 8 \end{bmatrix} + \begin{bmatrix} 7 \\ 9 \end{bmatrix}\right) = T \begin{bmatrix} 6+7 \\ 8+9 \end{bmatrix}$

$= T\begin{pmatrix} 13 \\ 17 \end{pmatrix} = T \begin{pmatrix} 10-17 \\ 10+17 \\ 2(10 \end{pmatrix}$

$= T\begin{pmatrix} -7 \\ 27 \\ 20 \end{pmatrix}$

<u>2nd condition</u>:-

$T(cv) = cT(v)$

let $u = \begin{bmatrix} a1 \\ a2 \end{bmatrix} = T\left[c\begin{bmatrix} a1 \\ a2 \end{bmatrix}\right) = T\begin{bmatrix} c \cdot a1 \\ c - a2 \end{bmatrix}$

$= \begin{bmatrix} ca1 - ca2 \\ ca1 + ca2 \\ 2ca1 \end{bmatrix}$

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Q3) Let V be a set where elements are called vector and F be a field, where elements are called scaler. The set V is called vector space or linear space are field F if V(F) satisfy the following condition.

(i) (V,+) is a belian group w.r.t internal composition.

(ii) V(F) is closed under scaler multiplication w.r.t "."

i-e $\forall a \in F, \forall \alpha \in V \Rightarrow a.\alpha = \alpha.a \in V$

(iii) (V, +.) satisfy (b) $a\alpha \in V, \forall a \in F$ & $\forall \alpha \in V$

(i) $a.(\alpha + B) = a\alpha + aB \wedge \forall a \in F$ & $\alpha, B \in V$

(ii) $(a+b)\alpha = a\alpha + b\alpha \quad \forall a, b \in F \; \alpha \in V$

(iii) $a(b\alpha) = (ab)\alpha$
$1 \cdot \alpha = \alpha$
$\Big\} \forall 1, a, b \in F$
$\alpha \in V$

(E) . is a vector space because its satisfy all condition of a vector space.

(b) It is not a vector space because its not satisfy the closure property of a vector space.

(Q4) Determinants

Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2x2 matrix

$$A = \begin{bmatrix} ID1 & ID1 & ID1 \\ ID2 & ID3 & ID2 \\ ID4 & ID1 & ID5 \end{bmatrix}$$

Sol

Part a The following matrix have inverse exist.

$M_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $M_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Part b $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $M_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$M_3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Part c $M_1 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, $M_2 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $M_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

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Part D

$$A = \begin{bmatrix} 6 & 6 & 6 \\ 8 & 4 & 9 \\ 9 & 6 & 0 \end{bmatrix}$$

$$|A| = 6 \begin{vmatrix} 4 & 8 \\ 6 & 0 \end{vmatrix} - 6 \begin{vmatrix} 8 & 8 \\ 9 & 0 \end{vmatrix} + 6 \begin{vmatrix} 8 & 4 \\ 9 & 6 \end{vmatrix}$$

$$= 6(0-48) - 6(0-72) + 6(48-36)$$

$$= -282 + 432 + 72 = 216 \text{ Ans}$$