* ID : 13847
Final Exam
Biostatics
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* QUESTION ANSWERS :-

QUESTION NO : 1
(a) Calculate the correlation coefficient between x and Y

Price (x)	3	4	5	6	7	8	9	10	11	13
Demand (Y)	25	24	20	20	19	17	16	13	10	8

* Ans:

X	Y	XY	X^2	Y^2
3	25	75	9	625
4	24	96	16	576
5	20	100	25	400
6	20	120	36	400
7	19	133	49	361
8	17	136	64	289
9	16	144	81	256
10	13	130	100	169
11	10	110	121	100
13	8	104	169	64
$\Sigma x = 76$	$\Sigma y = 172$	$\Sigma xy = 1148$	$\Sigma x^2 = 670$	$\Sigma y^2 = 3240$

$$r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{(n\Sigma x^2 - (\Sigma x)^2)(n\Sigma y^2 - (\Sigma y)^2}}$$

Hence

$n = 10$

$\Sigma xy = 1148$

$\Sigma x = 76$

$\Sigma y = 172$

$\Sigma x^2 = 670$

$\Sigma y^2 = 3240$

Putting the values

$$r = \frac{(10)(1148) - (76)(172)}{\sqrt{[(10)(670) - (76)^2][(10)(3240) - (172)^2]}}$$

$$r = \frac{11480 - 13072}{\sqrt{(6700 - 5776)(32400 - 29584)}}$$

$$r = \frac{-1592}{\sqrt{(924)(2816)}}$$

$$r = \frac{-1592}{\sqrt{2601984}}$$

$$r = \frac{-1592}{1613.06}$$

$r = -0.98$ Ans.

Day: M T W T F S Date: __/__/20

(b) Given the following set of values

X	20	11	15	10	17	18	21	25	28
Y	5	15	14	17	8	9	12	16	18

(a) Determine the equation of the least squares regression line of Y on X and X on Y.

(b) Find the predicted values of Y for X = 20, 11, 15, 25, 28 and X = 5, 15, 9, 12, 16, 18.

* Ans-

X regration line on Y

$$Y = a + b$$

Now the two normal equations are

$$\Sigma y = na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

eq (i) $\Sigma Y = na + b\Sigma x$

$$\Sigma y - b\Sigma x = na$$

$$na = \Sigma y - b\Sigma x$$

diving ÷ "n" ob.side

Day: M T W T F S Date: __/__/20

$$\frac{na}{n} = \frac{\Sigma y}{n} - \frac{b\Sigma y}{n}$$

$$\frac{\cancel{n}a}{\cancel{n}} = \frac{\Sigma y}{n} - \frac{b\Sigma y}{n}$$

$$a = \bar{y} - b\bar{x}$$

dividing ÷ eq (i) on b.s by Σx
eq (ii) by "n" and then
subtraction

$$\Sigma x \Sigma y = na\cancel{\Sigma x} + b(\Sigma x)^2$$

$$\ominus n \Sigma xy \ominus \cancel{na}\Sigma x \pm bn\Sigma x^2$$

$$\Sigma x 2y - n\Sigma xy = b(\Sigma x)^2 - bn\,\Sigma x^2$$

$$\cancel{+}\frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \cancel{+}b\frac{\cancel{[n\Sigma x^2 - (\Sigma x)^2]}}{\cancel{n\Sigma x^2 - (\Sigma x)^2}}$$

$$b = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

X	Y	X^2	Y^2	XY	X	Y	X^2	Y^2	XY
20	5	400	25	100	28	18	784	324	
11	15	121	225	165	$\Sigma x = 165$	$\Sigma y = 114$	$\Sigma x^2 = 3304$	$\Sigma y^2 = 1604$	$\Sigma xy = 2099$
15	14	225	196	210					
10	17	100	289	170					
17	8	289	64	136					
18	9	324	81	162					
21	12	441	144	252					
25	16	625	256	400					

Then $n = 9$, $\Sigma XY = 2099$, $\Sigma X = 165$, $\Sigma Y = 114$

$\Sigma X^2 = 3309$

Now,

$$b = \frac{n\Sigma XY - \Sigma X \Sigma Y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b = \frac{(9)(2099) - (165)(114)}{(9)(3309) - (165)^2}$$

$$b = \frac{18891 - 18810}{29781 - 27225}$$

$$b = \frac{81}{2556}$$

$$b = 0.03$$

Now (B)

$$a = \bar{Y} - b\bar{X}$$

$$a = 12.66 - (0.03)(18.33)$$

$$a = 12.66 - 0.55$$

$$a = 12.11$$

Now the fitted Regration line Y on x

$$\hat{Y} = a + bx$$
$$\hat{Y} = 12.11 + 0.03x$$

Now fitted regration line x on Y is-

$$\hat{x} = a + bY$$

Two normal equations are

$$\Sigma x = na + b\Sigma Y \quad —(3)$$
$$\Sigma xY = a\Sigma Y + b\Sigma Y^2 \quad —(4)$$
$$na = \Sigma x - b\Sigma Y$$

"n" ÷ on b.side

$$\frac{na}{n} = \frac{\Sigma x}{n} - \frac{b\Sigma Y}{n}$$

$$a = \bar{x} - b\bar{y}$$

xing eq (3) on b.side by ΣY and equation by "n" and subtraction

$$\Sigma x\Sigma y = \cancel{na\Sigma Y} + b(\Sigma Y)^2$$
$$\pm n\Sigma xY = \cancel{\pm na\Sigma Y} \pm nb\Sigma Y^2$$
$$\Sigma x\Sigma y - n\Sigma xy = b(\Sigma y)^2 - nb\Sigma y^2$$

$$\frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma y^2 - (\Sigma y)^2} = b\left[\frac{(\cancel{n\Sigma y^2 - nb\Sigma y^2})}{\cancel{n\Sigma y^2 - (\Sigma y)^2})}\right]$$

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$$b = \frac{n\Sigma XY - \Sigma X \Sigma Y}{n\Sigma Y^2 - (\Sigma Y)^2}$$

Now $n = 9$, $\Sigma XY = 2099$, $\Sigma X = 165$, $\Sigma Y = 114$,

$\Sigma X = 1604$

$$b = \frac{(9)(2099) - (165)(114)}{(9)(1604) - (114)^2}$$

$$b = \frac{18891 - 18810}{14436 - 12996}$$

$$b = \frac{81}{1440}$$

$$b = 0.05$$

$$\bar{X} = \frac{\Sigma X}{n} = \frac{165}{9} = 18.33$$

eq (E) $a = \bar{X} - b\bar{Y}$

$$\bar{Y} = \frac{\Sigma Y}{n} =$$

$$a = 18.33 - (0.05)(12.66)$$

$$a = 18.33 - 0.633$$

$$a = 17.69$$

$$\hat{X} = a + bY$$

$$\hat{X} = 17.69 + 0.05Y$$

Day: M T W T F S   Date: __/__/20

Part "B"

Find the Predicted values of Y for X = 20, 11, 15, 25, 28 and X - for 5, 15, 9, 12, 16, 18-

X	
	$\hat{Y} = 12.11 + 0.03X$
20	$\hat{Y} = 12.11 + 0.03(20) = 11.51$
11	$\hat{Y} = 12.11 + 0.03(11) = 11.78$
15	$\hat{Y} = 12.11 + 0.03(15) = 11.66$
25	$\hat{Y} = 12.11 + 0.03(25) = 11.36$
28	$\hat{Y} = 12.11 + 0.03(28) = 11.27$

X on Y

Y	
	$\hat{X} = 17.63 + 0.05Y$
5	$\hat{X} = 17.63 + 0.05(5) = 17.94$
15	$\hat{X} = 17.63 + 0.05(15) = 18.44$
9	$\hat{X} = 17.63 + 0.05(9) = 18.18$
12	$\hat{X} = 17.63 + 0.05(12) = 18.29$
16	$\hat{X} = 17.63 + 0.05(16) = 18.49$
18	$\hat{X} = 17.63 + 0.05(18) = 18.59$

Q no 2

Find the following

(a) A fair coin is tossed 5 times. Find the probabilities of obtaining various number of head.

Ans: let us regard the tossing of a coin as a experiment. Then we observed that

$P = 1/2$

$P = q - 1$

$q = P - 1$

$q = 1 - 1/2$

$q = 1/2$

Therefore the r.v. X which denote the number of heads (successes) has a binomial probability distribution with $p = 1/2$ and $n = 5$, $X =$ number of head

$X = 0, 1, 2, 3, 5,$ and 5

Now

$$P = (X = x) = \binom{n}{x} p^x q^{n-x}$$

$$(X = x) = \binom{5}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x} \quad x = 0, 1, 2 \cdots 5$$

Put $x = 0$ in eq (i)

* $P(\text{no head}) = P(X=0) = \binom{5}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 = 1\times\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

* $P(1\text{ head})\ P = (X=1) = \binom{5}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{5-1} = 5\times\left(\frac{1}{2}\right)^5 = \frac{5}{32}$

* $P(2\text{ head}) = P(X=2) = \binom{5}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{5-2} = 10\times\left(\frac{1}{2}\right)^5 = \frac{10}{32}$

* $P(3\text{ head}) = P(X=3) = \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3} = 10\times\left(\frac{1}{2}\right)^5 = \frac{10}{32}$

$P(4\text{ head}) = P(X=4) = \binom{5}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{5-4} = 5\times\left(\frac{1}{2}\right)^5 = \frac{5}{32}$

$P(5\text{ head}) = P(X=5) = \binom{5}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 = 1\times\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

These probability can also be obtained by expending the binomial $\left(\frac{1}{2}+\frac{1}{2}\right)^5$. The binomial probability distribution for the number of head obtained in 5 tosses of a fair coin.

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x :	0	1	2	3	4	5
$P(X=x)$:	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$.

* Part (B)

A and B play a game in which ---------------- 6 or more games-

* Ans:- (i) P: probability of winning players A

$$P = \frac{2}{3}$$

$$P + q = 1$$
$$q = 1 - p$$
$$q = 1 - \frac{2}{3}$$
$$q = 1/3$$

Now $P(X=x) = \binom{n}{x} p^x q^{n-x}$

$$P(X=x) = \binom{10}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{2}\right)^{10-x}$$

(i) P (at least 4 games)

$$P(X \geq 4) = 1 - P(X<4)$$

$$= 1 - \sum_{x=0}^{3} = \binom{10}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{10-x}.$$

$$= 1 - \left[\binom{10}{0} \left(\frac{2}{3}\right)^{0} \left(\frac{1}{3}\right)^{n-0} + \binom{10}{3} \left(\frac{2}{3}\right)^{3} \left(\frac{1}{3}\right)^{10-3} + \binom{10}{2} \left(\frac{2}{3}\right)^{2} \left(\frac{1}{3}\right)^{10-2} \right.$$

$$\left. + \binom{10}{3} \left(\frac{2}{3}\right)^{3} \left(\frac{1}{3}\right)^{10-3} \right]$$

$$= 1 - \left[\left(\frac{1}{3}\right)^{10} + 10 \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{9} + (45)\left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{8} + (120)\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{7} \right]$$

$$= 1 - \left[\frac{1}{59049} + \frac{20}{59049} + \frac{180}{59049} + \frac{960}{59049} \right]$$

$$= 1 - \left[\frac{1+20+180+960}{59049} \right]$$

$$= 1 - \left(\frac{1161}{59049} \right)$$

$$= 1 - 0.019$$

$$= 0.98.$$

(ii) $P(x = 4/10) = 0$

Because variable "X" is binomial distribution takes only one of the integer value $0, 1, 2 \text{ ---- } n$.

Day: M T W T F S

Date: ___/___/___

(iii) $P(x=11)=?$

Here the total number of game 10

$$P(X=11)=0$$

(iv) P [6 or more games] = ?

$$P[X\geq 6]=\sum_{x=6}^{10}\binom{10}{x}\left(\frac{2}{3}\right)^{x}\left(\frac{1}{3}\right)^{n-x}$$

$$P(X\geq 6)=\binom{10}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{10-6}+\binom{10}{7}\left(\frac{2}{3}\right)^{7}\left(\frac{1}{3}\right)^{10-7}+$$

$$\binom{10}{8}\left(\frac{2}{3}\right)^{8}\left(\frac{1}{3}\right)^{10-8}+\binom{10}{9}\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right)^{10-9}+\binom{10}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{0}$$

$$=\binom{21}{0}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{4}+(120)\left(\frac{2}{3}\right)^{7}\left(\frac{1}{3}\right)^{7}+(45)\left(\frac{2}{3}\right)^{8}\left(\frac{1}{3}\right)^{2}+10$$

$$\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right)^{1}+(1)\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{0}$$

$$P(X\geq 6)=\frac{13440}{59049}+\frac{15360}{59049}+\frac{11520}{59949}+\frac{5120}{59049}+\frac{1024}{59049}$$

$$P(X\geq 6)=\frac{13440+15360+11520+5120+1024}{59049}$$

$$P(X\geq 6)=\frac{46464}{59049}$$

$$=0.78 \text{ Ans-}$$

QUESTION NO 83

The following figures give the number of children born to 50 women.

2	6	1	5	4	3	3	8	10	1
4	3	3	0	5	2	1	4	10	3
5	3	3	6	3	3	2	2	7	4
1	4	2	4	4	4	6	8	10	7
7	5	6	5	3	2	3	9	2	2

(a) Construct the ungrouped frequency distribution of these data.

No of Children	Tally frequency	f
0	|	1
1	||||	4
2	||||| |||	8
3	||||| ||||| |	11
4	||||| |||	8
5	||||	5
6	||||	4
7	|||	3
8	||	2
9	|	1
10	|||	3
		50

Day: M T W T F S

Date: ___/___/20

Construct the grouped frequency distribution of these data-

Group	f-
0-1	5
2-3	19
4-5	13
6-7	7
8-9	3
10-above	3
	50

-End-