

UIC

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Paper

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Q No 1: Determine if the following system is consistent or not:

$$JD = 16284$$

$$x_1 - 2x_2 + x_3 = 0$$

$$1D_3 = 2$$

$$2x_2 - 8x_3 = 8$$

$$5x_1 - 5x_3 = 10$$

$$x_1 - 2x_2 + x_3 = 0$$

$$0x_1 - 2x_2 - 8x_3 = 8$$

$$5x_1 + 0x_2 - 5x_3 = 10$$

Now write in matrix form.

$$\begin{bmatrix} 1 & -2 & 1 & : & 0 \\ 0 & 2 & -8 & : & 8 \\ 5 & 0 & -5 & : & 10 \end{bmatrix}$$

We dividing Row 2 by 2.

$$\begin{bmatrix} 1 & -2 & 1 & : & 0 \\ 0 & 1 & -4 & : & 4 \\ 5 & 0 & -5 & : & 10 \end{bmatrix} \begin{array}{l} \frac{1}{2}R_2 \\ \frac{1}{2}R_3 \end{array}$$

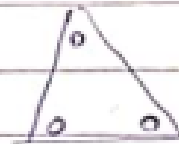
$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -1 & 5 \end{bmatrix}$$

consistent at least one solution

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$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 1 & 0 & -1 & 5 \end{bmatrix}$$

Lower triangle like  
are not zero



So the following system is  
not consistent.

b/c  $x_1 = 1, x_2 = 1, x_3 = -1$



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Q No 2:- Find the inverse of

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix} \text{ by adjoint method.}$$

$$\therefore |A| = 16234$$

Solution:

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|}$$

$$|A| = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= 3 \{ (-1 \times 7) - (-2 \times 8) \} - 4 \{ (2 \times 7) - (5 \times 8) \} + 5 \{ (2 \times -2) - (5 \times -1) \}$$

$$= 3(-7 + 16) - 4(14 - 40) + 5(-4 + 5)$$

$$= 3(9) - 4(-26) + 5(1)$$

$$= 27 + 104 + 5$$

$$|A| = 136$$

$$A_{11} = (-1)^{1+1} \times \begin{vmatrix} -1 & 8 \\ -2 & 7 \end{vmatrix}$$

$$= 1 \times (-7 + 16) = 9$$

$$\boxed{A_{11} = 9}$$

$$A_{12} = (-1)^{1+2} \times \begin{vmatrix} 2 & 8 \\ 5 & 7 \end{vmatrix}$$

$$= -1 \times (14 - 40) = 26$$

$$\boxed{A_{12} = 26}$$

$$A_{13} = (-1)^{1+3} \times \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$= 1 \times (-4 + 5) = 1$$

$$\boxed{A_{13} = 1}$$

$$A_{21} = (-1)^{2+1} \times \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix}$$

$$= -1 \times (28 + 10)$$

$$\boxed{A_{21} = -38}$$

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$$A_{22} = (-1)^{2+2} \times \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix}$$

$$= 1 \times (21 - 25) = -4$$

$$\boxed{A_{22} = -4}$$

$$A_{23} = (-1)^{2+3} \times \begin{vmatrix} 3 & 4 \\ 5 & 2 \end{vmatrix}$$

$$= -1 \times (-6 - 20) = 26$$

$$\boxed{A_{23} = 26}$$

$$A_{31} = (-1)^{3+1} \times \begin{vmatrix} 4 & 5 \\ -1 & 3 \end{vmatrix}$$

$$= 1 \times (32 + 5) = 37$$

$$\boxed{A_{31} = 37}$$

$$A_{32} = (-1)^{3+2} \times \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix}$$

$$= -1 \times (24 - 10) = -14$$

$$\boxed{A_{32} = -14}$$

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$$A_{33} = (-1)^{3+3} \times \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$$

$$= 1 \times (-3 - 8) = -11$$

$$A_{33} = -11$$

$$\text{adj}(A) = \begin{bmatrix} 9 & 26 & 1 \\ -38 & -4 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

$$\text{adj } A^T = \begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{1}{136} \begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{9}{136} & \frac{-38}{136} & \frac{37}{136} \\ \frac{26}{136} & \frac{-4}{136} & \frac{-14}{136} \\ \frac{1}{136} & \frac{26}{136} & \frac{-11}{136} \end{bmatrix}$$

Ans.

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Q No 6:- Reduce the matrix to normal form and find its rank

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Solution:-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - R_1 \end{array} \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$-2R_3 - R_2 \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$\begin{array}{l} C_2 - 3C_1 \\ C_3 - 4C_1 \\ C_4 - 3C_1 \end{array} \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



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Swap  $C_4$  with  $C_2$  we get

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide  $C_2$  by  $-6$ , we have

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 2

Answer

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Q No 3:- Solve the following system of linear equations by Gauss-jordan method.

Solution:-

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{bmatrix}$$

$$R_1 = \frac{1}{2} R_1 \quad \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{bmatrix}$$

$$\begin{aligned} R_2 &= R_2 - R_1 \\ R_3 &= R_3 - 3R_1 \end{aligned} \quad \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -6 & 13 \end{bmatrix}$$

$$R_2 = \frac{1}{2} R_2 \quad \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -6 & 13 \end{bmatrix}$$

$$R_3 = R_3 + 2R_2 \quad \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -6 & 9 \end{bmatrix}$$

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$$\begin{array}{l} R_1 = R_1 - R_2 \\ R_3 = \frac{-1}{6}R_3 \end{array} \left[ \begin{array}{cccc} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -\frac{3}{2} \end{array} \right]$$

$$R_1 = R_1 - 2R_3 \left[ \begin{array}{cccc} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -\frac{3}{2} \end{array} \right]$$

$$x + 0y + 0z = 10$$

$$0x + y + 0z = 2$$

$$0x + 0y + z = -\frac{3}{2}$$

$$x = 10$$

$$y = 2$$

$$z = -\frac{3}{2}$$

Ans.

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Q No 5:- Determine the following  
homogenous has a non-trivial solution.  
then describe the solution set.

$$3x_1 + 5x_2 - 4x_3 = 0$$

$$-3x_1 - 25x_2 - 4x_3 = 0$$

$$6x_1 + x_2 - 8x_3 = 0$$

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & 8 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 5 & -4 & : & 0 \\ 0 & 20 & 0 & : & 0 \\ 6 & 1 & 8 & : & 0 \end{bmatrix} \quad R_2 + R_1$$

$$A = \begin{bmatrix} 1 & 5/3 & -4/3 & : & 0 \\ 0 & 20 & 0 & : & 0 \\ 6 & 1 & 8 & : & 0 \end{bmatrix} \quad R_1 \times 3$$

$$A = \begin{bmatrix} 1 & 5/3 & -4/3 & : & 0 \\ 0 & 20 & 0 & : & 0 \\ 0 & 0 & 0 & : & 0 \end{bmatrix} \quad R_3 - 6R_1$$



From  $R_2$  :-

$$20x_2 = 0$$

$$x_2 = 0$$

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⇒ From  $R_1$  :-

$$x + \frac{5}{3}x_2 - \frac{4}{3}x_3 = 0$$

$$x = t$$

$$S.S = \{t, -t, t\}$$

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Q No 4:-

Show that this matrix is  
Diagonalisable.

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Let  $(A - \lambda I) = 0$

$$\begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix} = 0$$

$$\begin{array}{c|cc|c|cc|cc} 4-\lambda & 3-\lambda & 2 & -2 & -5 & 2 & -5 & 3-\lambda \\ & 4 & 1-\lambda & -2 & -2 & 1-\lambda & -2 & 4 \end{array}$$

$$4-\lambda [(3-\lambda)(1-\lambda) - (8)] - 2[(1-\lambda)(1-\lambda) + 4] - 2(-20 + 6 - 22)$$

$$4-\lambda (3-3\lambda-\lambda+2\lambda-8) - 2(-5+5\lambda+4) - 2(-20+6-22)$$

$$(\lambda^3 + 8\lambda^2 - 11\lambda - 20) + (10 - 10\lambda + 8) - 40 - 12 + 44$$

$$\lambda^3 + 8\lambda^2 - 11\lambda - 20 + 10 - 10\lambda - 8 - 40 - 12 + 44$$

$$\lambda^3 + 8\lambda^2 - 17\lambda + 10 \quad \lambda/25 \quad \alpha$$

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END