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Sk

SUBJECT : CALCULAS AND Analytical geometry

DEPT :- Software Engineering

Q#1:

Sol (a) · let $y = \dfrac{3x^3 - 5x^2 + 5}{x^2 + 1}$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x^3 - 5x^2 + 5}{x^2 + 1}\right)$$

$$= \frac{(x^2+1)\frac{d}{dx}(3x^3 - 5x^2 + 5) - (3x^3 - 5x^2 + 5)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2+1)(9x^2 - 10x) - (3x^3 - 5x^2 + 5)(2x)}{(x^2+1)^2}$$

$$= \frac{9x^4 - 10x^3 + 9x^2 - 10x - 6x^3 + 10x^2 - 10x}{(x^2+1)^2}$$

$$= \frac{9x^4 - 6x^3 + 9x^2 - 20x}{(x^2+1)^2}$$

Q# 11 (b)

Sol: let $y = \dfrac{(x^2+1)^2}{x^2-1}$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\frac{(x^2+1)^2}{x^2-1}$$

using quotient rule

$$= \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2 - (x^2+1)^2\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2-1)\,2(x^2+1)2x - (x^2+1)^2(2x)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^4-1)\,4x - (x^4+1+2x^2)\,2x}{(x^2-1)^2}$$

$$= \frac{4x^5 - 4x - 2x^4 - 2x - 4x^3}{(x^2-1)^2}$$

$$= \frac{4x^5 - 2x^4 - 4x^3 - 8x}{(x^2-1)^2}$$

Q # 2 (a)

$$y = (1+2\sqrt{x})^3 x^{2/3}$$

using chain rule

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}}$$

let $y = (1+2\sqrt{x})^3$

$u = x^{2/3}$

$$\frac{d}{du} y = \frac{d}{du}(1+2\sqrt{x})^3$$

$$\frac{dy}{dx} = \frac{2}{3}(1+2\sqrt{x})^{2/3-1} \frac{d}{dx}(1+2\sqrt{x})$$

$$= \frac{2}{3}(1+2\sqrt{x})^{\frac{2-3}{3}} \left(1+\frac{2\cdot 1}{2}x^{-1/2}\right)$$

$$= \frac{2}{3}(1+2\sqrt{x})^{-1/3}(1+x^{-1/2})$$

$$\frac{du}{dx} = \frac{d}{dx}(x^{2/3})$$

$$\frac{du}{dx} = \frac{2}{3}x^{\frac{2}{3}-1}$$

$$\frac{du}{dx} = \frac{2}{3}x^{\frac{2-3}{3}}$$

$$= \frac{2}{3}x^{-1/3}$$

Direct rule

$$\frac{dy}{dx} = \frac{d}{dx}(1+2\sqrt{x})^3 x^{2/3}$$

using product rule

$$= (1+2\sqrt{x})^3 \frac{d}{dx} x^{2/3} + x^{2/3} \frac{d}{dx}(1+2\sqrt{x})^3$$

$$= (1+2\sqrt{x})^3 \frac{2}{3}x^{\frac{2}{3}-1} + x^{2/3}\, 3(1+2\sqrt{x})^2 \frac{d}{dx}(1+2\sqrt{x})$$

$$= (1+2\sqrt{x})^3 \frac{2}{3}x^{-1/3} + x^{2/3}\, 3(1+2\sqrt{x})^2 \left(2\cdot\frac{1}{2}x^{-1/2}\right)$$

Q# 2 (b) $y = \sqrt{\frac{1-x}{1+x}}$

let $u = \frac{1-x}{1+x}$

$y = \sqrt{u}$

$\frac{dy}{du} = \frac{d}{du}\sqrt{u}$

$\frac{dy}{du} = \frac{1}{2} u^{\frac{1}{2}-1}$

$= \frac{1}{2} u^{-1/2}$

$= \frac{1}{2\sqrt{u}}$

$= \frac{1}{2\sqrt{\frac{1-x}{1+x}}}$

$u = \frac{1-x}{1+x}$

$\frac{du}{dx} = \frac{d}{dx}\frac{1-x}{1+x}$

$\frac{du}{dx} = \frac{(1+x)\frac{d}{dx}(1-x) - (1-x)\frac{d}{dx}(1+x)}{(1+x)^2}$

$\frac{du}{dx} = \frac{(1+x)(1) - (1-x)(1)}{(1+x)^2}$

$\frac{du}{dx} = \frac{1+x-1+x}{(1+x)^2}$

$= \frac{2x}{(1+x)^2}$

Using chain rule formula

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \times \frac{2x}{(1+x)^2}$

Q#3: (a) $\int \frac{1}{\sqrt{x^3}}\,dx$

$= \int \frac{1}{x^{3/2}}\,dx$

$= \int x^{-3/2}\,dx$

$= \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$

$= \frac{x^{\frac{-3+2}{2}}}{\frac{-3+2}{2}} + C$

$= \frac{x^{-1/2}}{-1/2} + C$

$= -\frac{2}{1}\, x^{-1/2} + C$

$= -2\,\frac{1}{\sqrt{x}} + C$

Q#3 (b)

$$\int \frac{1}{(6x+7)^6}\, dx$$

using

$$= \int (6x+7)^{-6}\, dx$$

$$= \frac{-6\,(6x+7)^{-6+1}}{(-6+1)\,(6)}\, dx$$

$$= \frac{-6\,(6x+7)^{-5}}{(-5)(6)} + C$$

$$= -\frac{(6x+7)^5}{-5} + C$$