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Question No: 01

(a)

Solution:

$y(n) - 3y(n-1) - 4y(n-2) = x(n) - 2x(n-1)$ — ①

the homogenous equation of the system

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

the characteristics eq of the system

$$\lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda = 4, \quad \lambda = -1$$

So

$$y_h(n) = C_1[-1]^n u[n] + C_2[4]^n u[n]$$

Since 4 is a characteristic root and the excitation is

$$x[n] = 4^n u[n]$$

Assume a particular solution

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$y_P[n] = Kn4^n u[n]$

than

$Kn4^n u[n] - 3K[n-1]4^{n-1}u[n-1] - 4K[n-2]4^{n-2}$
$u[n-2] = 4^n u[n] + 2(4)^{n-1} u[n-1]$

for $n = 2$

$K(32-12) = 4^2 + 8 = 24$

$K = 6/5$

the total solution is

$y(n) = y_P(n + y_h(n)$

$= [\frac{6}{5} n 4^n + C_1 4^n + C_2 (-1)^n] u(n)$

The solve for C_1, C_4, C_2 we assume

that $y(-1) = y(-2) = 0$

$y(0) = 1$ and $y(1) = 3y(0) + 4 + 2 = 9$

Hence $C_1 + C_2 = 1$

$\frac{24}{5} + 4C_1 - C_2 = 9$

$4C_1 - C_2 = 21/5$

$C_1 = 26/25$

$C_2 = -1/25$

the total solution is

$y[n] = \{\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} [-1]^n\} u(n)$

Question No: 01

(b)

Solution: $y[n] = 0.4y[n-1] - 0.08y[n-2]$
$+ x[n]$

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$y[n] - 0.6y[n-1] + 0.08y[n-2] = x[n]$

$\Rightarrow$ homogenious eq

$x[n] = 0$

$y[n] = 0.6y(n-1) + 0.08y(n-2) = 0$

$y_n[n] = \lambda^n$

So

$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$

$\lambda^{n-2}(\lambda^1 - 0.6\lambda + 0.08) = 0$

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$(\lambda - 0.2)(\lambda - 0.8) = 0$

$\lambda_1 = 0.2, \quad \lambda_2 = 0.4$

thus the general form of the solution to the homogenous equation.

$y_n[n] = C_1(\lambda_1)^n + C_2(\lambda_2)^n$

$y(n) = C_1(0.2)^n + C_2(0.4)^n$ — ①

$y(n) = C_1 \frac{1}{5}^n + C_2 (2/5)^n$

$y(0) = 1, \quad y(1) - 0.6y(0) = 0$

$y(1) = 0.6$

$\rightarrow C_1 = -1, \quad C_2 = 3$

therefore

$h[n] = [-(\frac{1}{5})^n + 2(2/5)^n] u(n)$

the step responce

$y(n) = \sum_{K=0}^{n} h(n-k), n>0$

$= \sum_{K=0}^{n} [2(2/5)^{n-K} - (\frac{1}{5})^{n-K}]$

$$= \left(\frac{1}{0.12}\left(\frac{2}{5}^{n+1} - 1\right) - \frac{1}{0.16}\left(\frac{1}{5}^{n+1} - 1\right) u(n) \right)$$

Question No: 02

(a):

Solution:

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4,\ B = -3,\ C = -1$

Hence $x(n) = \left[4(2)^n - 3 - n\right] u(n)$

Question No: 02

(b)

Solution: $x(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$

eliminate the -ve power by xing both numenator & denominator by z^2

$$x(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

$$\frac{x(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A}{z-1} + \frac{B}{z-0.5}$$

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$z(z-0.5)A + (z-1)B$ — ① ∴ LCM

Now set $z = P_1 = 1$ in eq ① we eliminate the term involving A.

$$1 = (1-0.5)A$$

$$\boxed{A = 2}$$

Return to eq ② $z = P_2 = 0.5$ than elementory the term involving A, So we have

$$0.5 = (0.5-1)B$$

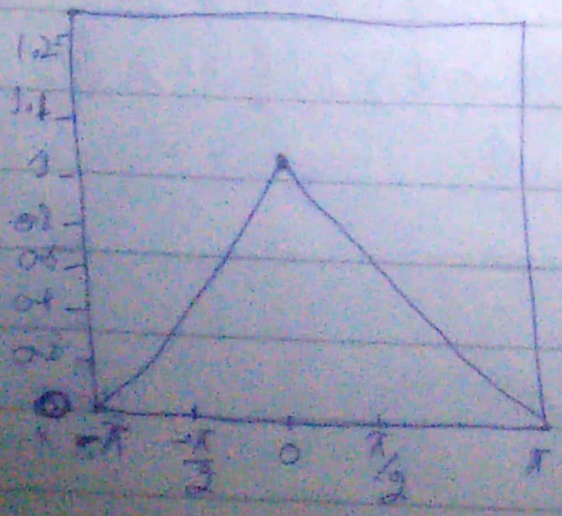
$$\boxed{B = -1}$$

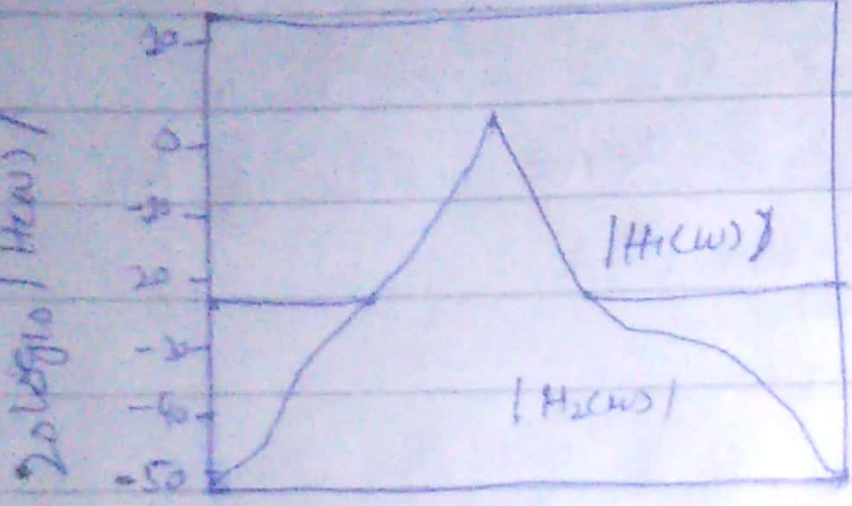
Question No: 03

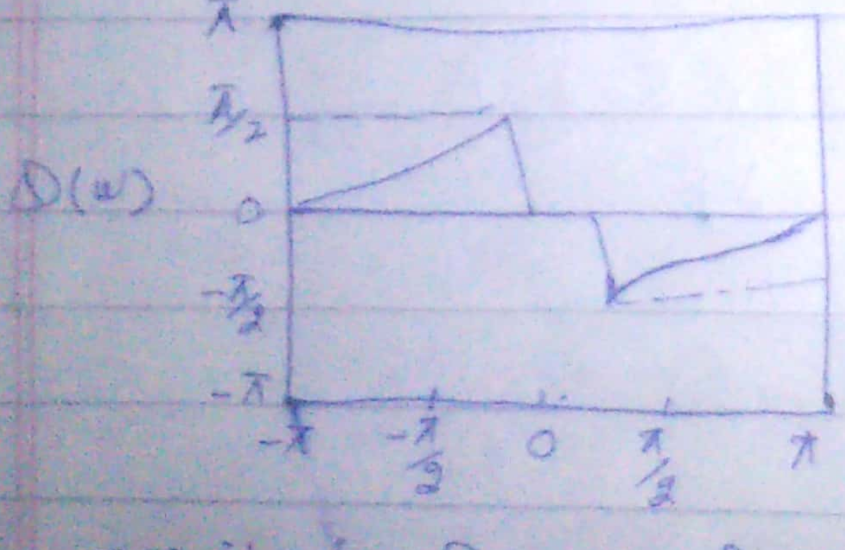
(a)

Solution:

linear time invarient system as frequency selective filters.







magnitude of and Phase response of (1) a single pole filter and (2) a one pole one zero filture.

$H_1(z) = (1-a)(1-az^{-1})$

$H_2(z) = [(1-a)/2][1+z^{-1}/(1-az^{-1})]$

$a = 0.9$

(b)

Solution: The filter must have poles at

$$p(z) = re$$

and zeros at $z = 1$ and $z = -1$,

consequently the system function is

$$H(z) = \frac{G(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

the gain factor is determined by evaluating the frequency response $H(w)$ of the filter at $w = \pi/2$

thus we have

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$$H\left(\frac{\pi}{2}\right) = G\,\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. thus we have

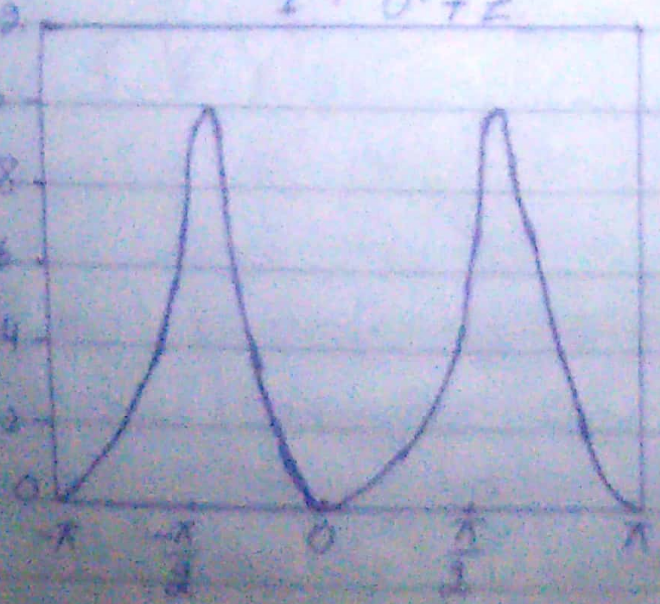
$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\,\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

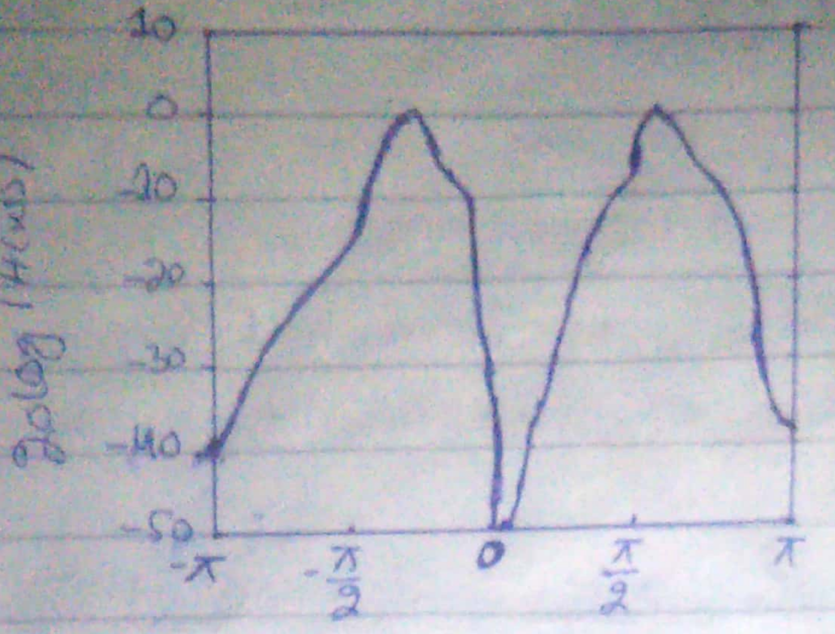
or equivalently

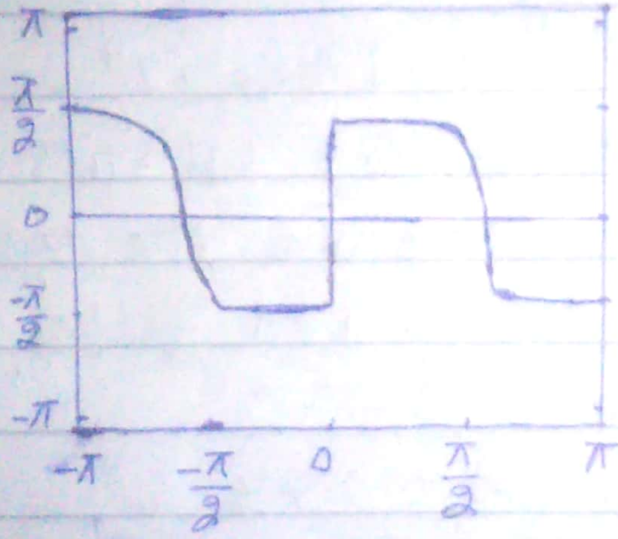
$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation. therefore the system function for the desired filter is

$$H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$$







Question No: 04

(a)

Solution: The Fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

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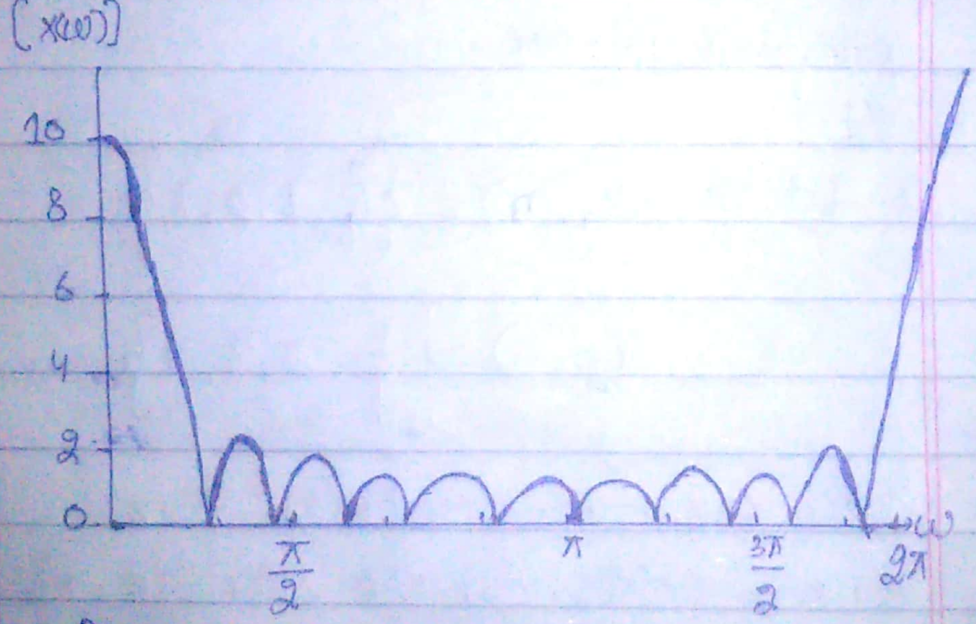
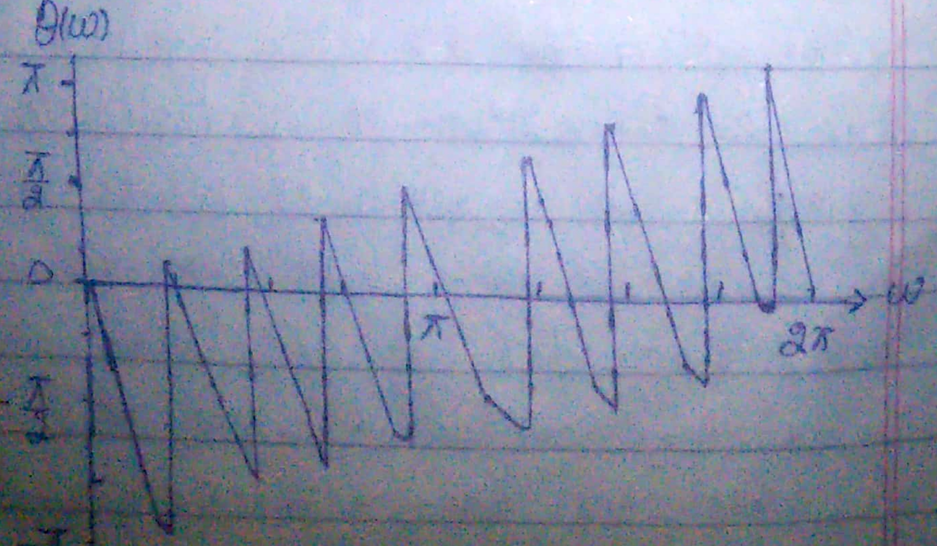
The magnitude and phase of $X(\omega)$ are illustrated below. For $L = 10$ the N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equality spaced frequencies $\omega_k = 2\pi K/N$, $K = 0, 1 \ldots N-1$. Hence

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}} \qquad K = 0, 1 \cdots N-1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





If N is selected such that $N = L$.
then the DFT becomes

$$X(K) = \begin{cases} L, & K = 0 \\ 0, & K = 1, 2 \cdots L-1 \end{cases}$$

Thus therefore there is only one nonzero value in the DFT. this is apparent from observation of $X(\omega)$. Since $X(\omega) = 0$ at the frequencies $\omega_K = 2\pi K/L$, $K \neq 0$.

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Question No: 04

(b)

Solution:

The first step is to determine the matrix w_4 be expoiting the periodicity propert of w_4

$$w_N^{K+N/2} = -w_N^F$$

$$w_4 = \begin{bmatrix} w_4^0 & w_4^0 & w_4^0 & w_4^0 \\ w_4^0 & w_4^2 & w_4^2 & w_4^2 \\ w_4^0 & w_4^2 & w_4^4 & w_4^4 \\ w_4^0 & w_4^3 & w_4^6 & w_4^1 \end{bmatrix}$$

$$w_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w_4^1 & w_4^2 & w_4^3 \\ 1 & w_4^2 & w_4^2 & w_4^2 \\ 1 & w_4^3 & w_4^2 & w_4^2 \end{bmatrix}$$

=> Than they

$$y_4 = w_4 \lambda_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

P — T — O

The DFT of λ_4 may be determined by conjugating the element in w_4 to obtain w_4^+ and than appling the formula.

"The End"

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